Some Infinite Products over the Continuous Interval between 0 and 1

$$\prod_{0,x\in R}^{1} e^{\gamma} \ln(1/x) = \sqrt{2} \qquad \gamma{=}0.5772157...$$

$$\prod_{0,x\in R}^{1} (\sqrt{2} * \sin^{x}(\pi * x)) = \sqrt{2}$$

$$\prod_{0,x\in R}^{1} e^{\frac{-\pi^2}{12}} * (1+x)^{\frac{1}{x}} = 1$$

$$\prod_{0,x\in R}^{1} (2 * \sin(\pi * x/2)) = \sqrt{2}$$

$$\prod_{0,x\in R}^{1} e^{\frac{-\pi}{2}} * (1+\cos(\pi * x))^{\frac{1}{\cos(\pi * x)}} = \tfrac{1}{2}$$

$$\prod_{0,x\in R}^{1} x\left[(2-x)^{\frac{12}{\pi^2(1-x)}}\right] = \sqrt{2}$$

$$\prod_{0,x\in R}^{1} \frac{e^{\sqrt{2}}}{(1+\sqrt{2})} x\left[x+\sqrt{x^2+1}\right] = \sqrt{2}$$

$$\prod_{0,x\in R}^{1} (2 * \sin(\pi * x)) = 2$$

$$\prod_{0,x\in R}^{1} x * e \uparrow (\sin^2(2\pi x)) = \sqrt{2}$$

$$\prod_{0,x\in R}^{1} \frac{2}{3} * (e \uparrow (\frac{x^2-x}{\ln(x)})) = 1$$

$$\prod_{0,x\in R}^{1} x * (e \uparrow (3x^2)) = 1$$

$$\prod_{0,x\in R}^{1} (\sqrt{e} * x) \uparrow x = 1$$

$$\prod_{0,x\in R}^{1} \frac{1}{4} * e^3 * x^2 * (2-x) = 2$$

$$\prod_{0,x\in R}^{1} (x + 0.54221...) = 1$$

$$\prod_{0,x\in R}^{1} (2x + 0.17696...) = 1$$

Using the following program…

```
10 Input N: P=1:E=2.718281828
20 For I = 1 to N
30 X=(2*I-1)/(2*N)
40 Gosub 1000
50 Next I
60 Print P
70 Goto 10

1000 T=(sqr(E)*X)^X: P=P*T:Return
```

… we get a finite product approximation of the infinite product. Change line 1000 for other functions (above we approximate for f(x)=(sqr(e)*x)^x). The larger the N, the closer the approximation –provided other types of errors, like truncation errors, are minimal.

Output for various functions were as follows:

$$\prod_{0, x\in R}^{1} (\sqrt{e} * x) \uparrow x = 1$$

N	Approximation of Product (N subintervals)
10	0.981037
100	0.997130
500	0.999291
1000	0.999616

$$\prod_{0, x\in R}^{1} e^{\gamma} \ln(1/x) = \sqrt{2} \qquad \gamma\text{=0.5772157...}$$

N	Approximation of Product (N subintervals)
10	1.310335846
100	1.343506217
200	1.349870725

$$\prod_{0, x\in R}^{1} (\sqrt{2} * \sin^{x}(\pi * x)) = \sqrt{2}$$

N	Approximation of Product (N subintervals)
10	1.41413562
100	1.41413563
500	1.41413544

$$\prod_{0,x\in R}^{1} e^{\frac{-\pi^2}{12}} * (1+x)^{\frac{1}{x}} = 1$$

N	Approximation of Product (N subintervals)
10	0.9987238865
100	0.9998721676
1000	0.9999873469

$$\prod_{0,x\in R}^{1} (2*\sin(\pi*x/2)) = \sqrt{2}$$

N	Approximation of Product (N subintervals)
1	1.414213562
2	1.414213562
3	1.414213562
10	1.414213562
100	1.414213563
200	1.414213564

$$\prod_{0,x\in R}^{1} e^{\frac{-\pi}{2}} * (1+\cos(\pi*x))^{\frac{1}{\cos(\pi*x)}} = \frac{1}{2}$$

N	Approximation of Product (N subintervals)
2	0.5227292201
10	0.501113427
100	0.5000113239
200	0.500002975
300	0.5000017122

$$\prod_{0, x\in R}^{1} x\left[(2-x)^{\frac{12}{\pi^2(1-x)}}\right] = \sqrt{2}$$

N	Approximation of Product (N subintervals)
10	1.406151846
100	1.413404699
200	1.413809068
400	1.414011295

$$\prod_{0, x\in R}^{1} (2*\sin(\pi * x)) = 2$$

N	Approximation of Product (N subintervals)
10	1.999999999
100	2.000000002
1000	2.000000017