Recent advances in refutations and validations using Meth8 modal logic model checker

In applied and theoretical mathematics, assertions are categorized in alphabetical order as: axiom; conjecture; definition, entry; equation; expression; formula; functor; hypothesis; inequality; metatheorem; paradox; problem; proof; schema; system; theorem; and thesis. We evaluate 130 objects for 519 assertions to validate 156 as tautology and 363 as not (70%). We use Meth8 that is a modal logic checker in five models.

The semantic content or predicate basis of some expressions on their face does not disqualify them from evaluation by Meth8 in classical modal logic. However, the rules of classical logic, as based on the corrected Square of Opposition by Meth8, apply to virtually any logic system. Consequently some numerical equations are mapped to classical logic as Meth8 scripts.

The rationale for mapping quantifiers as modal operators is in the Appendix based on satisfiability and reproducability of validation of syllogisms.

A table lists what was tested with separated results. The names are numbered in alphabetical order. Test results are Invalidated as Not Validated Tautology (nvt) or Validated as Tautology (vt). For a paradox, invalidated means it is not validated as true, that is, it is not a paradox or contradiction.

The experimental tests used variables for 4 propositions, 4 theorems, and 11 propositions. The size of truth tables are respectively for 16-, 256-, and 2048- truth values. One formula of Popper in 250-characters processed in 125-steps instantly due to recent advances in look up table indexing.

The Meth8 modal theorem prover implements the logic system variant VŁ4 which corrects the quaternary Ł4 of Łukasiewicz. There are two sets of truth values on the 2-tuple {00, 10, 01, 11} as respectively <False for contradiction, Contingent for falsity, Non contingent for truth, Tautology for proof> and <Unevaluated, Improper, Proper, Evaluated>. The model checker contains recent advances in parsing technology and is U.S. Patent Pending.

The mapping of formulas into Meth8 script was performed by hand, checked, and tested for accuracy of intent. (A semi-automation of that process is underway.) The Meth8 script uses literals and connectives in one-character. Propositions are p-z, and theorems are A-B. The connectives for <and, or, imply, equivalent> are <&; +; >; =). The negated connectives for <nand; nor; not imply; exclusive-or> are <\; -; <; @>. The operators for <not; possibility ◇ ∃; necessity □ ∀> are <~; %; #>. Some expressions are adopted for clarity such as: (p=p) for true; (p@p) for false; and (x<y) for x∈y. The expression "x less than or equal to y" is rendered in the negative as ~(x>y).

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

The designated proof value is T tautology. Note the meaning of (%p>#p): a possibility of p implies the necessity of p; and some p implies all p. In other words, if a possibility of p then the necessity of p; and if some p then all p. This shows equivalence and interchangeability of respective modal operators and quantified operators, as proved in Appendix.

For Meth8 an immediate further application to "validate as tautologous" is mapping sentences of natural language into logical formulas. The approach identifies parts of speech as nouns, verbs, and modifiers. These translate into logical symbols for literals, connectives, and operators. For example: the conjunction "and" becomes the connective "&"; and the modifier articles "the" and "a" become the modal box # and lozenge %. Expressions for consecutive sentences are linked by the imply connective to build paragraphs to form requirements documents.

No.	Name of object	Type of object	Results with instances
1	ABC	Conjecture	Invalidated
2	Alcoholics Anonymous BB: We agnostics, p 53	Traditions	Invalidated
3	Alexandroff correspondence	Conditional	Invalidated (3)
4	Anderson division by zero as nullity	Theorem	Invalidated
5	Axiomatizing category theory in free logic	Axioms	Invalidated
6	Banach-Tarski	Paradox	Invalidated
7	Barcan	Formula	Validated
8	Bayes rule	Rule	Invalidated (11) Validated(11)
9	Bell / CHSH / Spekken toy model	Inequalities	Invalidated
10	Berkeley	Paradox	Invalidated
11	Biscuit conditionals	System	Invalidated (13)
12	Bogdanov map, 2D conjugate of Hénon map	Formula	Invalidated
13	Borsuk-Ulam	Theorem	Validated
14	Borsuk-Ulam extensible, non-invertive	Theorem	Validated (2)
15	Branching quantifier (Hintikka)	System	Invalidated
16	Buridan's Ass	Paradox	Invalidated
17	Cantor pairing	Functor	Invalidated (2)
18	Category composition of morphisms	Definition	Invalidated (1)
19	Church	Thesis	Invalidated
20	Clifford tori 2D / Kanban cell neuron	Definition	Validated
21	Constructivistic logic	System	Invalidated (14) Validated (2)
22	Creative theories in degrees of unsolvability	Theorem	Invalidated
23	D Ultrafilter contra continuum problem	Equation	Invalidated (1)
24	Dedekind lattice identity	Axiom	Validated
25	Density of all Turing and truth table degrees	Formula	Invalidated (2)
26	Description logic	System	Invalidated (2)
27	Dialetheism	System	Invalidated (4)
28	Dialetheism: inconsistent	System	Invalidated (2)
29	Dichotomy of selection	System	Invalidated
30	Diverse double compiling	Schema	Invalidated
31	Doxastic logic	System	Invalidated (8) Validated (13)
32	Ehrenfeucft-Mostowski indiscernables	Theorem	Invalidated (1)
33	Epistemic coalition	Perfect recall	Invalidated (4) Validated (3)
34	Epistemic dynamic reasoning	System	Invalidated (2)

No.	Name of object	Type of object	Results with instances
35	Epistemic Hilbert substructure	System	Invalidated (5)
36	Epistemic navigation	System	Invalidated (8)
37	Epistemic quantifiers over agents	Conjecture	Invalidated (8) Validated (12)
38	Erdös-Strauss	Conjecture	Invalidated
39	FOL disjunctive normal forms (DNF): minimize	FOL Optimizer	Invalidated (2) Validated (1)
40	Gentzen proof of sequent System G-M	System	Invalidated (6) Validated (2)
41	Gettier (Justified true belief)	Conjecture	Validated
42	Gödel compactness	Theorem	Invalidated
43	Gödel completeness	Theorem	Invalidated
44	Gödel first incompleteness	Theorem	Invalidated
45	Gödel incompleteness	Theorems	Invalidated (15)
46	Gödel incompleteness theorem	Assistant tools	Invalidated (2) Validated (2)
47	Gödel incompleteness theorem	Schema	Invalidated (4)
48	Gödel-Löb	Axiom	Invalidated
49	Gödel recursion	Theorem	Validated
50	Gödel-Scott on God	Theorem schema	Invalidated
51	Goldbach's conjectures	Conjectures	Invalidated
52	Grassmannian (*recently discovered*)	Paradox	Invalidated
53	Henkin cyclic algebra and first order logic	System	Invalidated (9) Validated(6)
54	Applications to logic	Axioms	Invalidated (8) Validated(6)
55	Permutation model nonrepresentable	Assertion	Invalidated (1)
56	Herbrand semantics	System	Invalidated (6)
57	Heyting-Brouwer intuitionistic logic	Systems	Invalidated (9) Validated (1)
58	Hilbert #10 Diophantine universal solution	Formulas	Invalidated
59	Hilbert generalization	System	Invalidated
60	Huhn 2-distributive lattice identity	Formula	Invalidated
61	Ignorance of first choice	System	Invalidated
62	Inconsistent theory	Theorem	Invalidated
63	Extending the monad to a triad	Formulas	Invalidated
64	Kunen inconsistency	Theorem	Invalidated
65	Independence-friendly logic (Kreiselization)	System	Invalidated (2)
66	Indicative conditionals	Encyclopedia entry	Invalidated
67	Induction: Black raven (swan); Kripkenstein	System	Invalidated (3)
68	Inequality: 'arbitrarily' vs 'sufficiently large	Conjecture	Invalidated (2) Validated (1)

No.	Name of object	Type of object	Results with instances
69	Infinite set theory	Theorem	Invalidated
70	Jonsson positive logic: retromorphism	System	Invalidated (3)
71	Immanuel Kant: falsity of syllogistic figures	Theorems	Invalidated (8) Validated (2)
72	Karpenko, S.A.	System K-Ł4	Invalidated
73	Kuratowski–Zorn lemma (Zorn's lemma)	Lemma	Invalidated
74	Lachlan problem solution	Problem	Invalidated (4)
75	Leibniz' ontological proof	Proof	Invalidated (1) Validated (1)
76	Briefest known ontological proof of God	Proof	Validated (2)
77	Lemmon D	Axiom	Invalidated
78	Liar	Paradox	Invalidated
79	Prior rendition	Paradox	Invalidated
80	Kripke rendition	Paradox	Invalidated
81	Löb original, corrected	Theorem	Invalidated (1) Validated (1)
82	Lothar Collatz	Conjecture	Invalidated
83	Löwenheim–Skolem, Hilbert style	Metatheorem	Invalidated
84	Luce model (general)	Definitions/Axioms	Validated (5)
85	Marjorana's 'root'	Equations	Invalidated (5)
86	Meth8 versus Prover9 via Lifshitz	Problem	Validated
87	Modified divine command	Theory	Invalidated
88	Leonard Nelson's criticism of epistemology	System	Invalidated
89	von Neuman-Bernays-Gödel [NBG]	Theory	Invalidated (2) Validated (3)
90	P=NP	Conjecture	Invalidated
91	Paraconsistency, machine-assisted view	Axioms	Invalidated
92	Paraconsistent contradiction	Conext	Invalidated
93	Peano arithmetic 9, 1-8	Axioms	Invalidated (1) Validated (8)
94	Karl Popper on God	Proof	Validated
95	PowerEpsilon mathematical induction	Axiom	Validated (1)
96	Prover9 vs Meth8 differences	System	Invalidated
97	Ramsey's theorem for the 2-color case	Theorem	Validated (2)
98	Ranjan, A.	Problem	Validated
99	Reichenbach common cause / event-splitting	Principle	Invalidated
100	Roman Catholic Church (RCC)	(Dogma)	Invalidated (7)
101	Erasmus contra Luther	Controversy	Validated
102	Infallibility and the Historic Church	Pius IX	Invalidated (2)

No.	Name of object	Type of object	Results with instances
103	Magisterium	Paul VI	Invalidated (1)
104	Tradition above scripture	Pius IX	Invalidated (4)
105	Rota lattice theory, distributive	Axiom	Invalidated
106	Russell	Paradox	Invalidated
107	S5Π+ propositional quantification	System	Invalidated
108	Schrödinger's cat	Paradox	Invalidated
107	Sorites	Paradox	Invalidated
108	Square of Opposition Meth8 Corrected	System	Validated
109	Square of Opposition Modern Revised	System	Invalidated
110	Square of Opposition	Proportions	Invalidated (3)
111	Stone-Wales rotation transform reversibility	Theorem	Invalidated (2) Validated (1)
112	Time as God	Conjecture	Validated
113	Topological manifold transition	Function	Invalidated (1)
114	Veblen (corrected)	Axiom	Invalidated (1) Validated(1)
115	Veronoï regions (with "nonempty sets")	Definition	Invalidated
116	W (K4W)	Theorem	Invalidated
117	Well ordering property	Axiom	Invalidated
118	Wittgenstein's ab-notation	System	Invalidated (3)
119	Yalcin logic	Axioms	Invalidated (2)
120	Zadeh first operators on fuzzy logic	System	Invalidated (5)
121	Zermello-Fraenkel (ZFC):	(Axioms)	Invalidated (10) Validated (1)
122	ZFC Choice	Axiom	Invalidated
123	ZFC Empty set	Axiom	Invalidated
124	ZFC Extensionality	Axiom	Invalidated
125	ZFC Infinity	Axiom	Invalidated
126	ZFC Pairing	Axiom	Invalidated
127	ZFC Power set	Axiom	Invalidated
128	ZFC Regularity or foundation	Axiom	Invalidated
129	ZFC Schema of replacement	Axiom	Invalidated
130	ZFC Specification	Axiom	Validated
131	ZFC Union	Axiom	Invalidated
132	ZFC Well ordering	Axiom	Invalidated

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