

From: Sanders, S. 2017. Grilliot's trick in Nonstandard Analysis. [arXiv:1706.06663v1](https://arxiv.org/abs/1706.06663v1)

We evaluate two of the simpler equations for tautology which relate to Grilliot's trick and the standard extensionality trick via effective implication.

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. The designated *proof* value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	$p=p$	T	Tautology	proof	11	3
2	$p@p$	F	Contradiction	absurdum	00	0
3	$\%p>\#p$	N	Non-contingency	truth	01	1
4	$\%p<\#p$	C	Contingency	falsity	10	2

LET: & And; > Imply \rightarrow ; # \forall all; % \exists some
 LET: p q r s t uc_Phi Φ uc_Psi Ψ A B t

Results are proof table(s) of 16-values in row major horizontally (as fragments if repeated).

$$\text{"An implication } (\exists\Phi)A(\Phi) \rightarrow (\exists\Psi)B(\Psi) \text{ (proved in } RCA_0 \text{) ..."} \tag{3.1.1}$$

$$(\%p\&(r\&p)) > (\%q\&(s\&q)) ; \quad TTTT \ TFTF \ TTTT \ TFTT \tag{3.1.2}$$

$$\text{"... additionally } (\forall\Phi)[A(\Phi) \rightarrow B(t(\Phi))] \text{ (proved in } RCA_{\omega_0}) \tag{3.1.3}$$

$$(\#p\&(r\&p)) > (\#p\&((s\&r)\&p)) ; \quad TTTT \ TCTC \ TTTT \ TTTT \tag{3.1.4}$$

Definition 3.1. [Effective implication] An implication $(\exists\Phi)A(\Phi) \rightarrow (\exists\Psi)B(\Psi)$ (proved in RCA_0) is effective if there is a term t (in the language of RCA_{ω_0}) such that additionally $(\forall\Phi)[A(\Phi) \rightarrow B(t(\Phi))]$ (proved in RCA_{ω_0}). (3.1.5)

$$((\#p\&(r\&p)) > (\#p\&((s\&t)\&p))) > ((\%p\&(r\&p)) > (\%q\&(s\&q))) ; \tag{3.1.6}$$

TTTT TNTN TTTT TNTT, TTTT TNTN TTTT TFTT

Eqs. 3.1.2, 3.1.4, and 3.1.6, as rendered, are *not* tautologous.

According to VŁ4, this means Grilliot's trick, effective implication, and the subsequent non standard extensionality trick are *not* bivalent, but rather are a *vector space*.