Theorem of prime pair distribution

Let

$S_n = \{(A_1, B_1),\ (A_2, B_2),\ \cdots, (A_n, B_n)\}$

$$A_n = a_1 n + a_2$$

$$B_n = b_1 n + b_2$$

If A_n, B_n are not obviously composite,

S_p contains 2pair that contains factor p

Lenth of $3,3,5,5,\cdots,p,p$ is about $\frac{2p}{lnp}$

3 continuous pair (A_n, B_n) contains 2 pair that contains factor 3

$$(3,3,5),(3,3,5),\cdots,(3,3,p),(3,3,p),(3,3)$$

It's lenth not greater than $\frac{2p}{lnp} \cdot \frac{3+2}{3-2} < \frac{2p}{lnp} \cdot (\frac{3}{3-1})^4$

For p_i, p_i continuous pair (A_n, B_n) contains 2 pair that contains factor p_i makes it's lenth not greater than $\frac{2p}{lnp} \cdot (\frac{p_i}{p_i-1})^4$

Hence if $\frac{2p}{lnp} \cdot (\frac{3}{3-1})^4 \cdot \left(\frac{5}{5-1}\right)^4 \cdot \cdots \cdot \left(\frac{p}{p-1}\right)^4 < n$,

S_n doesn't contains (A_n, B_n) has factor $p_i \leq n$

but $(\frac{2-1}{2}) \cdot (\frac{3-1}{3}) \cdot \cdots \cdot (\frac{p-1}{p})$is about $\frac{1}{lnp}$,

$(\frac{3}{3-1})^4 \cdot (\frac{5}{5-1})^4 \cdot \cdots \cdot (\frac{p}{p-1})^4$ is about $(\frac{lnp}{2})^4$

Hence lence of (A_1, B_1), (A_2, B_2), $\cdots, (A_n, B_n)$ that isn't have factor $p_i \leq n$ is not greater than

$$\frac{2p}{lnp} \cdot \left(\frac{lnp}{2}\right)^4 = p \cdot \left(\frac{lnp}{2}\right)^3$$

Hence, if $\mathrm{n} > p \cdot \left(\frac{lnp}{2}\right)^3$, every $A_n, B_n < p^2$,

S_n contains (A_k, B_k) that both $A_k \; and \; B_k$ are prime.

From that, we can solve

1.

$$A_n = 2n + 1$$

$$B_n = -2n + 2N - 1$$

Goldbach's conjecture

2.

$$A_n = 2n - 1 + 2N$$

$$B_n = 2n + 1 + 2N$$

Twin prime conjecture,

And polignac’s conjecture, so on