A new concept for simultaneity in the STR (v2)

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Abstract. Simultaneity is a key concept in the special theory of relativity (STR), when two reference frames (RFs) are moving relative to each other at constant speed, *v*. We introduce a new approach to simultaneity and its relation to the clock reading and position along the *x* –axis. The two clocks at the origins play a crucial role. We synchronize these at time 0, and refer to this event as the 'point of initiation'. From symmetry reasons, we conclude that identical readings of these two 'basic clocks' – at least in some sense – represent simultaneity. The second element of our approach is a new version of the Lorentz transformation. We combine the two (clock, space) variables of both RFs at a specific position, giving a state variable of two orthogonal components. The absolute value of this two-dimensional time vector provides a measure for 'distance in time' from the 'point of initiation'. We utilise this distance to specify simultaneity of events; *i.e.* time vectors having same absolute value (='total distance') will correspond to simultaneous events. We can also give the time vector as a complex variable, where the real part equals the clock reading of the 'basic clocks' at the relevant position. The given description of simultaneity is well suited to analyse and conclude regarding the travelling twin case.

Key words: Time dilation, simultaneity, symmetry, Lorentz transformation, time vector, travelling twin.

1 Introduction

When reference frames (RFs) are moving relative to each other, the definition of *simultaneity* becomes crucial. Of course, we have the 'local simultaneity'; *i.e.* simultaneity of events occurring at the same instant *and* same location, but these are rather the same event, just seen in the perspective of two different RFs. For events at a distance, we can essentially observe simultaneity from the 'perspective' of a certain RF. When the synchronized clocks of a specific RF show the same readings, we have simultaneous events *in the perspective of* this RF.

In general, one may define simultaneity by use of light rays; *e.g.* see standard textbooks, [1], [2], but it seems not to be a unique definition of distant simultaneity. Here we present an approach - based on [3], [4] - with two main elements:

- There is a clock at the origins of each RF. We synchronized these 'basic clocks' at time zero (when they are at the same location); and due to symmetry, they remain synchronized. So we utilise the 'basic clocks' to define distant simultaneity.
- As we consider just one space coordinate, the two RFs will together have four (clock, space) variables at a specific location. These are related through the Lorentz transformation (LT). Here we combine these four variables in a new way and obtain two state vectors, related by an orthogonal transformation; *i.e.* a version of the LT.

Then we specify time as a two-dimensional vector; thus, pointing to the important distinction between time and clock reading. We can formulate this vector as a complex variable; and then the magnitude represents the 'distance' from the initial synchronization at time 0; *i.e.* from the 'point of initiation'. Events with the same magnitude for this time variable implies simultaneity. The real part of the time variable equals the clock reading of a 'basic clock' made available for the event. The imaginary part represents the distance in space from the 'point of initiation'- measured by the time required for a light signal. The given approach directly provides a tool to handle the travelling twin case, and we present a solution to this paradox.

2 Foundation

We here present some basic assumptions, and give both the standard and a new version of the Lorentz transformation. Throughout we restrict to consider one space coordinate.

2.1 Basic assumptions and notation

We start out with a reference frame (RF), K_0, where the position along the x-axis is denoted x_0 and the clock reading (of any of its synchronized clocks) is denoted, t_0.

Further, there is a reference frame, K_v (with the same orientation), moving along the x-axis of K_0 at velocity v. So K_0 is moving along the x-axis of K_v at velocity $-v$. On K_v we have

x_v = The position (along the x-axis), that corresponds to (has same location as) x_0 when the clock reading on K_0 equals t_0.

t_v = Clock reading at position x_v on K_v at the instant when x_v corresponds to x_0.

Observers (observational equipment) on both of these two RFs agree on these four observations. Further

- There is a complete *symmetry* between the two reference frames K_0 and K_v; the frames being identical in all respects.
- The clock at $x_v = 0$ and the clock at $x_0 = 0$ will when $t_v = t_0 = 0$ be at the same location, and they are then synchronized. We refer to this as the 'point of initiation', and the clocks as 'basic clocks'.
- We will choose the *perspective* of one RF - here K_0 - and refer to this as the *primary* system. The clock reading on this primary RF is at any position, x_0 given as a constant, t_0. In contrast, at a certain time, t_0 on the primary system, the observation, t_v on the other ('secondary') RF will depend on the location where the clock reading is carried out.

2.2 The Lorentz transformation (LT) and time dilation

The LT represents the fundament for our discussion of time dilation. From the above notation the LT takes the form

$$t_v = \frac{t_0 - \frac{v}{c^2}x_0}{\sqrt{1-(\frac{v}{c})^2}} \tag{1}$$

$$x_v = \frac{x_0 - vt_0}{\sqrt{1-(\frac{v}{c})^2}} \tag{2}$$

Two special cases are of particular interest. Recall that the first clock comparison is carried out at the at origins $x_v = x_0 = 0$ when $t_v = t_0 = 0$. Now we consider two specific choices for time and location for a second time comparison.

First we compare the clock located at $x_v = 0$ on K_v with the passing clocks on K_0, showing time t_0. This clock on K_0 must according to (2) have position $x_0 = vt_0$, and (1) gives the relation between the two clock readings at this position. In summary, for $x_v = 0$, (*i.e.* follow the basic clock at the origin of K_v) we get:

$$t_v = t_0\sqrt{1-(v/c)^2} \tag{3}$$

This equals the standard 'time dilation formula'. Secondly, we can compare the clock located at $x_0 = 0$ on K_0 with a passing clock on K_v (actually at position $x_v = -vt_v$). Thus, for $x_0 = 0$, (*i.e.* follow the basic clock at the origin of K_0) we get

$$t_v = t_0 / \sqrt{1-(v/c)^2} \tag{4}$$

as the relation between t_0 and t_v. These relations, (3), (4) are apparently contradictory; *eq.* (3) tells that the clock on K_v goes slower, and (4) tells that the clock on K_0 goes slower. Thus, the time dilation is not a feature of the RF, but follows from which single clock we choose to follow when we perform the second clock comparisons. Therefore, it is often advantageous to formulate the time dilation formulas (3), (4) in compact form as

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2} \tag{5}$$

Here

t^{SC} = The clock reading on a single clock (SC), used both at the first and the second clock comparison; (*i.e.* the second observation of a clock located at the origin of one of the RFs).

t^{MC} = The clock reading on another clock than the one applied at the point of initiation; (*i.e.* clock reading on a RF using multiple clocks (MC) for this particular clock comparison.

Therefore, as seen from (3), (4), both of the RFs can apply a SC for a certain clock comparison, and thus conclude that it is a system where 'time goes slower'. However, the same RF can also apply MC for another clock comparison and then the conclusion would that 'time goes faster' on this RF. It is the observational principle, *i.e.* choice of clocks for the clock comparisons that matters.

2.3 An alternative formulation of the Lorentz transformation (LT)

We now proceed to replace v in the LT, (1), (2) with an angle, θ_v, given by

$$\sin\theta_v = v/c \tag{6}$$

implying that

$$\cos\theta_v = \sqrt{1-(\tfrac{v}{c})^2}.$$

Now the LT (1), (2) can be formulated as:

$$t_v \cdot \cos\theta_v = t_0 - (x_0/c)\cdot\sin\theta_v \tag{7}$$

$$x_v/c \cdot \cos\theta_v = x_0/c - t_0 \cdot \sin\theta_v \tag{8}$$

Thus, some rotation of the (time, space) coordinates is involved here. As we restrict to consider one space coordinate the LT involves just four state variables, t_0, x_0, t_v and x_v. If we now specify any two of these four variables, the other two will be given by the LT.

In particular, the standard version of the LT gives (t_v, x_v) expressed by (t_0, x_0), or *vice versa*. But similarly, we could reformulate the LT to give a relation between (t_0, t_v) and (x_0, x_v). And – as a third possibility – we can formulate the LT as a relation between (t_0, x_v) and (t_v, x_0). In the present work we follow up on this third possibility. First, by combining (7) and (8), we can replace (7) by

$$t_v = t_0\cos\theta_v - (x_v/c)\cdot\sin\theta_v \tag{9}$$

Now (8) and (9) give a new version of the LT, which in matrix form becomes

$$\binom{t_v}{x_0/c} = A_v\binom{t_0}{x_v/c} \tag{10}$$

Here the transformation matrix,

$$A_v = \begin{bmatrix}\cos\theta_v & -\sin\theta_v\\ \sin\theta_v & \cos\theta_v\end{bmatrix} \tag{11}$$

is orthogonal as

$$A_v^{-1} = A_v^{T} = \begin{bmatrix}\cos\theta_v & \sin\theta_v\\ -\sin\theta_v & \cos\theta_v\end{bmatrix}$$

Next we introduce the two 'time vectors'

$$\vec{t_1} = \binom{t_0}{x_v/c} \tag{12}$$

$$\overrightarrow{t_2} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} \tag{13}$$

and then write the relation (10) as

$$\overrightarrow{t_2} = \overrightarrow{A_v t_1} \tag{14}$$

which we denote the orthogonal version of the Lorentz transformation. A nice feature of this formulation is that it represents a rotation, θ_v, of the $(t_0,\ x_v/c)$ plane, (with the components t_0 and x_v/c being orthogonal). Similarly, the vector $\overrightarrow{t_1} = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix}$ will be given by $\overrightarrow{t_2} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix}$, using the same rotation in opposite direction, *i.e.* replacing v by $-v$, (and θ_v, by -θ_v,) in A_v.

We stress that the vectors, $\overrightarrow{t_1}$ and $\overrightarrow{t_2}$ provide identical information. They define what we see as the same event, related both to K_0 and K_v. The first component for both of these vectors equals the clock reading at one of the RFs. The second component equals the position of the other RF divided by c for the event in question, (*i.e* the time for a light flash to go the distance from the origin). Therefore, both components of the vectors $\overrightarrow{t_1}$ and $\overrightarrow{t_2}$ represent time. We could say that both time vectors of the specified event represent aspects of the ‘distance’ in time, as measured from the ‘point of initiation’.

We also observe that the present approach has some similarities with the Minkowski spacetime; however, the two approaches do indeed differ.

3 Simultaneity and the time vector

Within the framework of STR it seems most common to verify simultaneity across reference frames by use of light rays. This differs from the approach presented here.

3.1 Concepts of simultaneity

It hardly exists a concept of ‘global’ simultaneity within the STR. However, there are some types of simultaneity with limited applicability. First, we can refer to so-called *local simultaneity* of events, meaning that the events occur at the same instant *and* the same location. Actually, we may consider this to be the same event, just expressed by variables of different RFs. This local simultaneity occurs even when the relevant clocks on location give different time readings. So it is obvious that having events with identical clock readings is not the same thing as having simultaneous events! Thus, we should clearly distinguish between time and clock reading.

When the events occur at different locations one could refer to the weaker concept of *simultaneity by perspective.* One can say that events with the same clock reading (t) - as measured on a specific RF - are simultaneous in the perspective of this frame. However, the LT shows that this simultaneity depends on chosen RF.

Now in [3] we used an auxiliary reference frame as a tool to obtain such a simultaneity by perspective, and that proved useful. We simply postulate an auxiliary RF with origin always located at the midpoint between our two ‘basic’ RFs. Further, we utilize the symmetry, and argue that simultaneous clock readings of the auxiliary RF provides a valid symmetry for the two basic RFs. So this symmetry implies a certain ‘true’ simultaneity at a distance for the two ‘basic’ RFs. In particular, we strongly argued that this approach provides a solution to the travelling twin example.

Here we will pursue a slightly different approach based on the same symmetry, and also utilizing the discussions of Chapter 2. First we now point out that an essential requirement for the use of the LT is that we start out with three sets of synchronizations.

- First, all clocks on the first RF, K_0;
- Next, all clocks on the second RF, K_v;
- Third, the two clocks at the origins at time 0, (representing *local simultaneity*).

Usually, one assumes that all clocks on K_0 *remain* synchronized; as do also the clocks on K_v. However, we will here argue that the two ‘basic clocks’ at the origins of K_0 and K_v - being synchronized at time 0 - will remain synchronized. They are moving away from each other at speed, v, but in our model of complete symmetry, there is no way to claim that one of the two clocks goes faster than the other. We have the standard phrase 'moving clock goes slower', but that is when the 'moving clock' is compared with passing clocks, and not with the other clock at the origin; *cf.* detailed discussions in [3].

So our claim is that when the two 'basic clocks' at the origin show the same time, this corresponds to simultaneous events 'at a distance'. This is actually a much stronger form of simultaneity, as all observers can agree on this. We discuss this further in the next section, and present the main result in Section 3.3.

3.2 Moving clocks and simultaneity

We now apply the general result of Section 2.3 for two specific positions; that is the current positions of the two 'basic clocks'. Position I equals the origin of K_0, *i.e.* the location $x_0 = 0$. Position II is at the other origin, $x_v = 0$ of K_v. At both positions, we specify four variables; see illustration in Fig. 1.

Position I: The origin, $x_0' = 0$ of K_0.
Here *eq.* (4) is valid, and we specify the four variables at this position by the two vectors (see (12), (13)):

$$\vec{t_1'} = \begin{pmatrix} t_0' \\ x_v'/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ -v/c \end{pmatrix} t_v' = \begin{pmatrix} \cos\theta_v \\ -\sin\theta_v \end{pmatrix} t_v' \tag{15}$$

$$\vec{t_2'} = \begin{pmatrix} t_v' \\ x_0'/c \end{pmatrix} = A_v\, \vec{t_1'} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_v' \tag{16}$$

Position II: The origin, $x_v'' = 0$ of K_v.
Here *eq.* (3) is valid, and we specify the four variables at this position by the two vectors

$$\vec{t_1''} = \begin{pmatrix} t_0'' \\ x_v''/c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_0'' \tag{17}$$

$$\vec{t_2''} = \begin{pmatrix} t_v'' \\ x_0''/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ v/c \end{pmatrix} t_0'' = A_v\, \vec{t_1''} = \begin{pmatrix} \cos\theta_v \\ \sin\theta_v \end{pmatrix} t_0'' \tag{18}$$

First, we observe that the time vectors of (15), (16) are essentially identical to the two time vectors (17), (18); (just interchanging their order). There is a minor asymmetry here, as the position, $x_v' = -vt_v'$ in *eq.* (15), whilst the position $x_0'' = vt_0''$ (without the minus sign) in *eq.* (18). However, this difference appears since we here let K_0 and K_v have the *same* direction. Therefore, the clock at the origin of K_0 moves along the negative axis of K_v, (thus, $x_v' < 0$); while the origin of K_v moves along the positive axis of K_0, (thus, $x_0'' > 0$); but actually there is a complete symmetry here.

Next consider simultaneity. The two events (t_0', x_0') and (t_v', x_v') at Position I represent 'local simultaneity', *i.e.* they have 'same location, same time' (and are essentially identical). This is also the case for the two events (t_0'', x_0'') and (t_v'', x_v'') at Position II.

However, we will now also specify the events at Positions I and II in a symmetric way, and thus obtain simultaneity for all four events. So we assume that the two clocks at the origins show the same time at their respective positions, and let

$$t_0' = t_v'' \tag{19}$$

Then we will have the symmetric situation introduced in Section 3.1, and due to this symmetry we get that all four events are simultaneous (at least in some sense). Since also $x_0' = x_v'' = 0$, it further follows that $t_0'' = t_v'$. Then all four time vectors (15)-(18) can be expressed by t_0''. By further letting Position II represent an arbitrary position we also drop the double superscript; see illustration in Fig. 2. Also dropping superscripts for the time vectors, the expressions (15) - (18) for the symmetric situation become

$$\vec{t_1} = \begin{pmatrix} t_v \\ -x_0/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ -v/c \end{pmatrix} t_0 \tag{20}$$

$$\vec{t_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_0 \tag{21}$$

$$\vec{t_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_0 \tag{22}$$

$$\vec{t_2} = \binom{t_v}{x_0/c} = \binom{\sqrt{1-(v/c)^2}}{v/c} t_0 \tag{23}$$

At a given position it is sufficient to provide either $\vec{t_1}$ or $\vec{t_2}$ - then the other vector is given by the orthogonal LT. Also recall that in the present approach we apply the 'trick' to define the time vector by combining the clock reading at one RF with the position on the 'other' RF at the same location.

The clock reading, t_0 plays a special role in the time vectors. Here $t_0\sqrt{1-(v/c)^2}$ is the clock reading of a basic clock; and t_0 is the clock reading for the clock on the same location but on the other RF.

3.3 The general time vector

In the discussions of the previous section, we focused on the full symmetry between the two RFs, *cf. eq.* (19). However, we now proceed to define a time vector for a specific RF, and will thus analyze the situation in the perspective of this RF. We choose K_0 as this primary RF, and now specify an arbitrary event (t_0, x_0) on this RF; that is t_0 = clock reading at the position x_0. Next, we define the velocity

$$v = x_0/t_0$$

We further introduce another RF, K_v, moving relative to K_0 at this specified velocity, v. As before, this K_v has a clock at its origin, which at the point of initiation was synchronized with the clock on K_0. According to our choice of velocity, v, this 'basic clock' on K_v will at time t_0 be located at the position x_0 relative to K_0. Note that it is irrelevant whether this K_v really exists or is just an imagined RF!

In any case, this imagined clock at the origin of K_v would show the reading $t_0\sqrt{1-(v/c)^2}$ (*cf.* (3)), and following the discussion of sections 2.3 and 3.2, we can now define the following time vector for the event (t_0, x_0) on K_0:

$$\vec{t}(t_0, x_0) = \binom{\sqrt{1-(v/c)^2}\, t_0}{x_0/c} = \binom{\sqrt{t_0^2-(x\ \ /c)^2}}{x_0/c} \tag{24}$$

By our choice of v can also write this time vector as

$$\vec{t}(t_0, v) = \binom{\sqrt{1-(v/c)^2}}{v/c} t_0 \tag{25}$$

Here (20) - (23) come out as special cases. In particular (25) equals the $\vec{t_2}$ vector of K_0, given in (21), (23). It would also apply to K_v, as $\vec{t}(t_0, -v)$ equals the $\vec{t_1}$ vector of K_v given in (20), (22). Therefore, we have one single vector describing all the four events we considered in Section 3.2.

In summary, we interpret the two components of this time vector (24), (25) as follows:

- The first component, $\sqrt{1-(v/c)^2}\, t_0$ equals the clock reading of the 'basic' clock at the origin of the (imagined) RF, K_v, (now located at x_0 on K_0).
- The second component equals the relevant 'position', $x_0/c = (v/c)\, t_0$; measured as the time required for a light flash to go from the origin to this location.

It is implicit that the 'basic' clock on K_v was initially located at the same position as the origin of K_0, and was then synchronized with the clock on K_0. The absolute value of this time vector is independent of v and equals

$$\left|\vec{t}(t_0, v)\right| = t_0 \tag{26}$$

We note that this case of having the same absolute value equals the symmetric situation discussed In Section 3.2, where all the four relevant events are specified to be simultaneous; see Fig. 2. The two events at the origin of K_0 exhibit 'local simultaneity. This is also the case for the two events at the origin of K_v. Further, the when the clock readings at the origins are identical they represent simultaneity by symmetry ('distant simultaneity'). Therefore – as we also know from the LT - simultaneity can be present also when the involved clocks show different readings. So same clock reading should not be mixed up with simultaneity.

So as already indicated, rather than using clock readings, we will utilize the absolute value t_0 as a measure of simultaneity. Events with the same value for the absolute value of the time vector have the same 'distance' from the 'point of initiation', and are in this sense simultaneous. We note the restriction. The events are simultaneous *relative to the chosen point of initiation*.

3.4 Time formulated as a complex variable

We can also write the time vector at (t_0, x_0) on K_0 as a complex variable; written in polar form (25) becomes:

$$t(t_0, v) = t_0 e^{i\theta_v} \qquad (v = x_0/t_0)$$

As before, the magnitude, t_0 equals the clock reading at position, x_0. It also gives the distance from the point of initiation, and thus defines simultaneity of events. The argument, θ_v (*cf.* (6)) is given by the velocity, v relative to K_0 of an (imagined) 'basic' clock at the position in question. Having the argument, $\theta_v = 0$ corresponds to $v = 0$, and then the time variable becomes a real number.

The real part, $\mathrm{Re}(t(t_0,v)) = \sqrt{1-(v/c)^2}\,t_0 = \cos\theta_v\, t_0$, is the clock reading of the 'basic' clock at the position. The imaginary part, $\mathrm{Im}(t(t_0,v)) = (v/c)t_0 = \sin\theta_v t_0 = x_0/c$ equals the 'distance' from the 'point of initiation', measured by the time required for light to go this distance.

4 The travelling twin

As stated for instance in Mermin [2] the travelling twin paradox illustrates that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together. We will – as we also understand the approach of [2] – consider only the idealised, symmetric situation. Thus, we restrict to that part of the travel when TSR applies, *i.e.* no acceleration etc.; and therefore assume a complete symmetry between the two twins. We discussed the example at length in [3], and now just give a short comment.

We start out with two synchronized clocks at the origins of two reference frames; the RF of the earth, and the RF of the rocket of the travelling twin. This is exactly the situation described above. In our symmetry argument, we here conclude that the two clocks remain synchronized in the sense that identical readings of these imply simultaneity.

By the arrival of the twin to the 'star', both twins observe two clock readings, their own clock, and the clock on the other passing RF. The four (clock, space) observations at these two positions are identical, and at both places give rise to two 'time vectors', as discussed in Section 3.2.

Ch. 10 of Ref. [2], gives the following numerical example. The distance between the earth and the 'star' equals $x_0 = 3$ light years, and the rocket has speed, $v = (3/5)c$, giving $\sqrt{1-(v/c)^2} = 4/5$. It follows that in the RF of the earth/star, the rocket will reach the star at time, $t_0 = x_0/v = 5$ years. The clock on the rocket is located at $x_v = 0$, and at the arrival at the star this clock reads $t_v = t_0 \cdot \sqrt{1-(v/c)^2} = 4$ years.

Then ref. [2] – as seems common in the literature - claims that the travelling twin ages 4 years during 'the same time' as the earthbound twin ages 5 years. However, we conclude differently. We claim that this argument does not handle simultaneity properly. Of course it is correct that 'in the perspective' of one twin the other twin ages more slowly; but this argument is valid for both twins; *cf.* the so-called Dingle's question [5], [6]. As seen above, we will from symmetry reasons argue that the clock of the travelling twin, which shows 4 years (at his arrival to the star), occurs simultaneously with the event that the clock of the earthbound twin shows 4 years. However, when the twins compare their clock with the passing clocks on the other RF, they will *both* observe a time dilation. This will fully correspond to the length contraction they observe; actually being two sides of the same thing. In order to claim an asymmetric solution to this problem, one has to specify both the assumed asymmetry, and the model chosen to handle it. We further refer to the rather detailed discussion in [3].

5 Conclusions

The Lorentz transformation (LT) relates the (clock time, space) coordinates of one reference frame (RF) with the (clock time, space) coordinates at the same position on 'the other' RF. In the present work we define a state vector, which combines the clock reading of one RF with the space coordinate of the other RF. Thus, we obtain two vectors, related by an orthogonal version of the LT. We can interpret both components of these state vectors as aspects of time.

These vectors have nice features. They have orthogonal components and are related by an orthogonal transformation (rotation) in the two-dimensional time space. The first component of each vector equals a clock time. We can further interpret the absolute value as a measure of the overall distance in time from the 'point of initiation', (*i.e.* time 0 when the origins of the two RFs had a common location). We thus provide a means to define (a certain type of) simultaneity. That is: Events having time vectors with the same absolute value are simultaneous. As we know, simultaneity of events is not identical to having the same clock reading, and time is a much richer concept than given by clock readings alone.

The simultaneity result presented here is based on one fundamental assumption. Due to the inherent symmetry of the two RFs we claim that the two clocks at the origin of the two RFs at time 0 will remain 'synchronized'. We refer to these as 'basic clocks', and identical clock readings of these two symmetric clocks will represent simultaneity at a distance.

We further utilize this symmetry consideration in order to define the time vector of any 'stand alone' RF. For any event (t_0, x_0) on this RF, we just specify an 'imaginary' RF with its basic clock located at the position of the specified event, and then we utilize this clock reading to define the time vector.

We also present the time vector as a complex variable. The real part equals the clock time, and the imaginary part is a measure of the distance from the origin in terms of the speed of light. The magnitude still represents the 'distance' from the initial point of synchronization at time 0.

The given approach provides a conclusion regarding the travelling twin example. It seems that previous discussions of this case has not handled simultaneity properly.

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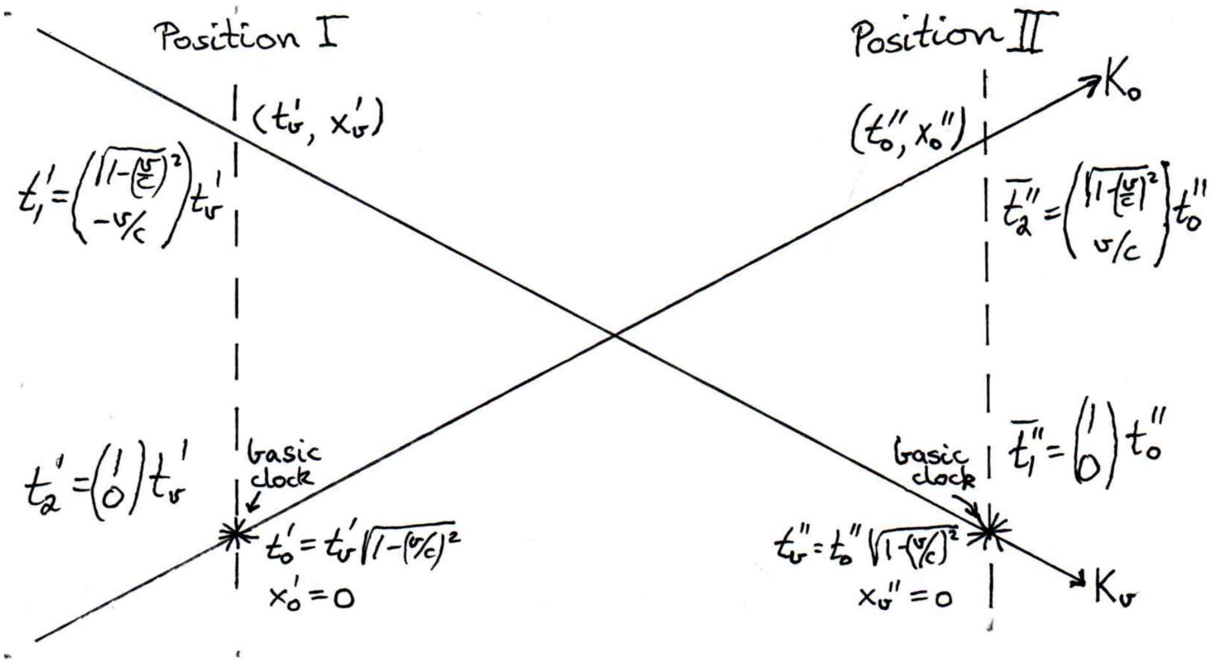

Figure 1 Illustrating observations and time vectors at the four events of Positions I and II

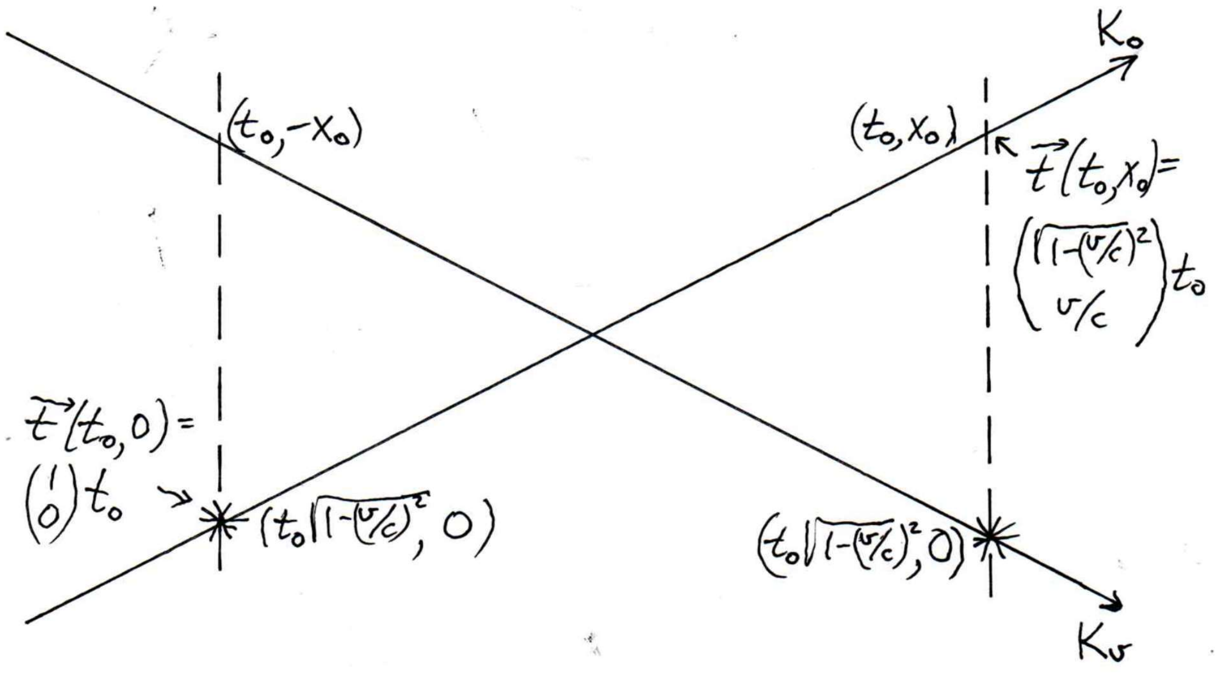

Figure 2 Restricting to the symmetric situation $t_0' = t_v''$ and $t_v' = t_0''$. Time vector $\vec{t}(t_0, x_0)$ for an arbitrary event (t_0, x_0) on K_0.