

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VL4. Meth8 allows to mix four logical values with four analytical values. The designated *proof* value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	$p=p$	T	Tautology	proof	11	3
2	$p@p$	F	Contradiction	absurdum	00	0
3	$\%p>\#p$	N	Non-contingency	truth	01	1
4	$\%p<\#p$	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to;
 @ Not equivalent to; # all; % some; (p@p) 00, zero; (p=p) 11, one

Results are the proof table of 16-values in row major, horizontally.

We define:

"a sentence" as p (0.0)
 p ; FTFT FTFT FTFT FTFT (0.1)

We assert for clarity an expression cast in the positive using *as* for a fragment of implication, instead of *is* for a sentence of equivalency, and injecting the modal operator of necessity:

"The necessity of 'This sentence as a proof.'" (1.1)
 $\#(p > (p=p)) ;$ NNNN NNNN NNNN NNNN (1.2)

Systems of two- or three-valued logic are insufficient to capture the complete informational content of Eq. 1.1 for subsequent discourse. We also avoid testing the more complicated instance forced by assignment of Eq. 1.1 to another variable by injecting the modal operator of possibility:

"Possibly a sentence implies the necessity of 'This sentence as a proof.'" (2.1)
 $\%p > \#(p > (p=p)) ;$ NNNN NNNN NNNN NNNN (2.2)

This means Eq. 2.1 is an axiom with a truth value of N for non-contingency (as opposed to a falsity value of C for contingency), but not a theorem with truth value of T for tautology. This contradicts Gödel's theorem of incompleteness, where Eq. 2.2 should a refutation with truth value of F for contradiction.

We test the common contra-example for 'This sentence as not a proof'. We rewrite Eqs. 1.1-2.2:

"The necessity of 'This sentence as not a proof.'" (3.1)
 $\#(p > \sim(p=p)) ;$ NFNF NFNF NFNF NFNF (3.2)

"Possibly a sentence implies the necessity of 'This sentence as not a proof.'" (4.1)
 $\%p > \#(p > \sim(p=p)) ;$ NFNF NFNF NFNF NFNF (4.2)

This means Eq. 4.2 is not an axiom or a theorem. This contradicts Gödel's theorem of incompleteness, where Eq. 4.2 should be a theorem with truth value of T for tautology.

Remark: In quantified terms, Eqs. 2.1 and 4.1 with the same results alternatively read:

"Some sentence implies all instances of 'This sentence as a proof'." (5.1)

"Some sentence implies all instances of 'This sentence as not a proof'." (6.1)

Our examples show the shortest refutation for Gödel's incompleteness theorem.