Refutation of Chaitin's theorem of incompleteness

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to;
@ Not equivalent to; # all; % some; (p@p) 00, zero; (p=p) 11, one;
pqrs KLns

Results are the proof table of 16-values in row major horizontally.

We evaluate Chaitin's incompleteness theorem of 1974 from

wikipedia.org/wiki/Kolmogorov_complexity#Chaitin.27s_incompleteness_theorem .

Martin Davis described it as "*a dramatic extension of Gödel's incompleteness theorem*" (Davis, 1978).

"Theorem: There exists a constant L ... such that there does not exist a string s for which the statement

(K(s) ≥ L) (as formalized in S) [This is equivalent to ~(K(s) < L).] (0.1)

can be proven within the axiomatic system S. Note that, by the abundance of nearly incompressible strings, the vast majority of those statements must be true. (1.1)

The proof is by contradiction. It the theorem were false [not a proof] then the following is a proof [tautology]:

Assumption (X): For any integer n there exists a string s for which there is a proof in [logic system] S of the expression "(K(s)≥ L)". (S is assumed to enumerate all formals proofs of S.) (2.1)

We render Eq. 0.1 as:

~((p&s)<q) ; `TTTT TTTT TFTT TFTT` (0.2)

Eq. 0.2 means that "~(K(s)<L) (as formalized in S)" is already not a proof (not a tautology) but is also is not a contradiction because the F value of contradiction is mixed twice into the resulting proof table.

Remark: Eq. 0.2 implies that Chaitin's constant L is suspicious.

We render Eq. 1.1 as:

%q>((~((p&s)<q)=(s=s))>~~%s) ; NNNN NNNN NTFF NTFF (1.2)

Eq. 1.2 means the theorem is not a tautology, and *not* a contradiction, with the proof table of a mixture of values for F, N, and T.

The refutation of the theorem could end here, however for to be comprehensive we continue the approach of the argument and render Eq. 2.1 as:

#r&(%s>(~((p&s)<r)>(s=s))) ; FFFF NNNN FFFF NNNN (2.2)

Eq. 1.2 means that Assumption (X) is not a contradiction because of the N value of truth mixed into the resulting proof table.

In an attempt to resuscitate Eq. 1.2, we rewrite it by distributing the universal quantifier over the antecedent and consequent as:

(#r&%s) > (#r&(~((p&s)<r)>(s=s))) ; TTTT TTTT TTTT TTTT (2.3)

In this case, Eq. 2.3 shows Assumption(X) is a proof, and therefore Eq. 1.1 should be a contradiction. However, we already showed Eq. 1.2 is *not* a contradiction, but rather contains some T value of tautology mixed with some F value of contradiction.

In either case of Eq. 0.2 with Eq. 2.2 or with Eq. 2.3, the approach of the conjecture is moot, and Chaitin's theorem of incompleteness is refuted.

Reference:

David, M. (1978). "What is a computation?". Steen, L.A. (ed.) Mathematics Today, Twelve informal essays. Springer. 1978. pp. 241/267. DOI: 10.1007/978-1-4613-9435-8_10.