

From:

Rin, B.G.; Walsh, S. (2016). Realizability semantics for quantified modal logic: generalizing Flagg’s 1985 construction. arxiv.org/pdf/1510.01977.pdf

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proof* value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	$p=p$	T	Tautology	proof	11	3
2	$p@p$	F	Contradiction	absurdum	00	0
3	$\%p>\#p$	N	Non-contingency	truth	01	1
4	$\%p<\#p$	C	Contingency	falsity	10	2

LET: \sim Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to;
 @ Not equivalent to; # all \forall ; % some \exists ($p@p$) 00 zero; ($p=p$) 11 one

Results are the repeating proof table(s) of 16-values in row major horizontally.

"The resulting semantics generalize the important but little-understood construction of Flagg (1985), whose goal was to provide a consistency proof of Epistemic Church’s Thesis together with epistemic arithmetic, a modal rendition of first-order arithmetic. *Epistemic Church’s Thesis* (ECT) is the following statement:

$$(1.1) [\Box (\forall n \exists m \Box \phi(n,m))] \Rightarrow [\exists e \Box \forall n \exists m \exists q (T(e,n,q) \wedge U(q,m) \wedge \Box \phi(n,m))]" \quad (1.1)$$

LET: pqtuwxy pqtuemn

$$\begin{aligned} & \#((\#y\&\%x)\&(\#p\&(y\&x))) > \\ & ((\%w\&\#(\#y\&(\%x\&(\%x\&\%q))))\&(((t\&(w\&(y\&q)))\&(u\&(q\&x)))\&(\#p\&(y\&x))))); \\ & TTTT TTTT TTTT TTTT, TCTC TCTC TCTC TCTC, TCTT TCTT TCTT TCTT \end{aligned} \quad (1.2)$$

"EZF ... is built from $Q_{eq}.S4$ by the addition of the following axioms: ...

$$\text{II. Induction Schema: } [\forall x((\forall y \in x \phi(y)) \Rightarrow \phi(x))] \Rightarrow [\forall x \phi(x)]" \quad (2.1)$$

LET: pxy ϕ xy

$$\begin{aligned} & ((\#x\&\#y)\&\#(\#y < (x\&(p\&y)))) > (\#x\&(p\&x)); \\ & TTTT TTTT TTTT TTTT, CTCT CTCT CTCT CTCT \end{aligned} \quad (2.2)$$

$$\text{"III. Scedrov’s Modal Foundation: } [\Box \forall x(\Box(\forall y \in x \phi(y)) \Rightarrow \phi(x))] \Rightarrow [\Box \forall x \phi(x)]" \quad (3.1)$$

We distribute the quantification in the antecedent to ensure clarity.

$$\begin{aligned}
 & ((\#x\&\#y)\&\#(\#y\langle(x\&(p\&y))) \rangle) > ((\#x\&\#y)\&\#(\#x\&(p\&x))) ; \\
 & \text{TTTT TTTT TTTT TTTT, CTCT CTCT CTCT CTCT}
 \end{aligned}
 \tag{3.2}$$

Eqs. 1.2, 2.2, and 3.2 as rendered are *not* tautologous.

We did not test subsequent axioms.

This means respectively that the following are not theorems: Epistemic Church's Thesis; EZF induction schema; and Scedrov's modal foundation.

What follows is that Flagg's construction, Goodman's intensional set theory, and epistemic logic are suspicious.