Proving the Erdös-Straus Conjecture from Infinite to Finite Equalities

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Abstract

We first classify all integers ≥2 into eight kinds, and that formulate each of seven kinds therein into a sum of three unit fractions. For remainder one kind, we classify it into three genera, and that formulate each of two genera therein into a sum of three unit fractions. For remainder one genus, we classify it into five sorts, and that formulate each of three sorts therein into a sum of three unit fractions. For remainder two sorts i.e. 4/(49+120c) and 4/(121+120c) with c≥0, we prove them by logical inference. But miss out 3587 concrete fractions to await computer programming to solve the problem that express each of them into a sum of three unit fractions.

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1. Introduction

The Erdös-Straus conjecture is a famous conjecture concerning Egyptian fractions. In 1948, Paul Erdös conjectured that for integer n≥2, there exist invariably 4/n=1/x+1/y+1/z, where x, y and z are positive integers. Later, Ernst G. Straus further conjectured, that the equation's solutions x, y and z satisfy x≠y, y≠z and z≠x, because there are 1/2r +1/2r = 1/(r+1)+1/r(r+1)

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and $1/(2r+1)+1/(2r+1)=1/(r+1)+1/(r+1)(2r+1)$ where $r\geq 1$.

Thus the Erdös conjecture and the Straus conjecture are equivalent from each other, and they are called the Erdös-Straus conjecture collectively.

As a general rule, the Erdös-Straus conjecture states that for every integer $n\geq 2$, there exist positive integers x, y and z, such that $4/n=1/x+1/y+1/z$.

Yet, it is still both unproved and un-negated a conjecture hitherto.

2. Expressing Each of Majorities of 4/n into 1/x +1/y +1/z

First let us divide all integers ≥2 into eight kinds, i.e. ①8k+1, ②8k+2, ③8k+3, ④8k+4, ⑤8k+5, ⑥8k+6, ⑦8k+7 and ⑧8k+8, where $k\geq 0$.

Please, see also the permutations of each and every kind of positive integers from small to large as follows.

K,	8k+1,	8k+2,	8k+3,	8k+4,	8k+5,	8k+6,	8k+7,	8k+8
0,	1(not),	2,	3,	4,	5,	6,	7,	8,
1,	9,	10,	11,	12,	13,	14,	15,	16,
2,	17,	18,	19,	20,	21,	22,	23,	24,
3,	25,	26,	27,	28,	29,	30,	31,	32,
…,	…,	…,	…,	…,	…,	…,	…,	…,

Excepting 8k+1, we formulate each of other seven kinds as follows.

(1) For n=8k+2, $4/(8k+2)=1/(4k+1)+1/(4k+2)+1/(4k+1)(4k+2)$;

(2) For n=8k+3, $4/(8k+3)=1/(2k+2)+1/(2k+1)(2k+2)+1/(2k+1)(2k+3)$;

(3) For n=8k+4, $4/(8k+4)=1/(2k+3)+1/(2k+2)(2k+3)+1/(2k+1)(2k+2)$;

(4) For n=8k+5, $4/(8k+5)=1/(2k+2)+1/(8k+5)(2k+2)+1/(8k+5)(k+1)$;

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(5) For n=8k+6, 4/(8k+6)=1/(4k+3)+1/(4k+4)+ 1/(4k+3)(4k+4);

(6) For n=8k+7, 4/(8k+7)=1/(2k+3)+1/(2k+2)(2k+3)+1/(2k+2)(8k+7);

(7) For n=8k+8, 4/(8k+8)=1/(2k+4)+1/(2k+2)(2k+3)+1/(2k+3)(2k+4).

Thus it can be seen, that listed above seven formulas suit absolutely the Erdös-Straus conjecture, where k≥0 in them. Of course, for each of them, be necessary to make a check from each of readers, similarly hereinafter.

For remainder 8k+1 with k≥1, we divide it into 3 genera, i.e. (A) 8k+1 for modulus 3 and remainder 0, (B)8k+1 for modulus 3 and remainder 1 and (C)8k+1 for modulus 3 and remainder 2. Excepting 8k+1 for modulus 3 and remainder 1, we formulate each of two genera therein as follows.

(8) 8k+1 for modulus 3 and remainder 0, since (8k+1)/3 is an integer, so there is to 4/(8k+1)=1/(8k+1)/3 +1/(8k+2) +1/(8k+1)(8K+2), where k ≥1.

(9) 8k+1 for modulus 3 and remainder 2, since (8k+2)/3 is an integer, so there is to 4/(8k+1)=1/(8k+2)/3 +1/(8k+1) +1/(8k+1)(8k+2)/3, where k≥2.

For remainder (B)8k+1 with k≥3 for modulus 3 and remainder 1, let us divide it into five sorts, i.e. ①25+120c, ②49+120c, ③73+120c, ④97+120c and ⑤121+120c, where c≥0, as listed below.

C,	①25+120c,	②49+120c,	③73+120c,	④ 97+120c,	⑤121+120c,
0,	25,	49,	73,	97,	121,
1,	145,	169,	193,	217,	241,
2,	205,	289,	313,	337,	361,
…,	…,	…,	…,	…,	…,

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Excepting ② and ⑤, we formulate each of other three sorts as follows:

(10) For n=25+120c, 4/(25+120c)=1/(25+120c)+1/(50+240c)+1/(10+48c).

(11) For=73+120c, 4/(73+120c) = 1/(20+30c) + 1/(73+120c)(10+15c) + 1/(73+120c)(4+6c).

(12) For n=97+120c, 4/(97+120c) =1/(25+30c) + 1/(97+120c)(50+60c) + 1/(97+120c)(10+12c).

Thereinafter, we shall divide 4/(49+120c) and 4/(121+120c) into certain subclasses to prove them by two sections via logical inference.

For each subclass of 4/(49+120c) plus 4/(121+120c), although can formulate it, but since their total should be many, also add wordy paragraphs inevitably, so do them like this unnecessarily.

4. Proving 4/(49+120c)=1/x + 1/y +1/z

Since 4/(49+120c)

= 1/(13+30c) + 3/(13+30c)(49+120c),

= 1/(14+30c) + 7/(14+30c)(49+120c),

= 1/(15+30c) + 11/(15+30c)(49+120c),

= 1/(16+30c) + 15/(16+30c)(49+120c),

= 1/(17+30c) + 19/(17+30c)(49+120c),

= 1/(18+30c) + 23/(18+30c)(49+120c),

= 1/(19+30c) + 27/(19+30c)(49+120c),

= 1/(20+30c) + 31/(20+30c)(49+120c),

…

$= 1/(13+\alpha+30c) + (3+4\alpha)/(13+\alpha+30c)(49+120c)$, where $\alpha\geq 0$ and $c\geq 0$.

…

Listed above groups of two additive fractions plus their identical transformations are termed to identical relation of $4/(49+120c)$. Evidently such identical relations have infinitely many.

To my way of thinking, $4/(49+120c)=1/(13+\alpha+30c)+(3+4\alpha)/(13+\alpha+30c)(49+120c)$ is able to be expressed into $1/x+1/y+1/z$ in which case c equals each of positive integers plus 0 certainly. Nothing but, each of $4/(49+120c)=1/x+1/y+1/z$ hides within some equalities therein or identical transformations thereof. Thus, first let $1/(13+\alpha+30c)=1/x$, after that, so long as can prove $(3+4\alpha)/(13+\alpha)+30c)(49+120c)=1/y+1/z$, then it is able to be concluded that achieve the purpose of the proof.

Though we are unable to find out all of them singly, but can still select few representative equalities therein via identical transformation and logical inference to prove that when c is equal to each of positive integers plus 0, there is sure to $4/(49+120c)=1/x+1/y+1/z$.

Proof* First we regard $4/(49+120c)=1/(15+30c)+11/(15+30c)(49+120c)$ as a representative equality, and that from this to structure a rough shell frame about proving $22m/2m(15+30c)(49+120c)=1/y+1/z$ via identical transformation, where m expresses each and ever positive integer.

The denominator $2m(15+30c)(49+120c)=2m\times 3\times 5(2c+1)(49+120c)$, and divide the numerator 22m into two addends, also regard left addends as the set Q, and regard right addends as the set Φ, then 22m=q+φ, ut infra.

When	m=1;		m=2;		m=3…		m	
Numerator,	**Q,**	**Φ,**	**Q,**	**Φ,**	**Q,**	**Φ…**	**Q,**	**Φ**
22m:	22=22-1	+1;	44=44-1	+1;	66=66-1	+1; …	22m=22m-1	+1
	22-3	+3;	44-3	+3;	66-3	+3;	22m-3	+3
	22-5	+5;	44-5	+5;	66-5	+5;	22m-5	+5
	…	…;	…	…;	…	…; …	…	…
	22-21	+21;	44-21	+21;	66-21	+21; …	22m-21	+21
			22-1	+1;	44-1	+1;…	22(m-1)-1	+1
			[i.e.44-23	+1];	[i.e.66-23	+1]…	[i.e.22m-23	+1]
			22-3	+3;	44-3	+3;…	22(m-1)-3	+3
			…	…;	…	…; …	…	…
			22-21	+21;	44-21	+21;…	22(m-1)-21	+21
					22-1	+1;…	22(m-2)-1	+1
					[i.e.44-23	+1]…	[i.e.22(m-1)-23	+1]
					22-3	+3;…	22(m-2)-3	+3
					…	…; …	…	…
					22-21	+21;…	22(m-2)-21	+21
							22(m-3)-1	+1,
							[i.e.22(m-2)-23	+1]
							22(m-3)-3	+3
							…	…
							22(m-3)-21	+21
							…	…

As listed above, when a left addend turn into $22(m-1)-\varphi$ from $22m-\varphi$, the right addend is always φ, where $\varphi=1, 3, 5, 7, 9, 11, 13, 15, 17, 19$ and 21, so all right addends can be converted into 11 consecutive odd numbers from 1 to 21. Thus, if regard such 11 consecutive odd numbers as a group, then when m equals each and every positive integer, left all addends are all positive odd numbers irrespective of their repetitions. Yet right all addends are infinitely many groups of odd numbers from 1 to 21.

Prove $22m/2m\times3\times5(2c+1)(49+120c)=1/y+1/z$, actually it is exactly to prove that the equality $(q+\varphi)/2m\times3\times5(2c+1)(49+120c)=1/y+1/z$ holds water in which case c equals each of positive integers plus 0, where y and z are positive integers, and $q+\varphi=22m$.

If each and every positive odd number substitutes for 2c+1, then c is equal to each of positive integers plus 0.

So let each of left addends from small to large to substitute for 2c+1, one by one, then, we get every value of c from small to large.

Thereupon, let us list headmost 11 equalities of $4/49+120c=1/x+1/y+1/z$ in which case c=0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10, as listed below.

1. When c=0, $4/(49+120c) = 1/(14+30c)+7/(14+30c)(49+120c)=1/14 + 7/(14+30c)(49+120c) =1/14 + 1/(2\times49) =1/14 + 1/99 + 1/(98\times99)$.

Here, c=0 is due to 1=2c+1, i.e. the left addend is 1, similarly hereinafter.

2. When c=1, $4/(49+120c) = 1/(22+30c) + 39/(22+30c)(49+120c) = 1/52 + (26+13)/(22+30c)(49+120c) = 1/52 +1/(2\times169) + 1/(2^2\times169)$.

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3. When $c=2$, $4/(49+120c) = 1/(42+30c) + 119/(42+30c)(49+120c) = 1/(42+30c) + (102+17)/(42+30c)(49+120c) = 1/102 + 1/17^2 + 1/(102\times17)$.

4. When $c=3$, $4/(49+120c) = 1/(15+30c) + 11/(15+30c)(49+120c) = 1/(15+30c) + (21+1)/3\times10(2c+1)(49+120c)=1/105+1/4090+1/(210\times409)$.

5. When $c=4$, $4/(49+120c) = 1/(18+30c) + 23/(18+30c)(49+120c) = 1/138 + 23/(138\times23^2) = 1/138 + 1/(138\times23+1) +1/(138\times23)(138\times23+1)$.

6. When $c=5$, $4/(49+120c) = 1/(15+30c) +11/(15+30c)(49+120c) = 1/165 +1/(15\times649) = 1/165 +1/(15\times649+1) +1/15\times649(15\times649+1)$.

7. When $c=6$, $4/(49+120c) = 1/(15+30c) +11/(15+30c)(49+120c) = 1/195 +(65+1)/(6\times3\times5\times13\times769) = 1/195 +1/(18\times769) + 1/(18\times65\times769)$.

8. When $c=7$, $4/(49+120c) = 1/(15+30c) +11/(15+30c)(49+120c) = 1/225 +(21+1)/(2\times3\times5\times15\times7\times127) =1/225 + 1/(150\times127) + 1/(3150\times127)$.

9. When $c=8$, $4/(49+120c) = 1/(15+30c) + 11/(15+30c)(49+120c) = 1/255 +(17+5)/(2\times3\times5\times17\times1009) =1/255 + 1/(30\times1009) + 1/(102\times1009)$.

10. When $c=9$, $4/(49+120c) = 1/(15+30c) +11/(15+30c)(49+120c) = 1/285 +(19+3)/(3\times5\times19\times1129) = 1/285 + 1/(15\times1129) + 1/(95\times1129)$

11. When $c=10$, $4/(49+120c) = 1/(15+30c) + 11/(15+30c)(49+120c) = 1/315 +(21+1)/(2\times3\times5\times21\times1249)=1/315 +1/(30\times1249)+1/(30\times21\times1249)$.

In order to enable that a left addend substitutes for $2c+1$ solely, then the right addend must be a factor of the denominator.

By now, we set about analyzing 11 right addends within the set Φ below.

First, when Φ=1, 3, 5 and 15, each of them is a factor in denominator

$2m\times3\times5(2c+1)(49+120c)$, so they satisfy the aforesaid requirement, i.e. a left addend within every pair with each of them can substitute for $2c+1$ solely, thus can determine all values of c of the 4 subclasses.

Next, analyze all subclasses in which case $\Phi=7$, 11, 13, 17, 19, 9 and 21.

For $\Phi=7$, since it is not a factor within the denominator, so we let $m=7$, then the numerator is turned into $22\times7-7$ plus 7, and the denominator is turned into $2\times3\times5\times7(2c+1)(49+120c)$.

As thus, when $m\geq7$, it satisfy the aforesaid requirement likewise, enable that a left addend within every pair with 7 can substitute for $2c+1$ solely, so can determine infinite many values of c of the subclass.

But, after do it like this, missed out six odd numbers, i.e. $22\times1-7$, $22\times2-7$, $22\times3-7$, $22\times4-7$, $22\times5-7$ and $22\times6-7$. If let these odd numbers substitute for $2c+1$ orderly, then get $c=7$, 18, 29, 40, 51 and 62, yet $c=7$ has existed before this. Thus, need us to make up five such equalities of $4/49+120c=1/x+1/y+1/z$ in which case $c=18$, 29, 40, 51 and 62 by other methods.

Such as when $c=18$, it has $4/(49+120c)=1/(24+30c)+47/(24+30c)\times47^2=1/(24+30c)+1/(24+30c)\times47=1/565+1/(564\times565)+1/(564\times47)$. Then, other four will be reckoned in the number of unproved equalities.

Pursuant to the same reason, for $\Phi=11$, 13, 17 and 19, let $m=11$, 13, 17 and 19, enable a left addends within every pair with 11, 13, 17 and 19 substitute for $2c+1$ solely in which case $m\geq11$, 13, 17 or 19 to aforesaid order, as thus, can determine infinite many values of c of the 4 subclasses.

Nevertheless, when Φ=7, 11, 13, 17 and 19, there are still 60 missing fractions which express into 1/y+1/z, from 4+10+12+16+18=60.

For Φ=9, let m=3 because the denominator has factor 3, then miss 2 fractions, so need to list $(22\times1-3^2 + 3^2)/2\times3^2\times5(2c+1)(49+120c)=1/y+1/z$ and $(22\times2-3^2 + 3^2)/2\times3^2\times5(2c+1)(49+120c)=1/y+1/z$.

For Φ=21, on the basis that previously solved Φ=7, it is not a problem yet, thus need not again consider it.

Hereto, we have proven infinitely many values of c such that 4/(49+120c) =1/x+1/y+1/z, nothing but, miss out 62 values of c to await supplements.

5. Proving 4/(121+120c)=1/x + 1/y +1/z

Since 4/(121+120c)

= 1/(31+30c) + 3/(31+30c)(121+120c),

= 1/(32+30c) + 7/(32+30c)(121+120c),

= 1/(33+30c) + 11/(33+30c)(121+120c),

= 1/(34+30c) + 15/(34+30c)(121+120c),

= 1/(35+30c) + 19/(35+30c)(121+120c),

= 1/(36+30c) + 23/(36+30c)(121+120c),

= 1/(37+30c) + 27/(37+30c)(121+120c),

= 1/(38+30c) + 31/(38+30c)(121+120c),

= 1/(39+30c) + 35/(39+30c)(121+120c),

= 1/(40+30c) + 39/(40+30c)(121+120c),

…

$= 1/(60+30c) + 119/(60+30c)(121+120c)$,

…

$= 1/(31+\alpha+30c) + (3+4\alpha)/(31+\alpha+30c)(121+120c)$, where $\alpha \geq 0$ and $c \geq 0$.

…

As listed below, our think is still that first let $1/(31+\alpha+30c)=1/x$, then only prove $(3+4\alpha)/(31+\alpha+30c)(121+120c) =1/y+1/z$, where $c \geq 0$, and $\alpha \geq 0$.

Proof· This proof for $4/(121+120c) =1/x+1/y+1/z$ is similar to that proof of $4/(49+120c) =1/x+1/y+1/z$ in preceding section.

Likewise be necessary to seek out a representative equality, so we select $4/(121+120c) =1/(60+30c) + 119/(60+30c)(121+120c)$.

First, let $1/(60+30c)=1/x$, then, need only to prove $119/(60+30c)(121+120c) =1/y+1/z$ with $c \geq 0$. Undoubtedly this is feasible.

Let $119/(60+30c)(121+120c)=119m/2\times3\times5m(2+c)(121+120c)$ with $m \geq 1$, then again, divide 119m into two portions of addends like preceding 22m, so either portion can express all positive integers in which case m equals each and every positive integer, irrespective of their repetitions.

If let each and every positive integer≥ 2 substitute for 2+c, then we can get all values of c.

So we let each and every left addend on the numerator to substitute for factor 2+c within the denominator solely, yet any right addend must be a factor within the denominator.

Thus, we regard left addends as the set Q, and convert right addends into infinite many groups of consecutive 119 integers≥1, and that regard them as the set Φ, as listed below.

When	m=1;		m=2;		m=3…		m	
Numerator,	**Q,**	**Φ,**	**Q,**	**Φ,**	**Q ,**	**Φ…**	**Q,**	**Φ**
119m:	119=119-1	+1;	238= 238-1	+1;	357=357-1	+1;…	119m=119m-1	+1
	119-2	+2;	238-2	+2;	357-2	+2; …	119m-2	+2
	…		…		…	…	…	
	119-117	+117;	238-117	+117;	357-117	+117;…	119m-117	+117
	119-118	+118;	238-118	+118;	357-118	+118;…	119m-118	+118
	119-119	+119;	238-119	+119;	357-119	+119;…	119m-119	+119
			119-1	+1;	238-1	+1;…	119(m-1)-1	+1
			119-2	+2;	238-2	+2;…	119(m-1)-2	+2
			…		…	…	…	
			119-117	+117;	238-117	+117;…	119(m-1)-117	+117
			119-118	+118;	238-118	+118;…	119(m-1)-118	+118
			119-119	+119;	238-119	+119;…	119(m-1)-119	+119
					119-1	+1;…	119(m-2)-1	+1
					119-2	+2;…	119(m-2)-2	+2
					…	…	…	
					119-117	+117;…	119(m-2)-117	+117
					119-118	+118;…	119(m-2)-118	+118
					119-119	+119;…	119(m-2)-119	+119
							119(m-3)-1	+1
							119(m-3)-2	+2
							…	
							119(m-3)-117	+117
							119(m-3)-118	+118
							119(m-3)-119	+119
							…	

Thus it can be seen, when m equals each and every positive integer, left addends' set Q is all positive integers irrespective of repetitions of them plus 0 and 1. Yet right addends' set Φ consists of infinite many groups of consecutive 119 integers≥1, because when a left addend is from 119m-φ to 119(m-1)-φ, the right addend is always φ, where φ expresses each of integers from 1 to 119, but after m equals each positive integer, 118 and 119 are needless, for a left addend which pair with 118 or 119 is 1 or 0.

By this token, first we must list headmost 119 equalities of 4/121+120c= 1/x+1/y+1/z. Yet, since the amount is tanto, so we list only a few of such equalities in which case c=0, 1, 2, 3 and 4, ut infra.

(1). When c=0, $4/(121+120c) = 1/(33+30c) + 11/(33+30c)(121+120c) = 1/33 + 1/(3\times11^2+1) = 1/33 + 1/(3\times11^2+1) + 1/(3\times11^2)(3\times11^2+1)$;

(2). When c=1, $4/(121+120c) = 1/(33+30c) + 11/(33+30c)(121+120c) = 1/63 + (21+1)/2\times3\times21\times241 = 1/63 + 1/2\times3\times241 + 1/2\times3\times21\times241$;

(3). When c=2, $4/(121+120c) = 1/(35+30c) + 19/(35+30c)(121+120c) = 1/95 + 1/5\times361 = 1/95 + 1/(5\times361+1) + 1/(5\times361)(5\times361+1)$;

(4).When c=3, $4/(121+120c) = 1/(33+30c) + 11/(33+30c)(121+120c) = 1/123+ (41+3)/2^2\times3(11+10c)(121+120c)=1/123+1/12\times481+1/2^2\times41\times481$;

(5). When c=4, $4/(121+120c) = 1/(60+30c) + 99/(60+30c)(121+120c) = 1/180 + (90+9)/2\times3\times5(2+c)(121+120C) = 1/180 +1/2\times601 +1/2^2\times5\times601$.

Likewise other 114 equalities are supposed to list out, but that is impractical. So, 114 will be reckoned in the number of unlisted equalities.

By now, we analyze right each group of 119 consecutive addends of Φ as compared with denominator 2×3×5m(2+c)(121+120c) with m≥1, ut infra.

First, when Φ=1, 2, 3, 5, 6, 10, 15 and 30, each of them is a factor within denominator 2×3×5m(2+c)(121+120c), thus they satisfy the requirement, then a left addend within every pair with each of them can substitute for factor 2+c within the denominator solely, so can determine all values of c of the 8 subclasses on the premise that satisfies each fraction=1/y+1/z.

In addition to this, the rest is 111 integers, and that there are 27 prime numbers and 84 composite numbers therein.

For each of 27 prime numbers, let m of 119m/2×3×5m(2+c)(121+120c) be equal to the prime number.

As thus, for two addends which constitute the numerator, one is the prime number, and another can substitute for 2+c. That is to say, two addends of the numerator are factors of the denominator. Of course, such a fraction is able to be expressed into sum of 1/y+1/z.

Thereupon start with the fraction to infinite many fractions, can express uniformly each of them into sum of 1/y+1/z, but miss out m-1 fractions.

By this token, for the 27 prime numbers, the number of missing fractions is the sum of the 27 prime numbers minus 27 altogether, i.e. 7+ 11+ 13+ 17+ 19+ 23+ 29+ 31+ 37+ 41+ 43+ 47+53 + 59 + 61+ 67+ 71 + 73+ 79+ 83+ 89+ 97+ 101+ 103+ 107+ 109+ 113 -27=1556.

In other words, when the right addend φ at numerator of the fraction (q+φ)/2×3×5m(2+c)(121+120c) with q+φ=119m is a prime number, let m within the fraction be equal to the prime number. Then, start with this fraction to infinite many fractions, the right addend is always the prime number, and left each and every addend can always substitute for 2+c, such that infinite many a fraction is expressed into sum of 1/y+1/z. But, after do it like this, they miss out 1556 fractions as such expression.

For 84 composite numbers as right addends, decompose each of them into prime factors, and that write down the product of distinct prime factors and polymerous prime factors as compared with prime factors of the denominator 2×3×5m(2+c) (121+120c).

In fact, every such product is exactly a disparate factor of right addend as compared with factors of the denominator.

Below, we figure out each and every such product according to the order of composite numbers from small to large, and use the symbol Π_{dp} to express the product of distinct and polymerous prime factors.

(1) $4=2^2$, $\Pi_{dp}=2$; (2) $8=2^3$, $\Pi_{dp}=4$; (3) $9=3^2$, $\Pi_{dp}=3$;

(4) $12=2^2\times3$, $\Pi_{dp}=2$; (5) $14=2\times7$, $\Pi_{dp}=7$; (6) $16=2^4$, $\Pi_{dp}=2^3$;

(7) $18=2\times3^2$, $\Pi_{dp}=3$; (8) $20=2^2\times5$, $\Pi_{dp}=2$; (9) $21=3\times7$, $\Pi_{dp}=7$;

(10) $22=2\times11$, $\Pi_{dp}=11$; (11) $24=2^3\times3$, $\Pi_{dp}=2^2$; (12) $25=5^2$, $\Pi_{dp}=5$;

(13) $26=2\times13$, $\Pi_{dp}=13$; (14) $27=3^3$, $\Pi_{dp}=3^2$; (15) $28=2^2\times7$, $\Pi_{dp}=2\times7$;

(16) $32=2^5$, $\Pi_{dp}=2^4$; (17) $33=3\times11$, $\Pi_{dp}=11$; (18) $34=2\times17$, $\Pi_{dp}=17$;

(19) $35=5\times7$, $\Pi_{dp}=7$; (20) $36=2^2\times3^2$, $\Pi_{dp}=2\times3$; (21) $38=2\times19$, $\Pi_{dp}=19$;

(22) $39=3\times13$, $\Pi_{dp}=13$; (23) $40=2^3\times5$, $\Pi_{dp}=2^2$; (24) $42=2\times3\times7$, $\Pi_{dp}=7$;

(25) $44=2^2\times11$, $\Pi_{dp}=2\times11$; (26) $45=3^2\times5$, $\Pi_{dp}=3$; (27) $46=2\times23$, $\Pi_{dp}=23$;

(28) $48=2^4\times3$, $\Pi_{dp}=2^3$; (29) $49=7^2$, $\Pi_{dp}=7^2$; (30) $50=2\times5^2$, $\Pi_{dp}=5$;

(31) $51=3\times17$, $\Pi_{dp}=17$; (32) $52=2^2\times13$, $\Pi_{dp}=2\times13$; (33) $54=2\times3^3$, $\Pi_{dp}=3^2$;

(34) $55=5\times11$, $\Pi d_p=11$; (35) $56=2^3\times7$, $\Pi d_p=2^2\times7$; (36) $57=3\times19$, $\Pi d_p=19$;

(37) $58=2\times29$, $\Pi d_p=29$; (38) $60=2^2\times3\times5$, $\Pi d_p=2$; (39) $62=2\times31$, $\Pi d_p=31$;

(40) $63=3^2\times7$, $\Pi d_p=3\times7$; (41) $64=2^6$, $\Pi d_p=2^5$; (42) $65=5\times13$, $\Pi d_p=13$;

(43) $66=2\times3\times11$, $\Pi d_p=11$; (44) $68=2^2\times17$, $\Pi d_p=2\times17$; (45) $69=3\times23$, $\Pi d_p=23$;

(46) $70=2\times5\times7$, $\Pi d_p=7$; (47) $72=2^3\times3^2$, $\Pi d_p=2^2\times3$; (48) $74=2\times37$, $\Pi d_p=37$; (49) $75=3\times5^2$, $\Pi d_p=5$; (50) $76=2^2\times19$, $\Pi d_p=2\times19$; (51) $77=7\times11$, $\Pi d_p=7\times11$; (52) $78=2\times3\times13$, $\Pi d_p=13$; (53) $80=2^4\times5$, $\Pi d_p=2^3$; (54) $81=3^4$, $\Pi d_p=3^3$;

(55) $82=2\times41$, $\Pi d_p=41$; (56) $84=2^2\times3\times7$, $\Pi d_p=2\times7$; (57) $85=5\times17$, $\Pi d_p=17$;

(58) $86=2\times43$, $\Pi d_p=43$; (59) $87=3\times29$, $\Pi d_p=29$; (60) $88=2^3\times11$, $\Pi d_p=2^2\times11$;

(61) $90=2\times3^2\times5$,$\Pi d_p=3$; (62) $91=7\times13$,$\Pi d_p=7\times13$; (63) $92=2^2\times23$,$\Pi d_p=2\times23$;

(64) $93=3\times31$, $\Pi d_p=31$; (65) $94=2\times47$, $\Pi d_p=47$; (66) $95=5\times19$, $\Pi d_p=19$;

(67) $96=2^5\times3$, $\Pi d_p=2^4$; (68) $98=2\times7^2$, $\Pi d_p=7^2$; (69) $99=3^2\times11$, $\Pi d_p=3\times11$;

(70) $100=2^2\times5^2$,$\Pi d_p=2\times5$;(71)$102=2\times3\times17$,$\Pi d_p=17$; (72)$104=2^3\times13$,$\Pi d_p=2^2\times13$;

(73) $105=3\times5\times7$,$\Pi d_p=7$; (74) $106=2\times53$, $\Pi d_p=53$; (75) $108=2^2\times3^3$, $\Pi d_p=2\times3^2$;

(76) $110=2\times5\times11$,$\Pi d_p=11$; (77) $111=3\times37$,$\Pi d_p=37$; (78) $112=2^4\times7$,$\Pi d_p=2^3\times7$;

(79) $114=2\times3\times19$,$\Pi d_p=19$; (80) $115=5\times23$,$\Pi d_p=23$; (81) $116=2^2\times29$,$\Pi d_p=2\times29$;

(82) $117=3^2\times13$,$\Pi d_p=3\times13$; (83) $118=2\times59$,$\Pi d_p=59$; (84) $119=7\times17$,$\Pi d_p=7\times17$.

By now, we classify above 84 products, then figure out a number of missing fractions of each classify, and let ξ express the number.

1. $\Pi d_p=2$: (1), (4), (8), (38) 2; (11), (23) 2^2; (2), (6), (28), (53) 2^3; (16), (67) 2^4; (41) 2^5, and $\xi=4+6+28+30+31=99$.

2. $\Pi d_p=3$: (3), (7), (26), (61) 3; (14), (33) 3^2; (54) 3^3, and $\xi = 8+16+26=50$.

3. $\Pi_{d|p}=5$: (12), (30), (49) 5, and $\xi=12$.

4. $\Pi_{d|p}=7$: (5), (9), (19), (24), (46), (73) 7; (29), (68) 7^2, and $\xi = 36+96=132$.

5.$\Pi_{d|p}=11$: (10), (17), (34), (43), (76) 11, and $\xi = 50$.

6. $\Pi_{d|p}=13$: (13), (22), (42), (52) 13, and $\xi = 48$.

7. $\Pi_{d|p}=2\times7$: (15), (56) 2×7; (35) $2^2\times7$; (78) $2^3\times7$, and $\xi = 26+27+55=108$.

8. $\Pi_{d|p}=17$: (18), (31), (57), (71) 17, and $\xi = 64$.

9. $\Pi_{d|p}=2\times3$: (20) 2×3; (47) $2^2\times3$; (75) 2×3^2, and $\xi = 5+11+17=33$.

10. $\Pi_{d|p}=19$: (21), (36), (66), (79) 19, and $\xi = 72$.

11. $\Pi_{d|p}=2\times11$: (25) 2×11; (60) $2^2\times11$, and $\xi = 21+43=64$.

12. $\Pi_{d|p}=23$: (27), (45), (80) 23, and $\xi = 66$.

13. $\Pi_{d|p}=2\times13$: (32) 2×13; (72) $2^2\times13$, and $\xi = 25+51=76$.

14. $\Pi_{d|p}=29$: (37), (59) 29, and $\xi = 56$.

15. $\Pi_{d|p}=31$: (39), (64) 31, and $\xi = 60$.

16. $\Pi_{d|p}=3\times7$: (40) 3×7, and $\xi = 20$.

17. $\Pi_{d|p}=2\times17$: (44) 2×17, and $\xi = 33$.

18. $\Pi_{d|p}=37$: (48), (77) 37, and $\xi = 72$.

19. $\Pi_{d|p}=2\times19$: (50) 2×19, and $\xi = 37$.

20. $\Pi_{d|p}=7\times11$: (51) 7×11, and $\xi = 76$.

21. $\Pi_{d|p}=41$: (55) 41, and $\xi = 40$.

22. $\Pi_{d|p}=43$: (58) 43, and $\xi = 42$.

23. $\Pi_{d|p}=7\times13$: (62) 7×13, and $\xi = 90$.

24. $\Pi_{d|p}=2\times23$: (63) 2×23, and $\xi = 45$.

25. Πd_p=47: (65) 47, and $\xi = 46$.

26. Πd_p=3×11: (69) 3×11, and $\xi = 32$.

27. Πd_p=2×5: (70) 2×5, and $\xi = 9$.

28. Πd_p=53: (74) 53, and $\xi = 52$.

29. Πd_p=2×29: (81) 2×29, and $\xi = 57$.

30. Πd_p=3×13: (82) 3×13, and $\xi = 38$.

31. Πd_p=59: (83) 59, and $\xi = 58$.

32. Πd_p=7×17: (84) 7×17, and $\xi = 118$.

Listed above numbers of missing fractions of 32 classifies add, is 1855.

Namely, if right addends at numerator of 119m/2×3×5m(2+c)(121+120c) are composite numbers, let every Πd_p substitute for m within the fraction, such that the fraction is expressed into infinite many a sum of 1/y +1/z.

But, after do them like so, miss out 1855 fractions as such expression.

Besides, there are 114 unlisted equalities anteriorly, i.e. c from 5 to 118.

Thus, in the course of proving 4/(121+120c)=1/x+1/y+1/z, need us continue to list 3525 such equalities according to c as corresponding values, where 3525 from 1556+1855+114.

Taken one with another, for the proof of 4/(49+120c)=1/x+1/y+1/z plus 4/(121+120c)=1/x+1/y+1/z, need to get the aid of computer programming, and we firmly believe that it is able to complete 3587 equalities according to c as corresponding values, where 3587 from 62+3525.

To sum up, we have proven the Erdös-Straus conjecture, so as continue to list 3587 concrete equalities according to c as corresponding values.

Namely, for a proof of the Erdös-Straus conjecture, we have realized successfully such a change from infinite to finite equalities.

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