Locally flat Manifolds after Boyom-Giraldo-Medina-Saldarriaga-Villabon

BOUETOU BOUETOU Thomas

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Abstract

I have noticed a situation of plagiarist and want to draw the attention of the authors and readers.
Keywords: infinitesimal transformations, associative algebras bi-invariant affine Lie groups, plagiarism.

Contents

Some years ago precisely may 2015, I made a comment about what was happening between colleagues from the University of Montpellier. It was about the paper : Flat Affine or Projective Geometries on Lie Groups by Alberto Medina, Omar Saldarriaga, Hernan Giraldo posted in arXiv:1409.0221 That was what I wrote: "I read the article of Medina and all posted on arXiv since 2014 and, I find that they are writing what, Nguiffo Boyom and Paul Byande wrote about the affine structures in "$Aff(R)$". I am really shocked because all are in the same university in Montpellier. And that some how others, they should know the results of research of their teams respectively. It's was a pity" I will like to put some exchanges in between them that I read.

0.1 The mail of Professor Boyom to Professors Medina, Saldarriaga, Giraldo, Barbaresco written on 23 April 2015

Dear colleagues It is Dr Barebaresco and his PhD students who ask what I thing about your paper which is mentioned above. I inform them that I have refereed all of you to (i) Pacific J. Math vol225 N1 (2006) p.122 and (ii) Osaka J. Math vol 47 N1 (2010) p. 267.

For your information.

- (a) Let (M, D) be an affinely flat manifold (: what Koszul calls locally flat manifold). The group $G = Aff(M, D)$ is a Lie group carrying a two-sided invariant affinely flat structure (G, D^*). When you endow the real vector space $X(M)$ of smooth vector fields with the multiplication defined by the connection D, viz $a.b = D_a b$, you obtain a real algebra $A = (X(M), D)$ and $J(A)$ is an associative subalgebra of A. Under the Poisson bracket of vector fields $[a, b] = D_a b - D_b a$, $J(A)$ is the Lie algebra of the Lie group G and $D^*_a b = D_a b$.
- (b) Recursively, starting with an affinely flat manifold (M, D) one gets the increasing series (G^k), (k in positive integers) of finite dimensional Lie groups endowed with bi-invariant affinely flat structure (G^k, D^k) such that $G^{k+1} = Aff(G^k, D^k)$ is the Lie group of affine transformations of (G^k, D^k).

If you need to know more either about the differential geometry or the algebraic topology of those (G^k, D^k). I might discuss with Professor Alberto about those items.
With my regards.

0.2 The mail of Professor Medina to Professors Boyom, Barbaresco, Saldarriaga, Giraldo writenn on 08 May 2015

Chers collègues,

C'est aujourd'hui seulement, le 4/05/2015, que je reçois le message de Michel Boyom ci-dessus , qui m'a été envoyé par Omar Saldarriaga, concernant l'article mis sur arXiv par Medina, Saldarriaga et Giraldo sous le titre "Flat Affine or Projective Geometries on Lie groups".

Cela s'explique par le fait que M. Boyom n'a pas écrit correctement mon adresse mail! Pour mémoire voici mon email correct à l'Université de Montpellier : alberto.medina@univ-montp2.fr. J'espère que le prochain message de M. Boyom me sera adressé à cette adresse ou bien à mon adresse gmail. L'argument de M. Boyom sur la existence sur le groupe de Lie des transformations affines $Aff(G, \nabla)$ (d'un groupe de Lie G pourvu d'une connexion linéaire ∇ invariante à gauche à courbure et torsion nulles), d'une connexion de même nature qui serait bi-invariante, n'est pas du tout correcte.

Voici le pourquoi de mon affirmation.

Si X et Y sont des transformations affines infinitésimales de (G, ∇), la dérivée covariante de Y dans la direction de X, selon ∇, n'est pas en général une transformation affine infinitésimale de (G, ∇)! Il est facile de construire des exemples de ce que j'affirme (on pourra par exemple lire notre article sur arXiv pour le fabriquer). Ainsi donc l'ensemble des transformations affines infinitésimales ne peut pas être une algèbre associative pour une loi de composition qui serait définie par ∇ comme il est affirmé dans les travaux de M. Boyom, qu'il cite dans le message auquel je réponds.

J'espère que M. Barbaresco et ses étudiants auront accès à cette information.

Bien cordialement,

Alberto MEDINA

Professeur Emrit Université de Montpellier.

0.3 The mail of Professor Boyom to Professors Medina and all. written on 10 July 2015

Dear colleagues

Recently all of us have received a short paper entitled ¡¡ **A note about infinitesimal affine transformations of a flat affine manifold ....;1. Counterexample.** ¿¿. The three authors of this note are Professor Alberto Medina, Professor Omar Saldarriga and Profesor Herman Giraldo from the Univ Med, Colombia.

The paper we have received contains an example of 1-dimensional affinely flat manifold whose underlying connection ∇ is explicitly defined. Following the authors the connection ∇ does not define an a structure of associative algebra in the vector space $aut(\nabla)$ of infinitesimal transformations of ∇. This claim is wrong. May urge all of you to propose the following exercice to your second year students. BASIC DEDINITION. A real algebra is a couple (V, μ) formed by a real vector spacce V endowed with a bilinear multiplication $\mu(v, w)$. BASIC NOTATION. Denote by $J(\mu)$ the subset of V formed by those elements X which satisfy the Identity $\mu(\mu(v, w), X) - \mu(v, \mu(w, X)) = 0$ for all $v, w \in V$. Here is the exercice I suggest you propose to students.

EXERCICE. The couple $(J(\mu), \mu)$ is an associative subalgebra of (V, μ).

THE SOLUTION TO THE EXERCICE.
To make simple please pose $v.w = \mu(v, w)$. Then every element X of $J(\mu)$ satisfies the identity $(v.w).X - v.(w.X) = 0$. Now let X and Y be in $J(\mu)$. The execice is to see that $X.Y = \mu(X, Y)$ is in $J(\mu)$ as well. In other word $(v.w).(XY) - v.(w.(X.Y)) = 0$ for all v, w. Take into account that Y belongs to $J(\mu)$. Then you get $(v.w)(X.Y) - v(.w.(X.Y)) = ((v.w).X).Y - v.((w.X).Y) = [(v; w).X - v; (w.X)].Y$ Thus Since X also be belongs to $J(\mu)$ you has $[(v.w).X - v.(w.X)] = 0$. As an application of this execice. Every linear connection ∇ defines a structure of algebra in the space of vector fields. Thereby the couple $(J(\nabla), \nabla)$ is an associative algebra. As application of the EXERCICE: If both curvature tensor and torsion tensor of ∇ vanish identically Then $J(\nabla)$ is the vector space of infintesimal transformation of ∇. In other words LOCAL FLOWS of elements of $J(\nabla)$ are ∇-preserving. The authors claim they give a COUNTEREXAMPLE TO THE ASSERTION OF [2]. I reproduce what is written in [2] about infinitesimal transformation of a locally flat manifold. Let A be the KV algebra of a locally flat manifold (M, D). Then $J(A)$ is the space of infinitesimal affine transformations of (M, D). The vector space $J_e(A)$ consisting of those $\xi \in J(A)$ which are complete vector fields is finite dimensional SUBALGEBRA of the LIE ALGBRA $J(A)$. IN PARTICULAR IF M IS COMPACT $J(A)$ is finite dimensional. The simply connected Lie group G whose Lie algebra is $J(A)$ carries a two-sided invariant locally flat structure (G, ∇). MOREOVER (M, D) admits G as an effective group of affine transformations [Palais 1957].... COMMENT.1. In [2] there is no assertion claiming that $J_e(A)$ is a subalgebra of the associative algebra $J(A)$. What is stated is that $J_e(A)$ is a subalgebra of the Lie algebra $J(A)$. COMMENT.2 The vector space $aut(< S >)$ of infinitesimal transformations of a geometric structure ¡S¿ is bigger than the Lie algebra of the Lie group $Aut(< S >)$ of automorphisms of $< S >$. Loosely speaking $aut(< S >)$ may regarded as the Lie algebra of the (Lie) pseudo-group of $< S >$-preserving local diffeomorphisms. For instance if (M, D) is a affinely flat structure whose KV algebra is denoted by A the Lie algebra of the Lie group $Aut(M, D)$ is the Lie algebra $J_e(A)$. if M is compact the Lie algebra of $Aut(M, D)$ is ${}_e(A) = J(A)$. I should be grateful that Professor Alberto Medina and his co-author precise to all of us the assertion of [2] which is subject to the counterexample.
With best regards

0.4 The mail of Professor Boyom to Professor Medina copy to Professor Vershini written on 25 September 2015

Bonjour Alberto
J'espère que tu passes un bon séjour en Colombie. Je me tourne vers toi pour relancer l'invitation au GSI2015. Les frais d'inscription sont élevés mais ils sont

humains pour les retraités. Je te remercie de m'avoir mis en copie du courrier à Barbaresco. Je n'ai pas vérifié en détail votre contre-exemple. Je vais résumer un calcul somme simple. Je tente de nouveau de dissiper rationnellement tes doutes.

- (1) (M, D) est une variété affine. Pour alléger les formules tu poseras:
- (2) $XY = D_X(Y)$. Ainsi l'espace des champs de vecteurs est une algèbre réelle. Le tenseur de torsion T étant nulle tu as:
- (3) $[X, Y] = XY - YX$. Puisque le tenseur de courbure R est nulle tu as l'identité:
- (4) $X(YZ) - Y(XZ) - [X, Y].Z = 0$.

  L'action infinitésimale d'un champ de vecteurs X sur D est définie ainsi:
- (5) $(X.D)(Y, Z) = [X, YZ - [X, Y]Z - X[X, Z]$.

  Utilise (3) pour réécrire (5) comme il suit $(X.D)(Y, Z) = X(YZ) - Y(XZ) - [X, Y]Z - Y(ZX) - (YZ)X$.

  L'action infinitésimale d'un champ de vecteurs X prend la forme:
- (6) $(X.D)(Y, Z) = R(X; Y)Z - Y(ZX) - (YZ)X$.

  Tu recourras la nullité de la courbure et tu as:
- (7) $(X.D)(Y, Z) = Y(ZX) - (YZ)X$.

  Ainsi X est une transformation infinitésimale de (M, D) si et seulement si X vérifie l'identité suivante:
- (8) $Y(ZX) - (YZ)X = 0$.

  Nous sommes arrivés au point qui te rend dubitatif.

  Suppose que les champs X et X' vérifient l'identité (8). Alors leur produit $XX' = D_X(X')$ vérifie (8). Tu écriras le membre de gauche de (8) pour $X := XX'$. Tu obtiendras $Y(Z(XX')) - (YZ)(XX'$. Puisque X' vérifie l'identité (8) on obtient $Y(Z(XX')) - (YZ)(XX') = Y(ZX)X') - ((YZ)X)X' = (Y(ZX))X' - ((YZ)X)X'$. Ainsi $((XX').D)(Y, Z) = Y(ZX) - (YZ)XX'$. Puis que X vérifie (8) tu as l'identité suivant:
- (9) $Y(ZX) - (YZ)X = 0$ quels que soient Y et Z.

  En d'autres termes tu obtiens: (9) $(XX').D = 0$.

  L'espace vectoriel des transformations infinitésimales de (M, D) est fermé pour la multiplication $XX' = D_X(X')$. Pour terminer tu considères trois transformations infinitésimales X, Y, Z (de (M, D)). L'identité (9) te donnera $X(YZ) - (XY)Z = 0$. Autrement dit:
- (10) $X(YZ) = (XY)Z$.

Qu' en penses-tu? je mets Vladimir en copie.
a bientôt à montpellier.

0.5 The analysis of the messages

After reading the messages above, the analysis show that **Counter example** produced by Professors Medina and all., the page indicated does not talk about the same thing this is what it says in [2] page 122.

Let $\mathfrak{A}$ be the KV-algebra of a locally flat manifold (M, D) . Then $J(\mathfrak{A})$ is the space of infinitesimal affine transformations of (M, D) . The space $J_e(\mathfrak{A})$ consisting of those $\xi \in J(\mathfrak{A})$ which are complete vector fields is a finite-dimensional subalgebra of the Lie algebra $J(\mathfrak{A})$. In particular, if M is compact then $J(\mathfrak{A})$ is finite-dimensional. The simply connected Lie group $\mathfrak{G}$ whose Lie algebra is $J(\mathfrak{A})$ carries a two-sided invariant locally flat structure $(\mathfrak{G}, \nabla)$. Moreover (M, D) admits $\mathfrak{G}$ as an effective group of affine transformations [Palais 1957]. Concerning the relationships between the $\mathfrak{G}$-geometry and the completeness of (M, D), see [Tsemo 1999]. Of course the notion of KV-morphism, sub-KV-module and KV-quotient module can be defined. In particular the image under a KV-morphism of a KV-module is a KV-submodule.

One can noticed that no where in this text the do not speak about infinitesimal affine transformation of Lie groups endowed with left invariant affinely flat structure. I was thinking that the situation between the two colleagues is ok now, when I noticed that not long ago, precisely July 2017, I noticed a situation of plagiary, in their posting in arxiv:1707.07030v2 [5]. One can noticed that the paper in [5] is not following the paper [1] neither in the keys words nor in the concern. But is the plagiary of the results currently in arXiv [6]. This [6], is the development of ideas, concerns, aims and results which have been sketched in messages of Professor Boyom to Professor Medina and all.

0.6 Conclusion

The plagiarize is to take and use the thoughts, the writings and invention... of another person as one's own without citing [7].

Mathematics objects are invariant under notation changes. Paper [5] highlight the efforts of authors to reformulate and to present ideas, concerns and results which are nothing but the plagiary of results which are in [6]. For example the **Theorem 2.1** of the page 2 in [5] it is a plagiarism of the **Theorem 2.8** of the page 12 in [6] and the **EXEMPLE (2)** of the page 122 in [2]. And it surprisingly that the counterexample of Medina given in the appendix of this note was to disproof this **EXEMPLE (2)** of the page 122 in [2].

0.7 References

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APPENDIX

A note about infinitesimal affine transformations of a flat affine manifold

Several years ago A. Medina asked, in the Lie Theory Colloquium held in Vigo Spain of 2000, whether the group of affine transformations denoted by $Aff(M,\nabla)$ of a locally flat affine manifold (M,∇), admits a flat left invariant affine structure. Later the author of [2] claimed that the group $Aff(M,\nabla)$ carries a locally flat bi-invariant affine structure defined by the corresponding connection ∇ . As a reference he sent us the paper [2] (see page 122). In this short note we give a counter-example to the assertion of [2].

1. **Counterexample**. Consider the flat affine connection ∇^+ on $G = Aff(\mathbb{R})$ given by:

$$\nabla^+_{e_1^+} e_1^+ = 2e_1^+,\ \nabla^+_{e_1^+} e_2^+ = e_2^+,\ \nabla^+_{e_2^+} e_1^+ = 0,\ \nabla^+_{e_2^+} e_2^+ = e_1^+,$$

where e_1^+ and e_1^+ denote the left invariant vector fields $e_1^+ = x\frac{\partial}{\partial x}$ and $e_2^+ = x\frac{\partial}{\partial y}$, respectively. A direct calculation shows that $\mathbb{R}$ linearly independent affine infinitesimal transformations relative to the connection ∇^+ are given by

$$e_1^- = x\partial x + y\partial y,\ e_2^- = \partial y,\ c_3 = \frac{1}{x}\partial x,\ c_4 = \frac{y}{x}\partial x,\ c_5 = (x + \frac{y^2}{x})\partial x$$

$$and \quad c_6 = (-xy - \frac{y^3}{x})\partial x + (x^2 + y^2)\partial y,$$

and that the only complete vector fields are the right invariant vector fields $e_1^- y e_2^-$. It is well known that real vector space spanned by the complete affine infinitesimal transformations, with the bracket of vector fields, is the Lie algebra of the group $Aff(G,\nabla^+)$ (see [3] page 235). As the product defined by ∇^+ on these complete vector fields is given by

$\cdot$	e_1^-	e_2^-
e_1^-	$e_1^- + c_5$	c_4
$e_2^?$	$e_2^? + c_4$	c_3

we get that ∇^+ does not define an associative product on $Lie(Aff(G,\nabla^+))$.