The 4th Spatial Dimension W

This paper attempts to provide a new vision on the 4th spatial dimension starting on the known symmetries of the Euclidean geometry. It results that, the points of the 4th dimensional complex space are circumferences of variable ray. While the axis of the 4th spatial dimension, to be orthogonal to all the three 3d cartesinan axes, is a complex line made of two specular cones surfaces symmetrical on their vertexes corresponding to the common origin of both the real and complex cartesian systems.

The 4th Spatial Dimension W

As we know, dimensions furter the 3 (three) are treated and used in mathematics as the extension of three dimensional spatial dimensions.

This means that if a P.to in 3d has coordinates: (x_1, x_2, x_3), then in the 4d space it will have coordinates: (x_1, x_2, x_3, x_4) and, in general, in the nd space it will have coordinates: (x_1, x_2 , x_3, ..., x_n).

Starting from this assumption, all mathematical analyzes using Cartesian coordinates are valid for 3d euclidean space and for those of higher dimension.

For example, for the distance between two points A and B we can use the following:

$$d_{1d}(A,B)=(x_B-x_A)$$
$$d_{2d}(A,B)=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}$$
$$d_{3d}(A,B)=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2+(z_B-z_A)^2}$$
....
$$d_{nd}(A,B)=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2+...+(n_B-n_A)^2}$$

But this seems to contrast with some assumptions concerning Euclidean geometry for which:

<u>1d)</u> In a single dimension space there is only one X-Cartesian coordinate corresponding to a straight line.

<u>2d)</u> Starting with dimension X, we can obtain the 2nd dimension Y by using a second straight line Y belonging to the bundle Y_n of straight lines orthogonals to the straight line X of the one-dimensional space 1d.

<u>3d)</u> With the same logic, the third dimension Z is obtained by using a straight line Z belonging to the bundle of straight lines Z_n which are orthogonals to the: 1th dimension X straight line, the 2nd dimension Y straight line, and the 2d XY plan generated by them.

<u>4d)</u> the fourth dimension, by symmetry, must correspond to the W line belonging to the bundle of straight lines W_n **orthogonals to the XYZ 3d-dimensional space volume**.

Thus, omitting the simple case 1d in which there is only one dimension X, we can state that:

<u>2d case:</u>

given the two straight lines in $\mathbb{R}^2$: XY

X : y = mx
Y : y = m’x

In $\mathbb{R}^2$ there exists one and only one Y_n parallel straight bundle that satisfies the following:

$$Y_n \perp X \text{ (1d straight line)}$$

and the 2nd dimension csn be represented by any of the Y straight lines:

$$Y \parallel Y_n$$

➔ Y $\perp$ X (1d straight line)

$$Y: y = -\frac{x}{m}$$

3d case:

Condidering the two straight lines in the XY plan:

$$X: y = mx$$
$$Y: y = m'x$$

and the two straight lines in the ZX and ZY planes:

$$Z: z = m''x$$
$$Z: z = m''y$$

And assuming that in $\mathbb{R}^3$ there is one and only one bundle of parallel lines Z_n that satisfies all of the following:

$Z_n \perp$ X (1d straight lines)
$Z_n \perp$ Y (1d straight lines)
$Z_n \perp$ XY (2d plan)

than the **3^{rd} dimension** is represented by any of the Z straight lines:

$Z \parallel Z_n$

$Z \perp XY$

and than, for the $\mathbb{R}^2$ plan: ZX $\perp$ XY, we have:

$Z \perp X$

$$➔ m = -\frac{1}{m''}$$

$$➔ Z: z = m''x = -\frac{x}{m}$$

While for the $\mathbb{R}^2$ plan: ZY $\perp$ XY

$Z \perp Y$

$$➔ m' = -\frac{1}{m''}$$

$$➔ Z: z = m''y = -\frac{y}{m'}$$

Therefore, for the **4^{th} dimension** we must have the following case:

4d case:

As for 3d space we can consider the two straight lines in the XY plan:

$$X: y = mx$$
$$Y: y = m'x$$

and the two straight lines in the ZX and ZY planes:

$$Z: z = m''x$$
$$Z: z = m''y$$

While adding for **4^{th}** dimension forrther three straight lines in the WX and WY and WZ planes which must be orthogonal to XYZ space:

W: $w = m_w x$, $w = m_w y$, $w = m_w z$

And assuming that in $\mathbb{R}$ $\nexists$ a bundle of straight lines W_n that satisfies all of the following:

$W_n \perp$ X
$W_n \perp$ Y
$W_n \perp$ Z
$W_n \perp$ XYZ (3d space) (1)

Than the W_n, to be the **4^{th} dimension**, and to be an Euclidean space greater than 3d, must meet these condictions:

$W_n \perp X$

$W_n \perp Y$

➔ $W_n \parallel Z$

$W_n \perp X$

$W_n \perp Z$

➔ $W_n \parallel Y$

$W_n \perp Y$

$W_n \perp Z$

$\rightarrow W_n \parallel X$

Then (1) implies the (2) :

$$W_n \parallel X \,,\; W_n \parallel Y \,,\; W_n \parallel Z \qquad (2)$$

and the **4th dimension** is represented by any of the W straight lines :

$$W \parallel W_n \qquad (3)$$

from which,

Considering the $\mathbb{R}^2$: WX $\perp$ XYZ

$W_n \perp X$
W : $w = m_w x$
W_n: $w = m_{Wn}\, x$

Considering the $\mathbb{R}^2$: WY $\perp$ XYZ

$W_n \perp Y$
W : $w = m_w y$
W_n: $w = m_{Wn}\, y$

Considering the $\mathbb{R}^2$: WZ $\perp$ XYZ

$W_n \perp Z$
W : $w = m_w z$
W_n: $w = m_{Wn}\, z$

and for the (3), (2), (1), we have:

$\rightarrow W \perp W_n \,,\; W \parallel W_n$

That, for simplicity, we can also write:

$$W \stackrel{\perp}{=} W_n \qquad (4)$$

where:

Def : $A \stackrel{\perp}{=} B$ means that A is both: parallel and orthogonal to B

Now, by applying the orthogonality and parallelism conditions between the generic straight lines W and the W_n beam, we obtain:

$m_w \cdot m_{w_n} = -1$ (orthogonality condition)

$m_w = m_{w_n}$ (parallelism condition)

$m_w m_w = -1$

$m_w^2 = -1$

$m_w = \sqrt{-1} = i$

And we finally obtain:

$\Rightarrow W_n$:

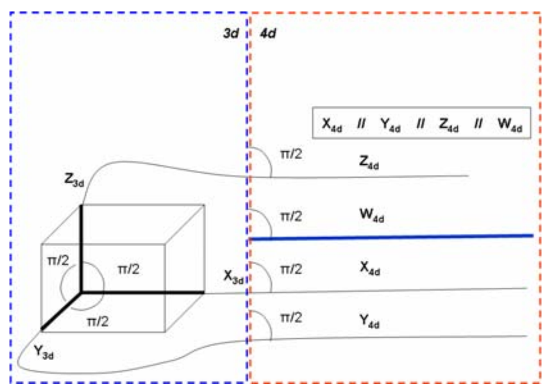
$w = m_{Wn} x$

$w = m_{Wn} y$

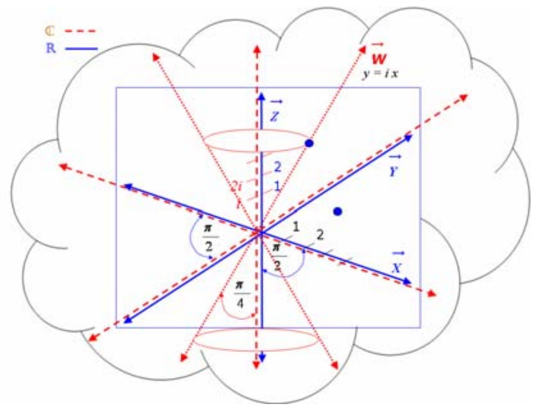
$w = m_{Wn} z$

which represents the bundle of straight lines in the complex $\mathbb{C}$ field corresponding to the W coordinate of the **4th dimension**.

As a consequence of (4) we have that orthogonal lines, that never meet in the $\mathbb{R}$ 3d space, become parallel in 4d space, an, vice versa, parallel lines of 4d space, meet in 3d space $\mathbb{R}$.



Graphically, by do coinciding the two Cartesian reference systems $\mathbb{R}$ (Real) and $\mathbb{C}$ (Complex) we get an indicative representation of the 4th dimension W:



As we can see, a single 4d Point named $\mathbf{P^{4d}}$, has the following coordinates:

$$P^{4d} = \left(x_P^{3d}, y_P^{3d}, z_P^{3d}, w_P^{4d}\right)$$

Which has the following spatial dimension corresponding to the a circumference whose radius is directly proportional to its 4d coordinate: w_P^{4d} :

$$\mathbf{P^{4d}\ dimension = 2\pi r_n = 2\ \pi\ n\ i}$$

We can also say that a 3d spatial Point does not have a spatial dimension, while a 4d spatial Point has a 2-dimensional spatial consistency, corresponding to a variable circumference with the ray proportional to its complex coordinate w_P^{4d} .

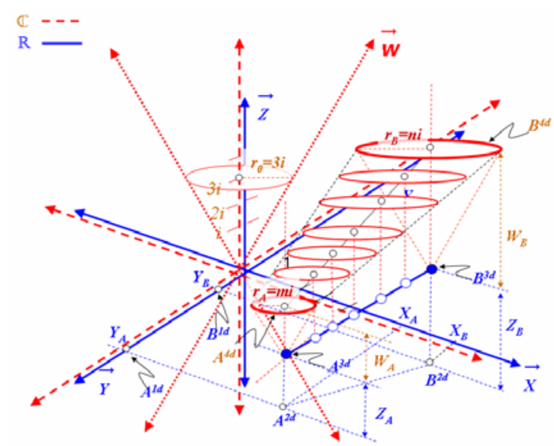
As a simple example, we can observe that 3d Points trasposed in 4d correspond to variable circumferences in complex space. While a segment AB in 4d space corresponds to a geometric figure formed by a continuous sequence of parallel and variable circumferences as shown in the following figure.

If we consider the segment: $\overline{\mathbf{A^{3d}\,B^{3d}}}$

$$\mathbf{A^{3d} = (X_A, Y_A, Z_A)}$$
$$\mathbf{B^{3d} = (X_B, Y_B, Z_B)}$$

and its transposed into 4d : $\overline{\mathbf{A^{4d}\,B^{4d}}}$

$$\mathbf{A^{4d} = (X_A, Y_A, Z_A, W_A)}$$
$$\mathbf{B^{4d} = (X_B, Y_B, Z_B, W_B)}$$



With the same logic of building of geometric figures in 4d, we could draw any geometric figure in 4d space.

The End