An unusual operation on a set of Poulet numbers which conducts to another set of Poulet numbers

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Abstract. In this paper I make the following conjecture on Poulet numbers: There exist an infinity of Poulet numbers P2 obtained from Poulet numbers P1 in the following way: let d1, d2, ..., dn be the (not distinct) prime factors of the number P1 – 1, where P1 is a Poulet number; than there exist an infinity of Poulet numbers P2 of the form (d1 + 1)*(d2 + 1)*...*(dn + 1) + 1. Example: for Poulet number P1 = 645 is obtained through this operation Poulet number P2 = 1729 (644 = 2*2*7*23 and 3*3*8*24 + 1 = 1729). Note that from more than one Poulet number P1 can be obtained the same Poulet number P2 (from both 1729 and 6601 is obtained 46657).

Conjecture:

There exist an infinity of Poulet numbers P2 obtained from Poulet numbers P1 in the following way: let d1, d2, ..., dn be the (not distinct) prime factors of the number P1 – 1, where P1 is a Poulet number; than there exist an infinity of Poulet numbers P2 of the form (d1 + 1)*(d2 + 1)*...*(dn + 1) + 1.

The set of Poulet numbers P2:
(ordered by the size of P1)

: 1729, obtained from P1 = 645 (644 = 2*2*7*23 and 3*3*8*24 + 1 = 1729);

: 46657, obtained from P1 = 1729 (1728 = 2^6*3^3 and 3^6*4^3 + 1 = 46657);

: 46657, obtained from P1 = 6601 (6600 = 2^3*3*5^2*11 and 3^3*4*6^2*12 + 1 = 46657);
(...)

Note that from more than one Poulet number P1 can be obtained the same Poulet number P2 (from both 1729 and 6601 is obtained 46657).

Note that the operation presented conducts sometimes to squares of primes which attests a special relation that I have often highlighted between the nature of Poulet numbers and the nature of squares of primes; example: from 3277 – 1 = 3276 = 2^2*3^2*7*13 is obtained 3^2*4^2*8*14 + 1 = 16129 = 127^2.

Observation:

Reversing the operation presented above (and allowing for d1, d2, ... dn to be not prime factors but complementary divisors), there seem to exist special numbers that are "roots" in obtaining multiple Poulet numbers. Example: such number is 36289 (not a Poulet number itself):

: 36289 – 1 = 36288 = 2^6*3^4*7, which can be written as:

: 2^3*3^3*7*24 which conducts to 1^3*2^3*6*23 + 1 = 1105, a Poulet number;

: 3^4*4*8*14 + 1, which conducts to 2^4*3*7*13 + 1 = 4369, a Poulet number;

: 3^3*4^2*6*14 + 1, which conducts to 2^3*3^2*5*13 + 1 = 4681, a Poulet number;

: 3^2*4^2*14*18 + 1, which conducts to 2^2*3^2*13*17 + 1 = 7957, a Poulet number.