Question 416 : Pi , Integral Representations

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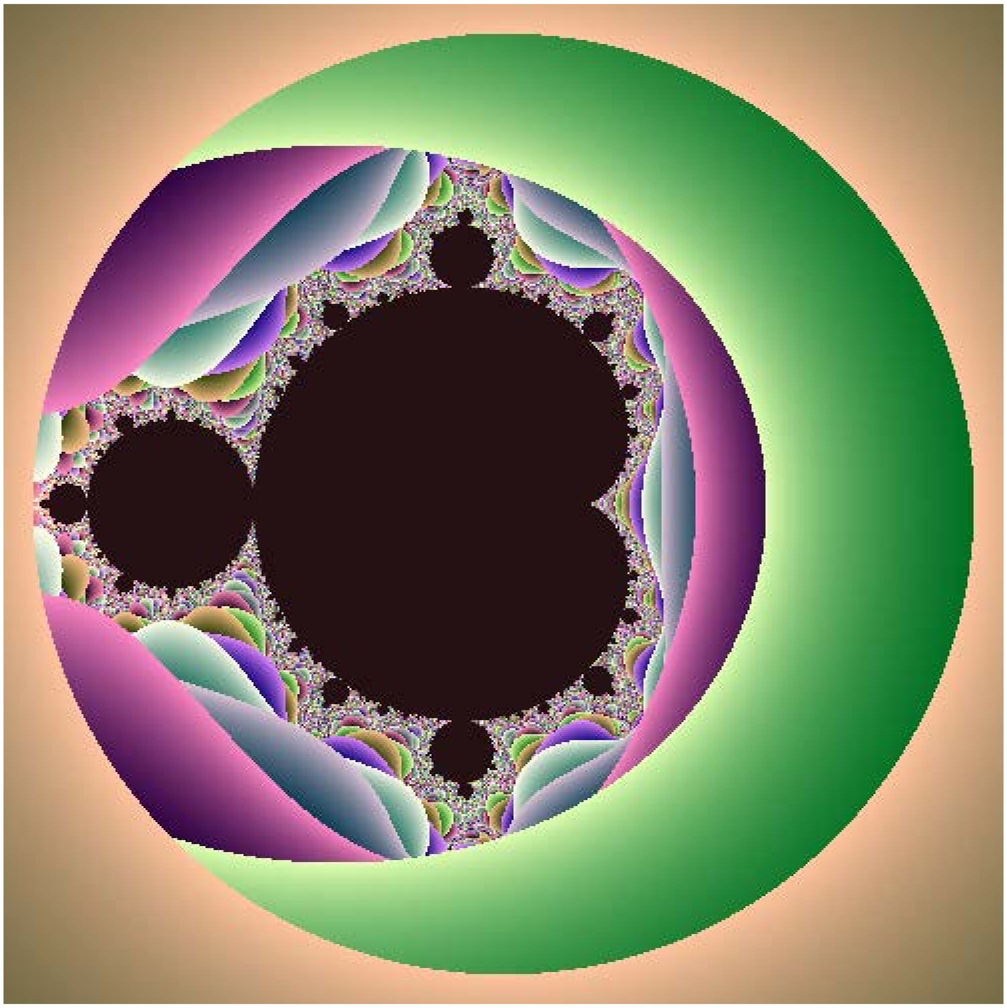
Abstract
This note presents some elementary integrals for pi.



Mandelbrot set , B.B. Mandelbrot , 1924 - 2010

Pi , Integral Representations

$$\pi=\frac{16}{\sqrt{3}}\int_0^{2\sqrt{3}/9}\left(\sqrt{1-2\cos\left(\frac{2\pi+\cos^{-1}f(x)}{3}\right)}-\sqrt{1-2\cos\left(\frac{4\pi+\cos^{-1}f(x)}{3}\right)}\right)dx \qquad (1)$$

$$f(x)=\frac{27x^2-2}{2}$$

$$\pi=\frac{8\sqrt{2}}{9}\int_{-1}^{1}\left(\sqrt{1-2\cos\left(\frac{2\pi+\cos^{-1}x}{3}\right)}-\sqrt{1-2\cos\left(\frac{4\pi+\cos^{-1}x}{3}\right)}\right)\frac{dx}{\sqrt{1+x}} \qquad (2)$$

$$\pi=\frac{8\sqrt{2}}{9}\int_{-1}^{1}\left(\sqrt{1-2\cos\left(\frac{5\pi-2\sin^{-1}x}{6}\right)}-\sqrt{1-2\cos\left(\frac{9\pi-2\sin^{-1}x}{6}\right)}\right)\frac{dx}{\sqrt{1+x}} \qquad (3)$$

$$\pi=\frac{8\sqrt{2}}{9}\int_0^{\pi}\left(\sqrt{1-2\cos\left(\frac{2\pi+x}{3}\right)}-\sqrt{1-2\cos\left(\frac{4\pi+x}{3}\right)}\right)\sqrt{1-\cos x}\,dx \qquad (4)$$

$$\pi=\frac{8\sqrt{2}}{9}\int_{-\pi/2}^{\pi/2}\left(\sqrt{1-2\cos\left(\frac{5\pi-2x}{6}\right)}-\sqrt{1-2\cos\left(\frac{9\pi-2x}{6}\right)}\right)\sqrt{1-\sin x}\,dx \qquad (5)$$

$$\pi=\frac{16}{9}\int_0^{\pi}\left(\sqrt{1-2\cos\left(\frac{2\pi+x}{3}\right)}-\sqrt{1-2\cos\left(\frac{4\pi+x}{3}\right)}\right)\sin\left(\frac{x}{2}\right)dx \qquad (6)$$

$$\pi=\frac{16}{3}\int_0^{\pi/3}\left(\sqrt{1+\cos x+\sqrt{3}\sin x}-\sqrt{1+\cos x-\sqrt{3}\sin x}\right)\sin\left(\frac{3x}{2}\right)dx \qquad (7)$$

$$\pi=\frac{32\sqrt{2}}{3}\int_0^{\pi/6}\left(\sqrt{1+\sqrt{3}\tan x}-\sqrt{1-\sqrt{3}\tan x}\right)\cos x\sin(3x)\,dx \qquad (8)$$

$$\pi=32\sqrt{2}\int_0^{1}\left(\sqrt{1+x}-\sqrt{1-x}\right)\frac{(9x-x^3)}{(3+x^2)^3}dx \qquad (9)$$

$$\pi = 32\int_0^{\pi/2}\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\frac{\left(9-(\cos x)^2\right)}{\left(3+(\cos x)^2\right)^3}\sin(2x)\,dx \tag{10}$$

$$\pi = \frac{8\sqrt{2}}{9}\int_{1/2}^{1}\left(\sqrt{1+x+\sqrt{3}\sqrt{1-x^2}}-\sqrt{1+x-\sqrt{3}\sqrt{1-x^2}}\right)\frac{\left(12x^2-3\right)}{\sqrt{1-3x+4x^3}}\,dx \tag{11}$$

$$\pi = \frac{8\sqrt{2}}{9}\int_0^{\sqrt{3}/2}\frac{\left(\sqrt{1+\sqrt{3}\,x+\sqrt{1-x^2}}-\sqrt{1-\sqrt{3}\,x+\sqrt{1-x^2}}\right)\left(9x-12x^3\right)}{\sqrt{\left(1-x^2\right)\left(1+\left(1-4x^2\right)\sqrt{1-x^2}\right)}}\,dx \tag{12}$$

References

1. Gradshteyn, I.S. and Ryzhik, I.M.: Table of Integrals, Series, and Products, seventh edition. Edited by Alan Jeffrey and Daniel Zwillinger, Academic Press, 2007.
2. Olver, F.W.J., Lozier, D.W., Boisvert, R.F., and Clark, C.W.: NIST Handbook of Mathematical Functions. Cambridge University Press, 2010.