Survival of small PBHs in the Very Early Universe

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Abstract

The formation of Primordial Black Holes is a robust prediction of several gravitational theories. Whereas the creation of PBHs was very active in the remote past, such process seem to be very negligible at the present epoch.

In this work, we estimate the effects from the radiation surrounding PBHs due to the absorption term in the equations that describe how their masses depend on time. The Hawking radiation contributes with mass loss and the absorption term contributes with gain, but a interesting competition between these terms is analysed.

These effects are included in the equations describing PBHs and its mass density as the universe evolves in time and the model is able to describes the evolution of the numerical density of PBHs and the mass evolution and comparisons with cosmological constraints set upper limits in their abundances.

We evaluate the effect of this accretion onto PBHs and we get some corrections for the initial masses that indicates some deviations from default values for the time scale for evaporation. The scale time of the PBHs in the early universe is modified due to the energy accretion and we can estimate how these contributions may alter the standard model of PBHs.

1 Introduction

Black holes are solutions to Einstein's equations, but the mechanism for their formation may be more generic than simply the final phase of stellar lives. The very early universe would be very turbulent and great fluctuations of the metric (at small scales) may be responsible for the origin of black holes, whose nature is different from that black holes formed by the stellar collapse. These black holes are named Primordial Black Holes, and these objects may compose a fraction of dark matter that exists today. However, the behaviour of PBHs is complicated by the fact of these objects evolve with time: the numerical density and mass density is dependent of the scale factor of expansion and the mass of each PBH is dependent on time due to accretion of energy and irradiation (due to quantum effects). Fluctuations of order unity are responsible for PBH formation, and some models predict a spectrum with several possibilities. The B.Carr's work is one of the first to note that some kind of fluctuations give rise to spectrum with power law behaviour, i.e. the numerical density is proportional to a power in mass [1]. Our general conclusions in this work do not depend on these details, and if we consider a collection of PBHs as a dilute gas, the evolution of each PBH is independent on other PBH (unless when two PBHs collide). In other work, B.Carr et al did consider the possibility that PBHs and wormholes could be growing as fast as the universe in the remote past [2].

Classically, black holes contain a singularity involved by a horizon (named event horizon) that serves as an one-way membrane, splitting the universe in two causally separated regions. The event horizon for Schwarschild black holes (that black holes described by mass)is not a solid surface: any form of matter and energy can be absorbed, simply crossing this surface. The light cone points to singularity when a particle crosses the event horizon, therefore, any form of energy and matter that enters on a black hole contributes to accretion of mass and the black hole mass rises. In other work, P.S.Custodio and J.E.Horvath did prove that in the most general conditions (quintessence and radiation), PBHs could not be growing as fast as the universe since these conditions are very particular and improbable, moreover, a simple analysis of the equations showed that the general solutions did not show this behaviour, [3] and [4]. This work show the general analysis and indicates the numerical approximations that support these conclusions, and we show some corrections to the time scale for evaporation if we consider the mass-energy that is absorbed by black holes.

2 Quantum evaporation and absorption of energy

Black holes are not eternal (they have a origin in the past) a subtly analysis made by S.Hawking [5] showed that these objects emit radiation and particles. The mare effects of the gravitational field are able to split the vacuum into particles and anti-particles and the black hole absorbs particles with negative energy at expenses of the emission of the same quantity of positive energy that reaches the observer at infinity.

The details of this discovery: black holes emit thermal radiation at temperature given by its mass. The quantum effects at the event horizon are responsible by created particles, and the temperature is proportional to surface gravity. Numerically, this effect is given by

$$T = \frac{\hbar c^3}{8\pi k_B G M} \sim \frac{6 \times 10^{-8} K}{(M/M_\odot)} \tag{1}$$

If we consider all details, the particle multiplicity must be taken account, since for higher temperatures this effect can create other particles. As consequence, the object loss mass, and this effect is proportional to area and the flux of particles. Since the spectrum is thermal, we consider the fourth power of the temperature (Stefan-Boltzmann's law): $\Phi(T) = \sigma T^4$. Inserting this relation (energy flux through the black hole surface) and converting by the factor c^2, we get

$$\frac{dM}{dt} = -\frac{\sigma T^4}{c^2} S \tag{2}$$

where the area is given by $S = 4\pi {r_g}^2$. Since the radius is given by $r_g = \frac{2GM}{c^2}$, then, inserting these relations into equations above we get

$$\frac{dM}{dt} = -\frac{A}{M^2} \tag{3}$$

Here, $A = \frac{4\sigma\hbar^4 c^8}{8^4\pi^4 {k_B}^4 G^2} = 2.65 \times 10^{21} g^3 s^{-1}$.

It is very easy solve the eq.(3) in terms of the initial mass M_i and time, and we get

$$M(t) = M_i(1 - (t/t_{evap}))^{1/3} \tag{4}$$

where $t_{evap}(M_i) = M_i^3/3A$ is the time scale for complete evaporation.

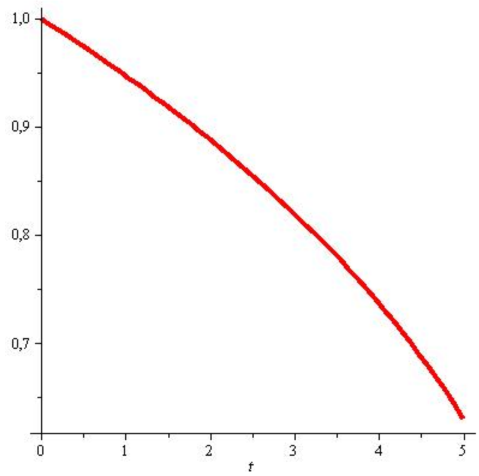

Figure 1: A PBH evaporating

However, the universe is filled with a thermal radiation whose origin is primordial. This energy will be absorbed as it crosses through the black hole horizon. This gain is proportional to area of the black hole and the flux, which is determined by the temperature T_{back}. Note that as the universe expands, it cools and its temperature falls, therefore the flux is dependent on the cosmological situation. In the Fig. 1 we can see the evolution of the PBH mass as function of time.

Therefore, the product of the black hole area and this flux is the positive term responsible for accretion of energy:

$$\frac{dM}{dt} = 4\pi {r_g}^2 \frac{\sigma {T_{back}}^4}{c^2} \tag{5}$$

where the temperature is given by the background and depends on the scale factor of expansion, $T_{back} = T_{back}(t)$.

The total mass variation is given by summing both terms above

$$\left(\frac{dM}{dt}\right)_{total} = \left(\frac{\sigma S}{c^2}\right)({T_{bh}}^4 - {T_{back}}^4) \tag{6}$$

$$\frac{dM}{dt} = -\frac{A}{M^2} + \lambda M^2 T^4 \tag{7}$$

with $\lambda = 5.4 \times 10^{-81} gs^{-1}$ if mass and temperature are measured in g and Kelvin, respectively. This equation predicts that the equilibrium between black holes and the radiation is determined by the temperature of the background (critical mass) and it is numerically given by

$$M_c(T) \sim \frac{2.62 \times 10^{25} g}{(T_{back}/K)} \quad (8)$$

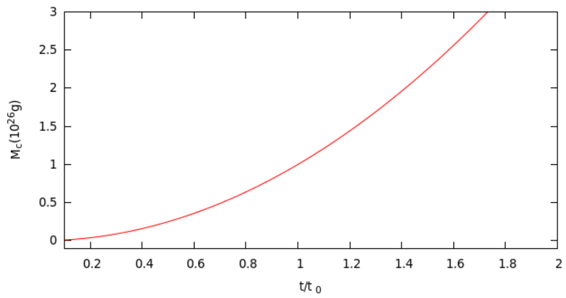

Figure 2: Critical Mass(T)

Therefore, stellar black holes (i.e. those black holes arising the death of stars) have masses very above this limit, and the Hawking evaporation is completely negligible for these objects. In fact, the second term, for $M = 5M_\odot$ and $T \sim 2.7K$ is responsible for the gain at rate $\dot{M}_{abs} \sim 2.86 \times 10^{-11} gs^{-1}$ and the Hawking evaporation term, for this object is numerically $\dot{M}_{evap} \sim -2.65 \times 10^{-47} gs^{-1}$, or several orders of magnitude smaller than the classical absorption. Any black hole above the critical mass must be described only by the second term, the absorption term, since the evaporation term decays very quickly for big black holes.

Now, in the next section, we must consider how one black hole evolves as the thermal radiation is considered when the universe was hotter than today.

In order to solve this dynamics we must know the cosmological initial data, and some simplifications are assumed.

3 Primordial Black Holes in the Radiation Era

The Hawking radiation describes how a PBH with initial mass loss energy as the time goes on. We do not know if this effect allows a remnant with mass close to the Planck scale since we do not have a quantum gravity theory yet, therefore we do not include this discussion here.

The absorption term written as a function of temperature can be expressed as

$$\frac{dM}{dt} = 4\pi {r_g}^2 F(T) = \frac{16\pi G^2 \sigma}{c^4} M^2 T^4 \quad (9)$$

But is more useful to represent it in terms of radiation density. Since $F(T) = \frac{c}{4} \varrho_{rad}(T)$ we shall write it as

$$\frac{dM}{dt} = \frac{16\pi G^2}{c^3} M^2 \varrho_{rad}(T) \quad (10)$$

Now, if we consider that a PBH was formed within the radiation era, the object will be immersed in a thermal bath, and it can absorbs the energy from the background. The complete evolution of this object may be described by the equation ($B = \frac{16\pi G^2}{c^3} \sim 8.36 \times 10^{-46} g^{-2} cm^3 s^{-1}$).

$$\frac{dM}{dt} = -\frac{A}{M^2} + BM^2 \varrho_{rad}(T) \tag{11}$$

the second term describes the energy that crosses the event horizon, and it is proportional to area, therefore it is proportional to mass at second power. Here $\lambda = aB$ as it was defined above.

Now, we can evaluate the equilibrium situation: the black hole mass that yields $\frac{dM}{dt} = 0$. This situation is a local and instantaneous equilibrium since the universe expands. Then, we can estimate the critical mass: the mas for which the black hole temperature = background temperature. At this moment, there is a equilibrium between the evaporation and the absorption term.

Solving $\frac{dM}{dt} = 0$ yields the critical mass as a function of $\varrho_{rad}(t)$.

$$M_c(t) = \left[\frac{A}{B\varrho_{rad}(t)}\right]^{1/4} \sim 4.2 \times 10^{16} g \varrho_{rad}(T)^{-1/4} \tag{12}$$

with ϱ_{rad} measured in gcm^{-3} as it should be.

The radiation era was very important in the very early universe. In this stage, the universe was filled with a thermal bath with density given by $\varrho_{rad}(t) = ag_* T^4$. g_* counts for the relativistic freedom degrees in particles at that moment, and $a = 7.56 \times 10^{-15} ergcm^{-3} K^{-4}$.

This era begins at the end of Inflation and it ends when the universe is matter-dominated. From this stage, the radiation is decoupled from the thermal equilibrium that existed previously.

The existence of PBHs in the early universe launches several constraints and astrophysical consequences. If the Hawking radiation is taken into account, then the smaller PBHs formed within the radiation era can be exploding today. Several considerations rise up the possibility that some GRB events tracks their behaviour, see D.Cline and W.Hong [6]. The radiation surrounding PBHs may be absorbed by these objects and the original prevision of PBHs with $10^{15} g$ exploding today must be modified by the energy absorbed in that period.

However, in this article we analyse the equations carefully, in order to split the approximations and we will show that some simplifications are very well justified in order to estimate the initial masses of the PBHs that may be exploding today. We can apply these ideas in order to evaluate the radiation that is injected in the medium if a PBH population was formed. Here we consider a model with PBHs with the same initial mass but this does not constitutes a loss of generality because we can integrate easily the results over the mass spectrum.

4 The Critical Mass and its Time Evolution

The radiation and temperature in the Radiation Era are given by $\rho_{rad}(t) = \frac{\pi^2}{30} g_* (\frac{T}{2.75K})^4$ and $T(t) \sim 1MeV(t/s)^{-1/2}$. We can combine these expressions to obtain $\rho_{rad}(t)$.

The Critical Mass is a very important parameter since it splits those PBHs that are cold $T_{pbh} < T_{rad}$, and they are absorbing energy (they are emitting too, but the second term dominates) from those hot PBHs

$T_{pbh} > T_{rad}(t)$. This PBH population injects radiation in the cosmic medium, and the contribution is proportional to mass, the time and a comparison between the temperatures: $T_{rad}(t)$ versus T_{pbh}.

Therefore, with the expressions above, the Radiation Era had $\rho_{rad}(t) \sim 2.5 \times 10^{-19}(t/s)^{-2} gcm^{-3}$, therefore the Critical Mass grows in time as

$$M_c(t) \sim 10^{15} g(t/1s)^{1/2} \tag{13}$$

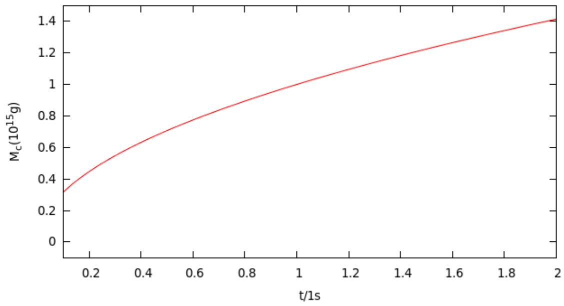

Figure 3: Critical Mass(T) at Radiation Era

For comparison effects, the Critical Mass at matter era depends on time as

$$M_c(t) \sim 10^{26} g(t/t_0)^{8/3} \tag{14}$$

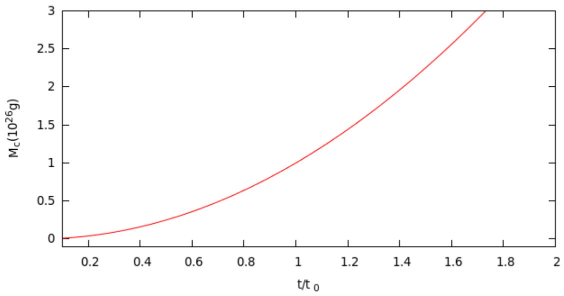

Figure 4: Critical Mass(T) at Matter Era

and $t_0 \sim 13.8 Gyr$. Therefore, black holes formed by the stellar deaths are very cold. With the definition above we cast the eq.(11) in a most suggestive form

$$\frac{dM}{dt} = \frac{A}{M^2}\left[(M/M_c)^4 - 1\right] \tag{15}$$

this relation is very useful since those black holes above the critical mass will be very cold, and these objects do not evaporate (the evaporation exists but the absorption term dominates the first term: $M > 10 * M_c$ implies $\frac{dM}{dt} >> 0$).

And, those PBHs satisfying $M < M_c/10$ will be very hot, and the evaporation is 10^4 most effective than the absorption. But it is equivalent to consider that the radiation is constant within the time scale for evaporation. Now, it is interesting to note that only those PBHs close to the Critical Mass will have both terms numerically closer.

But in this case, the absorption and evaporation will lead to $\frac{dM}{dt} \sim 0$, for PBHs where $M_c/10 < M < 10M_c$. Therefore, it is enough analyse the comparison between the terms in eq.(15) above.

In the same way, those PBHs above the $10M_c$ will be very cold, and the absorption is very relevant here. Then, only the PBHs whose initial mass falls within the interval $M_c/10 < M < 10M_c$ must be analysed solving both terms in the differential equation given by the eq.(15). But we note that this initial condition is a coincidence. In order to proceed, let us consider the Critical Mass in details, in order to estimate when we can use useful approximations.

5 Radiation Era and Critical Mass: which PBHs were able to arise at $t = t_i$?

The dynamics of the early universe must be solved before we analyse the Critical Mass, the PBH formation and the Horizon Mass. The fluctuations of the metric can induce the formation of PBHs if the scalar amplitude is greater then 1/3 as it is proven in B.Carr [1].

We know that the very early universe was very flat, therefore our approximations take into account that $\Omega = 1$ when the universe was very hot and dense. These fluctuations form PBHs with a mass spectrum, i.e. there is a probability of formation of PBHs as a mass dependence, and a invariant scale spectrum yields a PBH mass spectrum with $\Gamma(m) \propto m^{-2/3}$ as B.Carr showed in [1].

Now, our question is: how the radiation density will affect the time scale for complete evaporation if the Hawking mechanism dominates for those PBHs below the Critical Mass?

We consider three situations: a) PBHs form with $M > 10M_c$ at $t = t_i$, b) PBHs form with $M \sim M_c$ at $t = t_i$ and $M < M_c/10$ at $t = t_i$. The cases a) and c) are very interesting because there is bigger intervals for mass, but the case b) occurs with special conditions since the temperature of the cosmic radiation and the mass scale where this object form are independent.

First: In the cases above we will suppose that these objects are non-relativistic matter, therefore the mass density in PBHs is given by $\rho_{PBH} = mn_{PBH}$ where n_{PBH} is the numerical density in PBHs.

Second: $\rho_{rad} >> \rho_{PBH}$. This condition may be relaxed in models where the very early universe is filled with PBHs and no radiation. In these models, as discussed in Smolin [7], exist a primordial phase dominated by PBH matter and these objects decay very quickly, give rising to the radiation era. The radiation may be originated in the Hawking radiation in these exotic models!

5.1 PBHs sub-critical at t_i

For those PBHs formed in this phase where $m(t_i) < 0.1 * M_c(t_i)$ the time scale for evaporation is very smaller than the variation in the radiation due to the cosmic expansion (within the same period), remember that $\rho_{rad} \propto a^{-4}(t)$.

Therefore, we can suppose $M_c \sim cte$ since the PBH sees a background with the same radiation density if compared to its time scale for mass loss.

In this way, we can write for the eq.(15)

$$\frac{M^2}{A[(M/M_c)^4 - 1]} dM = dt \tag{16}$$

We can integrate this equation for estimate the effect of the radiation in the Hawking and how the radiation affects the life time for these objects.

Solving both sides above we get

$$t_{evap}(M, M_c) = t_i + \frac{1}{3A} \int_{M_i}^{0} \frac{3M^2 dM}{A[(M/M_c)^4 - 1]} \tag{17}$$

In the limit: $\rho_{rad} = 0$ we get $M_c \to \infty$ and we get the know time scale for evaporation: $t \propto M^3$ that it would be. Then, this formula is useful for estimate the initial mass M_i that is important for those PBHs that evaporate today, and these objects may be some kind of GRBs as its has been analysed in D.B.Cline et al in [6].

The right side may be written as

$$\frac{{M_i}^3}{3A} \int_0^1 \frac{3\mu^2 d\mu}{1 - C\mu^4} \tag{18}$$

where we have $\mu = \frac{M}{M_i}$ and $C = (\frac{M_i}{M_c})^4$.

The integral above can be expanded in a Taylor series. The integrand is expanded as a function of C, then we get

$$\frac{3x^2}{1 - Cx^4} = 3x^2 + 3Cx^6 + 3C^2x^{10} + O(C^3) \tag{19}$$

Inserting this term in the integral above we will get

$$t_{evap}(M_i, \rho) = t_i + \frac{{M_i}^3}{3A}\left[1 + \frac{3C}{7} + \frac{3C^2}{11} + ...\right] \tag{20}$$

Substituting C back into eq.(20) yields

$$t_{evap}(M_i, \rho) = t_i + \frac{{M_i}^3}{3A} + \frac{{M_i}^7}{7AM_c^4} + ... \tag{21}$$

Finally, inserting this expression in terms of ρ we have an approximation for the time scale for evaporation

$$t_{evap}(M_i, \rho_{rad}) \sim t_i + \frac{{M_i}^3}{3A} + \frac{M_i^7}{7A^2} B\rho_{rad} \tag{22}$$

Note that we recover the Hawking time scale for evaporation if $\rho_{rad} = 0$. In terms of the Critical Mass, we may define it as (use $t_i = 0$)

$$t_{evap}(M_i, \rho_{rad}) \sim \frac{{M_i}^3}{3A}\left[1 + \frac{3}{7}\left(\frac{M_i}{M_c}\right)^4\right] \tag{23}$$

which holds only if $M_i < M_c(t)$, where t means the formation time of the PBH and we remember that $M_c(t) \sim 10^{15} g(t/s)^{1/2}$.

Now, we may investigate how the dense and hot medium contributes to some gain of mass, and therefore, that PBHs with initial mass of order

$M_* \sim 5 \times 10^{14} g$ may not be exploding now. Let us define the scale of mass for PBHs that evaporates today (without consider the classical term of absorption). In this case we have the time scale as

$$t_0 \sim \frac{{M_*}^3}{3A} \sim 13.8Gyr \tag{24}$$

The constraint: $M_i < M_*$ holds, because those PBHs with initial mass bigger than this value will be living today ($M_i \geq M_*$) considering or not the existence of the thermal background!

Then, let us suppose that some PBH with initial mass M_i explodes today. We may simplify the expression eq.(23) above taking $t_{evap} \sim t_0$ and scaling this term as $\frac{{M_*}^3}{3A}$. Therefore, this equation becomes

$$\frac{M_*^3}{3A} = \frac{{M_i}^3}{3A}\left[1 + \frac{3}{7}\left(\frac{M_i}{M_c}\right)^4\right] \tag{25}$$

If we define

$$Y_i = \left(\frac{M_*}{M_i}\right) \tag{26}$$

and substituting the eq.(13) for the Critical Mass as function of time (at Radiation Era), we can write eq.(25) as

$${Y_i}^7 - {Y_i}^4 - 0.026(t/s)^{-2} = 0 \tag{27}$$

Now, we can solve this equation numerically, for some choices of the initial time of PBH formation, taking into account that $t > 1s$ because the Critical Mas must be greater than M_* in that moment, in the other hand the object is colder than the environment.

Solving the eq.(26) above we get a table of values. This table is interesting: we can choice some values for the formation time t that enters in the right side. For $t = 0.001s$ we get easily $Y_i = 4.298$ which implies $M_i = M_*/4.298$.

For $t = 0.01s$ we get $Y_i = 2.251$ and Y_i approaches to 1 rapidly as $t >> 1$. In the case $t = 1000s$ we have $Y_i = 1.000000009$. The results are not difficult to understand: only that PBHs formed very before than $t \sim 1s$ had gain mass in order to avoid the complete disintegration at $t = t_0$. Those PBHs formed after $t = 1s$ will find a medium rarefied, and the energy that these objects could be able to absorb is very small. For these cases, the default prevision $M_i = M_* \sim 10^{15}g$ holds and these PBHs will evaporate today, billions of years after the formation.

In the next section we show that the energetic conditions for a substantial growing require that the energy density at the formation of the PBH must be very higher. This conclusions are very robust and ruled out PBHs as possible candidate for the seeds for the AGNs.

5.2 PBHs super-critical at t_i will became super-massive?

This case is very interesting because these PBHs will be growing until these objects cross the Critical Mass at t. This time depends on the initial conditions (PBH mass and the temperature of radiation when this object arises.

Since the Critical Mass is the inverse of cosmic temperature,then there is a period when even very small PBHs can gain some mass and survive

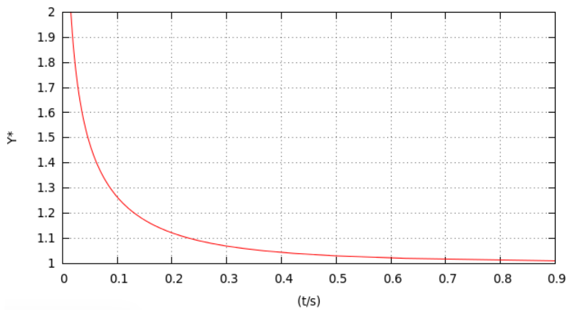

Figure 5: Mass critical as function time

against quantum effects. These objects will be evaporating after they have crossed the Critical Mass. We can evaluate these relations (time, masses, etc) in order to investigate what portion of PBHs can survive as dark matter today.

Now, the equation that describes the PBH evolution is very easy to solve in this limit. We will consider that $\frac{dM}{dt} \sim 0$ for all those PBHs whose initial mass is $M_i > 10M_c$ at $t = t_i$. We know that the PBH mass when the mass is close to Critical Mass will be almost constant. From this point, the object will be at an instantaneous equilibrium with the temperature. The universe, however, continues to be expanding and the Critical Mass will be different. Now, the PBH is hotter than the environment and the Hawking mechanism starts to gain the battle against the classical absorption term.

The differential equation describing the PBHs above the Critical Mass is

$$\frac{dM}{dt} = BM^2\rho_{rad}(t) \tag{28}$$

and this approximation is excellent while the PBH mass is above the Critical Mass.

The solution is very simply

$$\frac{1}{M_i} - \frac{1}{M(t)} = B\int_{t_i}^{t} \rho_{rad}(t)dt \tag{29}$$

whose solution can be expressed as

$$M(t) = \frac{M_i}{[1 - BM_i \int_{t_i}^{t} \rho_{rad}(t)dt]} \tag{30}$$

The Critical Mass $M_c(t)$ grows as $M_c(t) \propto t^{1/2}$ if and only if $\rho_{rad} >> \rho_{PBH} + \rho_{matter}$, i.e. in the Radiation Era.

Substituting $\varrho_{rad}(t) = \varrho_{rad}(t_i)t^{-2}$ into integral above and imposing that the mass is positive we get

$$\varrho_{rad}(t_i)\left(\frac{1}{t_i} - \frac{1}{t}\right) < \left(\frac{1}{M_iB}\right) \tag{31}$$

But, in later times the second term is negligible, therefore the causal constraint is written as

$$\varrho_{rad}(t_i)\left(\frac{1}{t_i}\right) < \left(\frac{1}{M_i B}\right) \quad (32)$$

This means that a PBH with mass bigger must not be described by the equations above, and it is not clear how to formulate the accretion into very big PBHs. A comparison with the cosmological horizon may launch some light into this question.

The ineq.(32) above can be written as

$$M_i < \left[\frac{t_i}{B\varrho_{rad}(t_i)}\right] \sim 4.7 \times 10^{63} g(t_i/s)^3 \quad (33)$$

But this value must be smaller than the mass contained within the cosmic horizon, and we expect that this causal constraint must be obeyed. The horizon mass $M_{hor}(t)$ depends on time, and we can compare these values now. The horizon mass is of order $M_{hor} \sim 10^{42} g(t/s)$, then a simply comparison of these expressions lead us to conclude that the formation time for PBHs above the critical mass is given by $M_i < M_{hor}(t)$ which gives

$$t_i < 1.45 \times 10^{-11} s \quad (34)$$

Now, an interesting question arises. Is there some possibility for a small PBH became super massive? It is very known that AGNs are driven by immense black holes lurked in the centre of these galaxies. Let us suppose that some PBH (satisfying the causality constraints at its formation) arises at the moment t_i, where the radiation density can be estimated. The formula above can be inverted in order to estimate what radiation density is required in order to obtain a immense black hole that grows from the thermal flux around it.

In a test, let us suppose the following data: initial mass $M_i \sim 10^{20} g$ and the final mass at t is $M(t) \sim 10^6 M_\odot$.

Solving the differential equation above, considering that $\varrho_{rad}(t) \propto t^{-2}$, i.e. Radiation Era, we get

$$\varrho_{rad}(t_i) > \left[\frac{(1 - 0.5 \times 10^{-19})t_i}{B \times 10^{20}}\right] \quad (35)$$

where 0.5×10^{-19} is the inverse of the ratio M/M_i. Inserting the numerical value of B we deduce that the requirement for $\varrho_{rad}(t_i)$ is improbable or physically rare, given the conditions of temperature, pressures and cosmological data involved. In this way, the cosmological radiation that fills the universe (or even the cosmological constant) was not enough in order to feed the growing of PBHs to super massive black holes that exist in the AGNs. The only possibility is that these objects formed by the fusion from smaller black holes.

In fact we can invert the ineq.(33) above to get the energy density enough to obtain very big PBHs:

$$\varrho_{rad}(t_i) > 1.2 \times 10^{25} (t_i/s) g cm^{-3} \quad (36)$$

where we did substitute B, M, M_i in the numerical example above.

These PBHs were able to grow only if they formed before

$$t_i < 5.71 \times 10^{-14} s \quad (37)$$

inverting the inequalities above.

The inequalities (34) and (37) are very important: let us summarise: (1) the ineq.(34) sets an upper limit for those PBHs that would be formed above the critical mass, and therefore in growing regime. In the other hand, (2) the ineq.(37) sets an upper limit for those PBHs that formed above the critical mass and moreover were in conditions to gain very much mass. These restrictions appear due the fact of the critical mass grows in time. But we can estimate the initial mass of these PBHs now. In fact, these PBHs must be smaller than the horizon, therefore the initial mass of PBHs in that conditions is given by solving the inequality: $M_i = M_{hor}(t \sim t_i)$, with $t_i \sim 5.71 \times 10^{-14}s$. This relation lead us to conclude that only PBHs smaller than

$$M_i < 5.71 \times 10^{28} g \tag{38}$$

may be formed in the very early universe in conditions to get some considerable mass.

This result ruled out the possibility of we get super-massive black holes, unless we choice very extremal conditions: the t_i very close to the Planck epoch, but this condition is delicate because the universe could be in an inflationary state and the scale factor and the temperature of the radiation is very different (compared to the Radiation Era of the standard model of cosmology).

We must observe the causal constraint that the horizon sets because the fluctuations may form PBHs whose PBH mass is contained within the horizon. Then, the formula that describes the PBH mass given by the eq.(29) has a limited range since we must consider cosmological constraints and all approximations used.

The estimates evaluated above are general and this study deserves interest, because their study may place constraints on the physics, even if these objects never existed [7].

The evaluations made above do not alter even we consider the accretion effect from the background energy, the mass-energy that one black hole can absorb due its gravity.

The models of accretion taking into account quintessence lead to the same conclusions, as can be seen in P.S.Custodio and J.E.Horvath [3]: in this case, small PBHs could not be very massive (absorbing the energy from quintessence) and their final masses are similar to the initial values.

Now, we can show that PBHs above the Critical Mass does not gain much more mass even at $t >> t_i$. If we consider that the formation time satisfies $t < 1.25 \times 10^{-44/3}s$, the horizon mass was $M_{hor} \sim 10^{42} g(t/s)$ and the PBHs had mass close to the horizon mass, we can insert these values into equations above and solving the relation that describes the mass of PBH that was super-critical at its formation time. In this way we get the following equation:

$$M(t) \sim \frac{2.5 \times 10^{27} g}{\left[1 - 5.2 \times 10^{-37}\left(\frac{1}{t_i} - \frac{1}{t}\right)\right]} \tag{39}$$

Since $t >> t_i \sim 1.25 \times 10^{-44/3}s$ we can write it as

$$M(t) \sim \frac{2.5 \times 10^{27} g}{1 - 5.2 \times 10^{-22.3}} \tag{40}$$

or, the gain was very small even in the most reasonable circumstances! Note that we do not need integrate the complete eq.(15) because the accretion term is bigger than the evaporation term. In this case the ratio of both terms is given by the expression: $(\frac{M}{M_c})^4 >> 1$. There is a fine-tuning here: these PBHs were super-critical and they were very above the Critical Mass, soon, the Hawking process is completely negligible. In the other hand, the medium is not hot and dense enough to feed these objects in a rate that leads these PBHs to achieve great masses at the end of Radiation Era! These objects remain with the same mass as they born, with a very small accretion over that period.

The mass of PBHs that were formed early and were super-critical was almost constant. Therefore, these objects will cross the Critical Mass in the future. This interval of time is given by

$$t_{cross}(M_i) \sim 1s\left(\frac{M_i}{10^{15}g}\right)^2 \tag{41}$$

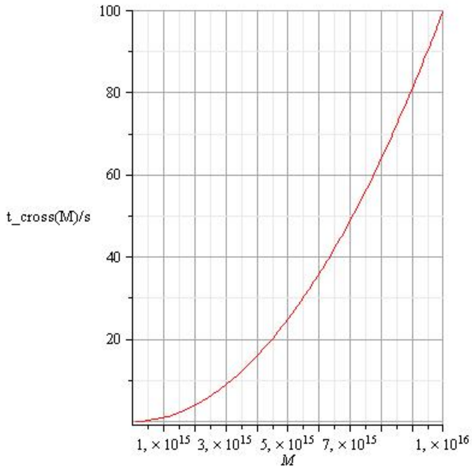

Figure 6: Lapse for a PBH achieve equilibrium, R.E.

in the Radiation Era and it is given by

$$t_{cross}(M_i) \sim t_0\left(\frac{M_i}{10^{26}g}\right)^{3/8} \tag{42}$$

in the Matter Era.

From these values, the PBH crosses the Critical Mass and become hotter than the environment, the Hawking process begins to dominate. It is interesting to estimate the lapse when PBHs satisfying the physical requirements given by eq.(38) holds above the critical mass. If we insert the upper limit $M_i \sim 5.71 \times 10^{28}g$ into eq.(41) above, we get $t_{cross} \sim 3.2 \times 10^{27}s$. During these long period these PBHs gain mass but in a very slow rate.

6 Primordial Black Holes and Cosmology

In order to get the complete behaviour of PBHs we must consider the scale factor $a(t)$. The radiation that falls onto the surface of a PBH is driven by

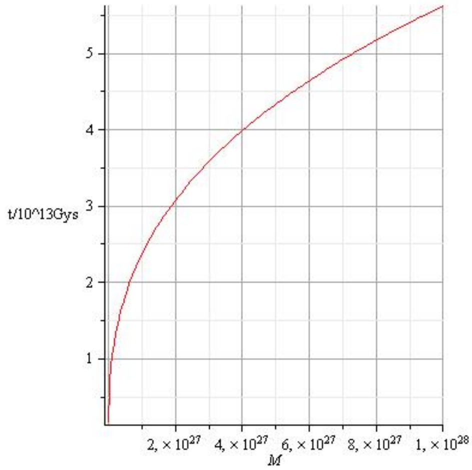

Figure 7: Lapse for a PBH achieve equilibrium, M.E.

the expansion of the universe, and therefore, the curvature is important. Now, the evolution of PBHs depends on some hypothesis: first, we will consider that $\rho_{rad} >> \rho_{pbh}$. This hypothesis is very reasonable, mainly in the radiation era (by definition). Second, the dynamics of the universe was very well described by the Friedmann's equations, therefore, the scale factor will not be affected by the disintegration of PBHs due to quantum effects or its growing as the time goes on.

Then, the scale factor $a(t)$ satisfies the Friedmann's equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \tag{43}$$

and

$$\left(\frac{\ddot{a}}{a}\right)^2 = -\frac{4\pi G}{3}(\rho + 3P) \tag{44}$$

Here $\rho = \rho_{rad} + \rho_{pbh} \sim \rho_{rad}$ and $P = \frac{\rho_{rad}}{3}$.

Now, we know that PBHs compose non-relativistic matter, and its mass density is given by $\rho_{PBH} = n_{PBH}m$, with n_{PBH} stands for the numerical density. The pressure for this matter is zero, and we shall consider that $K = 0$, because the universe was very flat in the beginning.

6.1 Monochromatic spectrum: all PBHs had the same mass!

This formula is very useful if we consider that the PBHs have a monochromatic spectrum, i.e. all they have same mass. The case $m_i < M_c(t = t_i)$ is solved analytically. The time derivative of ρ is (let us remove the index PBH from now)

$$\dot{\rho} = \dot{n}m + n\dot{m} \tag{45}$$

But we know that $\dot{m} = -Am^{-2}$, then if we develop the right side we get

$$\dot{\rho} = -\left(3H + \frac{A}{m^3}\right) nm \tag{46}$$

or

$$\frac{d\rho}{\rho} = -\left(3H + \frac{A}{m^3}\right) dt \tag{47}$$

Solving this simply equation yields

$$\rho(t) = \rho(t_i) a(t)^{-3} exp - \left[A \int_{t_i}^{t} m^{-3}(t) dt \right] \tag{48}$$

with $m^3(t) = m_i^3 \left[1 - \frac{t}{t_{evap}(m_i)} \right]$.

Solving the integral above yields the result

$$\rho(t) = \rho(t_i) a(t)^{-3} \left[\frac{t - t_{evap}(m_i)}{t_i - t_{evap}(m_i)} \right] \tag{49}$$

which holds for $t < t_{evap}(m_i)$ as it should be. Since the spectrum is monochromatic, for $t > t_{evap}$ we get $\rho = 0$ (some theories consider that the evaporation is not complete, leaving a relic at the Planck scale).

We summarize all these results in a formula

$$\rho(t) = \rho(t_i) a(t)^{-3} \left[\frac{t - t_{evap}(m_i)}{t_i - t_{evap}(m_i)} \right] \Gamma(t, t_{evap}) \tag{50}$$

where $\Gamma(t, t_{evap}) = 0$ for $t > t_{evap}(m_i)$.

Note that the initial conditions are satisfied: $a(t_i) = 1, \Gamma(t = t_i, t_{evap}) = 1$, since the PBH begins to evaporate from this epoch. This formula was able to decouple the dilution due to the cosmological expansion and the quantum disintegration of these objects in a closed form. We can proceed to evaluate the mass density for those PBHs that formed above the Critical Mass at t_i in a similar way.

The case $m_i > M_c$ yields a similar formula, in this case, we can ignore the evaporation and the PBH mass density is given by $\rho(t)$ will be given by

$$\rho(t) = \rho(t_i) a(t)^{-3} exp \left[B \int_{t_i}^{t} m(t) \rho_{rad}(t) dt \right] \tag{51}$$

This function holds only if $\rho_{pbh} << \rho_{rad}$, or the radiation of the PBHs will alter the global dynamics and the radiation density also.

6.2 Integrating the monochromatic spectrum

In fact, the formula given by the eq.(47) is specified at the each initial mass m_i.

Here, there is some source of unknown parameters and details because the very early universe is not known at all of its details. In fact we may speculate that the initial parameter $\rho(t_i)$ is independent on the mass, but the term $t_{evap} = t_{evap}(m_i)$ depends on the mass. If this case is valid, the eq.(47) must be integrated over the mass, but these integration depends on the choice of the mass spectrum. In some models all PBHs appear at the beginning with mass dependent on the time and constrained by the cosmological mass horizon:

$$M_{hor}(t) = \frac{c^3}{G} t \sim 10^{42}(t/s)g \tag{52}$$

If we contrast with the eq.(13) we have that PBHs formed close to $t \sim 1s$ were very above the Critical Mass (that mass was $\sim 10^{15}g$). These PBHs were in accretion regime and they will achieve the equilibrium only much later. If we consider the expressions above, only PBHs formed before $t < 6 \times 10^{-56}s$ were sub-critical necessarily. From these evaluations, we can estimate the lapse for a PBH that crosses the Critical Mass in the future, much later than it was formed by the pressure fluctuations. It is clear that before the Planck time, $t_{pl} \sim 10^{-43}s$, any evaluation is very imprecise and speculative.

Moreover, the constraints that were used in the Literature in order to put some stringent limits to the PBHs are useful only in a limited range of PBH masses, because there is a big interval for PBHs that did not emit radiation, since $M_{hor}(t) >> M_c(t)$ and PBHs arise with $M \sim M_{hor}$. Other mechanisms could be responsible to PBH formation in a very broad range of mass different from eq.(43) above, as collisions between cosmic strings, inflationary fluctuations and more.

All results written in the sections above are valid only in the approximation $\rho_{rad} >> \rho_{pbh}$ in whose case the analytical solutions could be described.

The system is very complicated if the approximation above does not hold. In principle a gas of PBHs can inject much energy and the radiation density may be altered by the evaporation. A simply scenario consists in a dilute gas of PBHs that evaporates and leads radiation as a relic: in this case, we can estimate the numerical density or the mass density in small PBHs that give rise to the radiation filling the universe. Moreover, small fluctuations in the PBH distribution may lead to the corresponding fluctuations in the background radiation when these objects evaporate.

A collection of PBHs plus radiation in the early universe may be described by the set of equations (considering a homogeneous universe in the average)

$$H^2 = \frac{8\pi G}{3}(\rho_{rad} + \rho_{pbh}) - \frac{K}{a^2} \tag{53}$$

$$\left(\frac{\ddot{a}}{a}\right)^2 = -\frac{4\pi G}{3}(\rho + 3P) \tag{54}$$

$$\frac{dM}{dt} = -\frac{A}{M^2} + B\rho_{rad} M^2 \tag{55}$$

and the energy conservation law may be written as

$$\dot{\rho}_{rad} + 4H\rho_{rad} = -\dot{\rho}_{pbh} \tag{56}$$

with

$$\dot{\rho}_{pbh} = \rho_{pbh}\left[mB\rho_{rad} - \frac{A}{M^3} + \frac{\dot{n}}{n}\right] \tag{57}$$

In the other hand, the numerical density n depends on the mass, energy and the cosmic time, defining the distribution function $f(E, M, t)$. Here, the details of evaporation and absorption are relevant also.

The numerical density in PBHs is given by

$$n(M,t) = \frac{g}{(2\pi)^3} \int dE f(E,M,t) \tag{58}$$

The complete set of equations depends on micro-physical details given by the kinetic theory of a black hole gas and at present we do not have all details yet. Remember that each particle depends on time, due to the evaporation effects and the absorption of energy. The usual kinetic theory of particles is simpler: the particles are created or destroyed in collisions but they do not depend on time explicitly. The eq.(15) that describes the behaviour of PBHs is valid in the approximation $M_{pl} < M < M_{hor}$. The lower limit sets the quantum effects that are unknown yet, and the upper limit sets the causal constraint determined by the universe itself. The metric that describes a black hole makes sense only if the space-time is flat far away from the PBH.

7 Conclusions

The most primitive eras in the very early universe may be prolific for PBH-formation.

If some PBHs survive against quantum effects of evaporation, the very hot phase of the early universe was able to set these objects with mass derivative positive, therefore with gain of energy. Some PBHs may survive against the evaporation due to this initial mass, but some gain of energy may be responsible for a longer duration. Black holes with stellar mass will be above the Critical Mass, therefore the Hawking effect is negligible for a long time. If PBHs formed with mass close to the horizon mass, they were in the accretion regime, because the conditions for evaporate happen very below that scale $M_c << M_{hor}$. But we could show that in these conditions, the Hawking evaporation term is negligible, therefore, those PBHs will be absorbing energy for a long time.

However, the energy density was not enough for a rapid growing regime, and we could demonstrate that PBHs will not be very massive at the final phase of growing.

A PBH with mass $M \sim M_\odot$ does not become a super-massive black hole energizing some AGN. In this work we consider some cosmological situations.

The background is responsible to determine the energy that is absorbed, and the behaviour of the PBHs is dependent on the cosmological constraints.

Primordial Black Holes that formed below the Critical Mass were hotter than the environment and we can obtain a new estimate for the duration of these objects different from the usual formulae because there was a energy accretion onto these PBHs. This effect deviates the usual numerical values for PBHs that explode today compared to the initial mass around $M \sim 10^{15} g$. We could show that very small PBHs may be formed before $t \sim 1s$ since the background was very hot and some gain of energy was able to alter the time scale for evaporation. This result is interesting because if very small PBHs formed at $t << 1s$ they were able to survive until now! These objects may comprise some fraction of the dark matter.

In flat or open models, the future of PBHs is uncertain, but if the universe is closed, the Hawking evaporation does not affects these objects until the universe contracts to a point again. We conclude that these objects will survive in closed universes. If quantum effects lead to stable Planck relics is a open question, and this is a plausible alternative.

We can put strong constraints to the initial abundance of PBHs from the present data: we know that the dark matter component is of order 0.30, and PBHs are a natural candidate for dark matter (cold) although they were not classified as barionic dark matter. The PBHs that could grow with a very strong rate could be formed only before $t \sim 1.25 \times 10^{-44/3} s$, therefore, if these PBHs had the mass of horizon, these PBHs must have mass $M_{pbh} \sim 2.5 \times 10^{27} g$, and these objects would be very cold until the present epoch.

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