An Alternate Proof of the Prime Number Theorem

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1) $\sum_{k=1}^{N} \frac{1}{k} \approx$ Ln(N) i.e. $\int_1^N \frac{1}{x}$ dx = Ln(N)
2) $Ln\left(Ln(N)\right) \approx$ Ln($\sum_{k=1}^{N} \frac{1}{k}$)
3) $\sum_{k=1}^{N} \frac{1}{k} = \prod_{p \le N} \frac{1}{1-\frac{1}{p}}$; p – prime number
4) $Ln\left(Ln(N)\right) =$ Ln ($\prod_{p \le N} \frac{1}{1-\frac{1}{p}}$) =
5) $\sum_{p \le N}^{N} Ln(\frac{1}{1-\frac{1}{p}})$
6) Ln($\frac{1}{1-\frac{1}{p}}$) = - Ln(1- $\frac{1}{p}$)
7) Solve $\int_0^x \frac{dt}{1-t}$
8) Let u = 1-t; du = -dt
9) $\int_0^x \frac{dt}{1-t} = \int_1^{1-x} \frac{-du}{u}$ = Ln(1) – Ln(1-x) = -Ln(1-x)
10) $x = \frac{1}{p}$
11) $\frac{1}{1-t}$ = 1 + t +t^2 + t^3 + …. + t^r as r →∞ geometric progression
12) $\int_0^{\frac{1}{p}} \frac{dt}{1-t} = \int_0^{\frac{1}{p}}(1 + t + t^2 + t^3 + \dots + t^r)$ dt =
13) $\frac{1}{p} + \frac{1}{2p^2} + \frac{1}{3p^3} + \dots + \frac{1}{rt^r} \sim \frac{1}{p}$
14) Ln($\frac{1}{1-\frac{1}{p}}$) $\approx \frac{1}{p}$
15) Therefore $\sum_{p \le N}^{N} Ln(\frac{1}{1-\frac{1}{p}}) \approx \sum_{p \le N} \frac{1}{p}$
16) And Ln(Ln(N)) $\approx \sum_{p \le N} \frac{1}{p}$
17) Solve $\int_e^N \frac{dt}{tLn(t)}$
18) Let u = Ln(t); du = $\frac{dt}{t}$
19) $\int_{Ln(e)}^{Ln(N)} \frac{du}{u}$ = Ln(u) = Ln(Ln(u)) = Ln(Ln(N)) – Ln(Ln(e)) = Ln(Ln(N)) – Ln(Ln((e)) = Ln(Ln(N))
20) $\sum_{p \le N} \frac{1}{p} \approx$ Ln(Ln(N)) = $\int_e^N \frac{dt}{tLn(t)}$
21) $\frac{1}{tLn(t)}$ is the probability density function of the sum of the reciprocals of prime numbers i.e. $\sum_{p \le N} \frac{1}{p}$
22) $\int_e^{100000} \frac{dt}{tLn(t)}$ = 2.831

23) The actual sum of the reciprocals of prime numbers up to (N = 100,000) = 2.705 (percentage difference of 4.66%

24) If $\frac{1}{tLn(t)}$ is the probability density function of the sum of the reciprocals of prime numbers then what is the probability density function of the actual sum of prime numbers?

25) Let y = $\frac{1}{x}$ (the reciprocal of the reciprocal i.e. p)

26) $pdf_y dy = pdf_x dx$

27) $pdy_y = pdf_x |\frac{dx}{dy}|$

28) $|\frac{dy}{dx}| = x^{-2}$; $|\frac{dx}{dy}| = x^2$

29) pdf_y (sum of primes) = $\mathbf{x^2 \; X} \; (\frac{\mathbf{1}}{\boldsymbol{xLn(x)}}) = \frac{\boldsymbol{x}}{\boldsymbol{Ln(x)}}$

30) $\int_e^{100000} \frac{xdx}{Ln(x)}$ = 455,059,956 ; $\sum_2^{p<100000} p$ = 454,495,540

31) % difference is 0.124% (very close)

32) $\frac{\int_e^N \frac{xdx}{Ln(x)}}{\int_e^N \frac{dx}{Ln(x)}}$ is the approximate mean value of prime numbers between e and N for a probability density of function of $\frac{1}{Ln(x)}$

33) $\frac{\sum_e^{p<N} p_i}{\int_e^N \frac{dx}{\ln(x)}}$ ≈ mean value of prime numbers less than N

34) $\frac{\int_e^N \frac{xdx}{Ln(x)}}{\int_e^N \frac{dx}{Ln(x)}} \approx \frac{\sum_e^{p<N} p_i}{\int_e^N \frac{dx}{\ln(x)}}$ ≈ mean value of primes less than N

35) $\int_e^N \frac{dx}{Ln(x)} \approx \frac{\sum_e^{p<N} p_i}{mean\ p} \approx \frac{N}{\ln(N)}$, which is the prime number theorem

References:

John Derbyshire.2003. Prime Obsession:Bernard Riemann and the Greatest Unsolved Problem in Mathematics. Washington, DC: Joseph Henry Press. https://doi.org/10.17226/10532