A new concept for simultaneity in the STR (v3)

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Abstract. Simultaneity is a key concept in the special theory of relativity (STR), considering two reference frames (RFs) moving relative to each other at constant speed, v. We introduce a new approach to simultaneity and its relation to the clock reading and position along the x –axis. The two clocks at the origins play a crucial role. We synchronize these at time 0, and refer to this event as the 'point of initiation'. Due to the symmetry we conclude that identical readings of these two 'basic clocks' represent a strong sense of simultaneity. The second element of the approach is a new version of the Lorentz transformation. We combine the two (clock, space) observations of each RF at a specific position, giving a time vector of two orthogonal components. The absolute value of this time vector provides a measure for 'distance in time' from the 'point of initiation', and events with the same 'distance' are simultaneous in some sense. We can further present the time vector as a comple x variable. The given description of simultaneity is well suited to analyse and conclude regarding the travelling twin case.

Key words: Time dilation, simultaneity, symmetry, Lorentz transformation, time vector, travelling twin.

1 Introduction

When reference frames (RFs) are moving relative to each other, the definition of *simultaneity* becomes crucial. Of course, we have the 'local simultaneity'; *i.e.* simultaneity of events occurring at the same instant *and* same location, but these are rather the same event, just seen in the perspective of two different RFs. For events at a distance, we can essentially observe simultaneity from the 'perspective' of a certain RF. When the synchronized clocks of a specific RF show the same readings, we have simultaneous events *in the perspective of* this RF.

In general, one may define simultaneity by use of light rays; *e.g.* see standard textbooks, [1], [2], but it seems not to be a unique definition of distant simultaneity. Here we present an approach - based on [3], [4] - with two main elements:

- There is a clock at the origins of each RF. We synchronized these 'basic clocks' at time zero (when they are at the same location); and due to symmetry, they remain synchronized. So we utilise the 'basic clocks' to define a distant simultaneity in this symmetric situation.
- When we consider just one space coordinate (x), the two RFs will together have four (clock, space) observations at a specific location. The Lorentz transformation (LT) provides a related between these. Here we combine these four variables in a new way and obtain two state vectors, related by an orthogonal transformation; *i.e.* a version of the LT. This provides an aid in the analysis of simultaneity.

We specify the two-dimensional state vector as a 'time vector'; depending on clock reading and position. We can also give it as a complex variable; where the magnitude represents the 'distance' from the initial synchronization at time 0; *i.e.* from the 'point of initiation'. The approach provides a tool to solve the travelling twin paradox.

2 Foundation

We here present some assumptions, and give both the standard and a new version of the LT.

2.1 Basic assumptions and notation

We start out with a reference frame (RF), K_0, where the position along the x-axis is denoted x_0. At virtually any position there are synchronized clocks with clock reading denoted, t_0.

Further, there is a reference frame, K_v (with the same orientation), moving along the x-axis of K_0 at velocity v. So K_0 is moving along the x-axis of K_v at velocity $-v$. On K_v we have

x_v = The position on K_v, being identical to the location x_0 on K_0 at a certain instant

t_v = Clock reading at position x_v on K_v, when x_v corresponds to x_0, and the clock on K_0 reads t_0.

Observers (observational equipment) on both of these two RFs agree on these four observations. Further,

- There is a complete *symmetry* between the two RFs K_0 and K_v; these being identical in all respects.
- The clock at $x_v = 0$ and the clock at $x_0 = 0$ will when $t_v = t_0 = 0$ be at the same location, and they are then synchronized. We refer to this as the 'point of initiation', and these clocks as 'basic clocks'.

2.2 The Lorentz transformation (LT) and time dilation

The LT represents the fundament for our discussion of time dilation. From the above notation the LT takes the form

$$t_v = \frac{t_0 - (v/c^2)x_0}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$x_v = \frac{x_0 - vt_0}{\sqrt{1-(v/c)^2}} \tag{2}$$

Two special cases are of particular interest. Recall that the first clock comparison is carried out at the origins $x_v = x_0 = 0$ when $t_v = t_0 = 0$. Now repeating arguments given in [3], [4], consider two specific choices for the time and location for a second time comparison.

First we compare the clock located at $x_v = 0$ on K_v with the passing clocks on K_0, showing time t_0. This clock on K_0 must according to (2) have position $x_0 = vt_0$, and (1) gives the relation between the two clock readings at this position. In summary, for $x_v = 0$, (*i.e.* follow the basic clock at the origin of K_v) we get:

$$t_v = t_0\sqrt{1-(v/c)^2} = \sqrt{t_0^2 - (x_0/c)^2} \tag{3}$$

This equals the standard 'time dilation formula'. Secondly, we can compare the clock located at $x_0 = 0$ on K_0 with a passing clock on K_v (actually at position $x_v = -vt_v$). Thus, for $x_0 = 0$, *i.e.* following the basic clock at the origin of K_0, we get the following relation between t_0 and t_v

$$t_0 = t_v\sqrt{1-(v/c)^2} = \sqrt{t_v^2 - (x_v/c)^2} \tag{4}$$

These relations, (3), (4) are apparently contradictory; *eq.* (3) tells that the clock on K_v goes slower, and (4) tells that the clock on K_0 goes slower. Thus, the time dilation is not a feature of the RF, but follows from which single clock we choose to follow when we perform the second clock comparisons. Therefore, it is often advantageous to formulate the time dilation formulas (3), (4) in compact form as

$$t^{BC} = t^{MC}\sqrt{1-(v/c)^2} \tag{5}$$

Here

t^{BC} = The clock reading of a basic clock (BC), *i.e.* clock located at the origin of a RF[1].

t^{MC} = The clock reading at the same location but on the other RF; *i.e.* clock reading on a RF using multiple clocks (MC) for clock comparisons with this basic clock.

Therefore, as seen from (3), (4), both of the RFs can apply a BC for a certain clock comparison, and then conclude that 'time goes slower' on this RF. However, the same RF would apply MC for the clock comparison with the basic clock on the other RF; and then we would conclude that 'time goes faster' on this RF. It is the observational principle, *i.e.* choice of clocks for the clock comparisons that matters.

[1] We have previously used t^{SC} (where SC = Single Clock) to denote this clock reading

2.3 An alternative formulation of the Lorentz transformation (LT)

We now proceed to replace v in the LT, (1), (2) with an angle, θ_v, given by

$$\sin\theta_v = v/c \tag{6}$$

implying that

$$\cos\theta_v = \sqrt{1-(v/c)^2}.$$

Now the LT (1), (2) can be formulated as:

$$t_v \cdot \cos\theta_v = t_0 - (x_0/c)\cdot\sin\theta_v \tag{7}$$

$$x_v/c \cdot \cos\theta_v = x_0/c - t_0 \cdot \sin\theta_v \tag{8}$$

As we restrict to consider one space coordinate the LT involves four state variables, t_0, x_0, t_v and x_v. If we specify any two of these four variables, the other two will be given by the LT. The standard version of the LT gives (t_v, x_v) expressed by (t_0, x_0), or *vice versa*. But similarly, we could reformulate the LT to give a relation between (t_0, t_v) and (x_0, x_v). And – as a third possibility – we can formulate the LT as a relation between (t_0, x_v) and (t_v, x_0). In the present work we follow up on this third possibility. First, by combining (7) and (8), we can replace (8) by

$$x_v/c = (x_0/c_0)\cdot\cos\theta_v - t_v\sin\theta_v \tag{9}$$

Now (7) and (9) give a new version of the LT, which we in matrix form can write[2]

$$\begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} = A_v \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} \tag{10}$$

Here the transformation matrix,

$$A_v = \begin{bmatrix} \cos\theta_v & \sin\theta_v \\ -\sin\theta_v & \cos\theta_v \end{bmatrix} \tag{11}$$

is orthogonal as

$$A_v^{-1} = A_v^{T} = \begin{bmatrix} \cos\theta_v & -\sin\theta_v \\ \sin\theta_v & \cos\theta_v \end{bmatrix}$$

Next we introduce the two 'time vectors' related to our RFs, K_0 and K_v

$$\vec{t}(v) = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} \tag{12}$$

$$\vec{t'}(v) = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} \tag{13}$$

and then we write the relation (10) as

$$\vec{t'}(v) = A_v \vec{t}(v) \tag{14}$$

which we denote the orthogonal version of the Lorentz transformation. A nice feature of this formulation is that it represents a rotation, θ_v, of the (t_v, x_0/c) plane, (with the components t_v and x_0/c being orthogonal). Similarly, the vector $\vec{t}(v) = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix}$ will be given by $\vec{t'}(v) = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix}$, using the same rotation in opposite direction, *i.e.* we replace v by $-v$, (and apply A_v^{T} rather than A_v. The vectors, $\vec{t}(v)$ and $\vec{t'}(v)$ provide identical information, as they define the same event, (relative to different coordinate systems).

For both time vectors, the first component equals the clock reading of a RF. The second component equals the position of the other RF for the event in question divided by c; so it equals the time for a light flash to go from the origin to this position. Therefore, both components of the vectors, $\vec{t}(v)$ and $\vec{t'}(v)$ represent time; what we could denote 'distance' in time, as measured from the 'point of initiation'.

[2] Again observe some change in the notation, as compared to previous versions

3 Simultaneity and the time vector

Within the framework of STR it seems most common to verify simultaneity across reference frames by use of light rays. This differs from the approach presented here.

3.1 Concepts of simultaneity

It hardly exists a concept of 'global' simultaneity within the STR. However, there are some types of simultaneity of limited applicability. First, we can refer to so-called *local simultaneity* of events, meaning that the events occur at the same instant *and* the same location. Actually, we may consider this to be identical events, just expressed by variables of different RFs. This local simultaneity occurs even when the relevant clocks on location give different time readings. So it is obvious that having events with identical clock readings is not the same thing as having simultaneous events, and we should clearly distinguish between time and clock reading.

When the events occur at different locations one could refer to the weaker concept of *simultaneity by perspective.* One can say that events with the same clock reading (t) - as measured on a specific RF - are simultaneous in the perspective of this frame. However, the LT shows that this simultaneity depends on chosen RF.

Now in [3] we used an auxiliary reference frame as a tool to obtain such a simultaneity by perspective, and that proved useful. We simply postulate an auxiliary RF with origin always located at the midpoint between our two 'basic' RFs. Further, we utilize the symmetry, and thus simultaneous clock readings of the auxiliary RF provides a valid symmetry for the two basic RFs. So this symmetry implies a certain 'true' simultaneity at a distance for the two 'basic' RFs. In particular, we strongly argued that this approach provides a solution to the travelling twin example.

Here we will pursue a slightly different approach based on the same symmetry, and also utilizing the discussions of Chapter 2. First we now point out that an essential requirement for the use of the LT is that we start out with three sets of synchronizations.

- First, all clocks on the first RF, K_0;
- Next, all clocks on the second RF, K_v;
- Third, the two clocks at the origins at time 0, (representing *local simultaneity*).

Usually, one assumes that all clocks on K_0 *remain* synchronized; as do also the clocks on K_v. However, we will here argue that the two 'basic clocks' at the origins of K_0 and K_v - being synchronized at time 0 - will remain synchronized. They are moving away from each other at speed, v, but in our model of complete symmetry, there is no way to claim that one of the two clocks goes faster than the other. We have the standard phrase 'moving clock goes slower', but that is when the 'moving clock' is compared with passing clocks, and not with the other clock at the origin; *cf.* detailed discussions in [3].

So our claim is that when the two 'basic clocks' at the origin show the same time, this corresponds to simultaneous events 'at a distance'. This is actually a rather strong form of simultaneity, as all observers can agree on this. We discuss this further in the next sections.

3.2 Moving clocks and simultaneity

We now apply the general result of Section 2.3 for two specific positions; that is the positions of the two 'basic clocks'. Position I equals the origin of K_0, *i.e.* the location $x_0 = 0$. Position II is at the other origin, $x_v = 0$ of K_v. At both positions, we specify four variables:

Position I; Events (t_0^I, x_0^I) and (t_v^I, x_v^I) are given as the origin of K_0, *i.e.* $x_0^I = 0$, (and so $x_v^I = -vt_v^I$ on K_v). Here *eq*. (4) is valid, and we specify the four variables at this position by our two vectors (12), (13):

$$\vec{t}(v) = \begin{pmatrix} t_v^I \\ x_0^I/c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_v^I \tag{15}$$

$$\vec{t'}(v) = A_v \vec{t}(v) = \begin{pmatrix} t_0^I \\ x_v^I/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ -v/c \end{pmatrix} t_v^I = \begin{pmatrix} \cos\theta_v \\ -\sin\theta_v \end{pmatrix} t_v^I \tag{16}$$

Position II; Events (t_0^{II}, x_0^{II}) and (t_v^{II}, x_v^{II}) are given as the origin, of $K_v, i.e. x_v^{II} = 0$ (and so $x_0^{II} = vt_0^{II}$). Here *eq*. (3) is valid, and we specify the four variables at this position by the two vectors

$$\vec{t}(v) = \binom{t_v^{II}}{x_0^{II}/c} = \binom{\sqrt{1-(v/c)^2}}{v/c} t_0^{II} = \binom{\cos\theta_v}{\sin\theta_v} t_0^{II} \tag{17}$$

$$\vec{t'}(v) = A_v\vec{t}(v) = \binom{t_0^{II}}{x_v^{II}/c} = \binom{1}{0} t_0^{II} \tag{18}$$

We observe that the time vectors of (15), (16) are essentially identical to the two time vectors (17), (18); (just interchanging their order). There is a minor asymmetry here, as the position, $x_v^I = -vt_v^I$ in *eq*. (16), whilst the position $x_0^{II} = vt_0^{II}$ (without the minus sign) in *eq*. (17). However, this difference appears since we here let K_0 and K_v have the *same* direction. Therefore, the clock at the origin of K_0 moves along the negative axis of K_v, (thus, $x_v^I < 0$); while the origin of K_v moves along the positive axis of K_0, (thus, $x_0^{II} > 0$); but actually there is a complete symmetry here.

Next consider simultaneity. The two (identical) events (t_0^I, x_0^I) and (t_v^I, x_v^I) at Position I represent 'local simultaneity', *i.e.* they have 'same location, same time'. This is also the case for the two events (t_0^{II}, x_0^{II}) and (t_v^{II}, x_v^{II}) at Position II.

However, we will now also specify the events at Positions I and II in a symmetric way, and thus simultaneity for all four events follows. So now the two clocks at the origins show the same time at their respective positions, that is:

$$t_0^I = t_v^{II} = t_0 \tag{19}$$

Then we will have the symmetric situation introduced in Section 3.1, and due to this symmetry we get that all four events are simultaneous (at least in some sense). Since also $x_0^I = x_v^{II} = 0$, it further follows that also $t_0^{II} = t_v^I = t_0$. Then all four time vectors (15) - (18) can be expressed by a common t_0, and the expressions (15) - (16) for Position I become:

$$\vec{t}(v) = \binom{1}{0} t_0 \tag{20}$$

$$\vec{t'}(v) = A_v\vec{t}(v) = \binom{\sqrt{1-(v/c)^2}}{-v/c} t_0 \tag{21}$$

Similarly, the expressions (17) - (18) for Position II become:

$$\vec{t}(v) = \binom{\sqrt{1-(v/c)^2}}{v/c} t_0 \tag{22}$$

$$\vec{t'}(v) = A_v\vec{t}(v) = \binom{1}{0} t_0 \tag{23}$$

An illustration is given in Fig. 1, showing the time vector, $\vec{t}(v)$ for Positions I and II in its coordinate system, $(t_v, x_0/c)$. We also show the vector, $\vec{t'}(v)$, (obtained by the orthogonal LT) in the rotated coordinate system, $(t_0, x_v/c)$. Note that the rotation from Position I to II is equal to the rotation performed by this LT. Further, we observe that $t_0\sqrt{1-(v/c)^2}$ is the clock reading of the basic clocks at these two positions; and that t_0 is the clock readings for the clocks at the same location but on the other RF.

We should mention that if there is a third RF, K_u moving relative to K_v at velocity, u, then the resulting rotation relative to K_0 would not be θ_{u+v}. Velocities do not simply add up in the STR, and the resulting argument would rather become, $\theta_{u\oplus v}$, where we define the operator $\oplus$ by

$$u \oplus v \stackrel{\text{def}}{=} \frac{u+v}{1+(u\cdot v)/c^2}$$

3.3 The general time vector

The result for time vector, $\vec{t}(v)$ of the previous section were obviously valid for two specific RFs, K_0 and K_v, moving relative to each other at speed, v. However, it suggests that we can introduce a more generic time vector related to one specific RF, say K_0. First we specify an arbitrary event (t_0, x_0) on K_0; that is t_0 = clock reading at the position x_0. Next, we define a parameter, w through the relation

$$x_0 = w \cdot t_0$$

We can interpret this w as the velocity relative to K_0 of a (possibly imagined) RF, K_w. Following the approach of the previous section we can now associate the following time vector to the event, (t_0, x_0)

$$\vec{t}(t_0, x_0) = \begin{pmatrix} \sqrt{1-(w/c)^2}\, t_0 \\ x_0/c \end{pmatrix} = \begin{pmatrix} \sqrt{t_0^2-(x_0/c)^2} \\ x_0/c \end{pmatrix} \tag{24}$$

Using our definition of w, we can also write this time vector as

$$\vec{t}(t_0, w) = \begin{pmatrix} \sqrt{1-(w/c)^2} \\ w/c \end{pmatrix} t_0 \tag{25}$$

We see that this is a generalization of the vector, $\vec{t}(v)$, given in (20) and (22) for the two Positions I and II, (and for a specified velocity, v). The more general vector (25) corresponds to a point on the semicircle with radius t_0 in Fig. 1. Note that we associate this time vector with K_0. However, the horizontal axis now equals t_w. Thus, polar coordinates may be more appropriate for this vector; also see next section.

In summary, we interpret the two components of the time vector (24), (25) as follows:

- The first component, $t_w = \sqrt{1-(w/c)^2}\, t_0$ equals the clock reading of the basic clock at the origin of the (possibly imagined) RF, K_w; (this clock now located at x_0 on K_0).
- The second component equals the 'position', $x_0/c = (w/c)\, t_0$, (that is the distance to the basic clock on K_0); measured as the time required for a light flash to go from the origin to this location.

Here we implicitly assume that the basic clock on K_w was initially synchronized with the basic clock on K_0. Again, note that the absolute value of the time vector is independent of w:

$$\left|\vec{t}(t_0, w)\right| = t_0 \tag{26}$$

First, this t_0 equals the clock reading at an arbitrary position, x_0 on K_0. Thus, the time vectors (25) refer to simultaneity in the perspective of K_0. Further, they refer to events with the same absolute value; *i.e.* same 'distance' from the 'point of initiation'. Thus, events with the same value, t_0 for the absolute value of the time vector are simultaneous in some sense. In particular, when we consider the events of two specified RFs, which exhibit the symmetry of Positions I and II described above, our claim is that we have a *strong* sense of simultaneity, relative to the chosen point of initiation.

3.4 Time formulated as a complex variable

We can of course write the time vector at (t_0, x_0) as a complex variable; in polar form (25) becomes:

$$t(t_0, w) = t_0 e^{i\theta_w}, \quad (w = x_0/t_0) \tag{27}$$

The magnitude, t_0 is defined as above. The argument, $\theta_w \in (-\pi/2,\ \pi/2)$, (*cf.* definition in (6)) is given by the velocity, w relative to K_0 of an (imagined) 'basic' clock at the position in question. When $\theta_w = 0$, (and $w = 0$) this basic clock is located on K_0 itself, and the time variable becomes a real number.

Also the real part, $\mathrm{Re}(t(t_0, w)) = \sqrt{1-(w/c)^2} t_0 = \cos\theta_w\, t_0$, and the imaginary part, $\mathrm{Im}(t(t_0, w)) = (w/c) t_0 = \sin\theta_w t_0 = x_0/c$ are defined above.

Finally, we can obviously generalize (27) to hold for a three-dimensional space, with coordinates (x_0, y_0, z_0). We then define w by $w = \sqrt{x_0^2 + y_0^2 + z_0^2}\, /\, t_0$, and still let $\sin\theta_w = w/c$. We observe that this formulation has similarities with the Minkowski spacetime; the two approaches do, however, differ.

4 The travelling twin

As stated for instance in Mermin [2] the travelling twin paradox illustrates that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together. We will – as we also understand the approach of [2] – consider only the idealised, symmetric situation. Thus, we restrict to that part of the travel when TSR applies, *i.e.* no acceleration etc., and therefore assume a complete symmetry between the two twins. We discussed the example at length in [3], and now just give a short comment.

We start out with two synchronized clocks at the origins of two reference frames; the RF of the earth, and the RF of the rocket of the travelling twin. This is exactly the situation described in Ch. 3. In our symmetry argument, we concluded that identical readings of the two clocks imply simultaneity.

By the arrival of the twin to the 'star', both twins observe two clock readings, their own clock, and the clock on the other passing RF. The observations of (clock reading, space) at these two locations are identical, and give rise to identical 'time vectors', as discussed in Section 3.2. We also refer to the illustration in Fig. 1: Positions I and II correspond exactly to the (clocks of) the two twins.

Ch. 10 of Ref. [2], gives the following numerical example. The distance between the earth and the 'star' equals $x_0 = 3$ light years, and the rocket has speed, $v = (3/5)c$, giving $\sqrt{1-(v/c)^2} = 4/5$. It follows that in the RF of the earth/star, the rocket will reach the star at time, $t_0 = x_0/v = 5$ years. The clock on the rocket is located at $x_v = 0$, and at the arrival at the star this clock reads $t_v = t_0 \cdot \sqrt{1-(v/c)^2} = 4$ years.

Then ref. [2] – as seems common in the literature - claims that the travelling twin ages 4 years during 'the same time' as the earthbound twin ages 5 years. However, we conclude differently. We claim that this argument does not handle simultaneity properly. Of course it is correct that 'in the perspective' of one twin the other twin ages more slowly; but this argument is valid for both twins; *cf.* the so-called Dingle's question [5], [6]. As seen above, we will from symmetry reasons argue that the clock of the travelling twin, which shows 4 years (at his arrival to the star), occurs simultaneously with the event that the clock of the earthbound twin shows 4 years. However, when the twins compare their clock with the passing clocks on the other RF, they will *both* observe a time dilation. This will fully correspond to the length contraction they observe; actually being two sides of the same thing. In order to claim an asymmetric solution to this problem, one has to specify both the assumed asymmetry, and the model chosen to handle it. We further refer to the rather detailed discussion in [3].

5 Conclusions

The Lorentz transformation (LT) relates the (clock, space) observations of one reference frame (RF) with the (clock, space) observations at the same location on another RF. In the present work we define a state vector, which combines the clock reading of one RF with the space coordinate of the other RF. Thus, we obtain two vectors, related by an orthogonal version of the LT. We can interpret both components of these vectors as aspects of time and refer to them as time vectors; specifying time as a richer concept than the clock reading alone.

The vectors have orthogonal components and are related by an orthogonal transformation (rotation) in the two-dimensional time space. We can interpret the absolute value as a measure of the overall distance in time from the 'point of initiation', *i.e.* time 0 when the origins of the two RFs had a common location. We thus provide a means to define (a kind of) simultaneity: Events having time vectors with the same absolute value are simultaneous.

The main simultaneity result presented here is based on one fundamental claim. Due to the inherent symmetry of the two given RFs we have that the two clocks at the origin of the two RFs at time 0 will remain 'synchronized'. We refer to these as 'basic clocks', and identical clock readings of these two symmetric clocks will represent simultaneity at a distance.

We define the time vector for an event $(t_0,\ x_0)$ on a specific RF. To obtain this we introduce an 'imagined' RF with its basic clock at the moment located at the chosen position, x_0, and then also utilize the clock reading of this 'basic clock'. Further, we can give the time vector as a complex variable. The real part equals the clock time for the 'basic clock' on location, and the imaginary part is a measure of the distance from the 'point of initiation'- measured by the time required for a light signal.

The given approach provides a conclusion regarding the travelling twin example. It seems that previous discussions of this case has not handled simultaneity properly.

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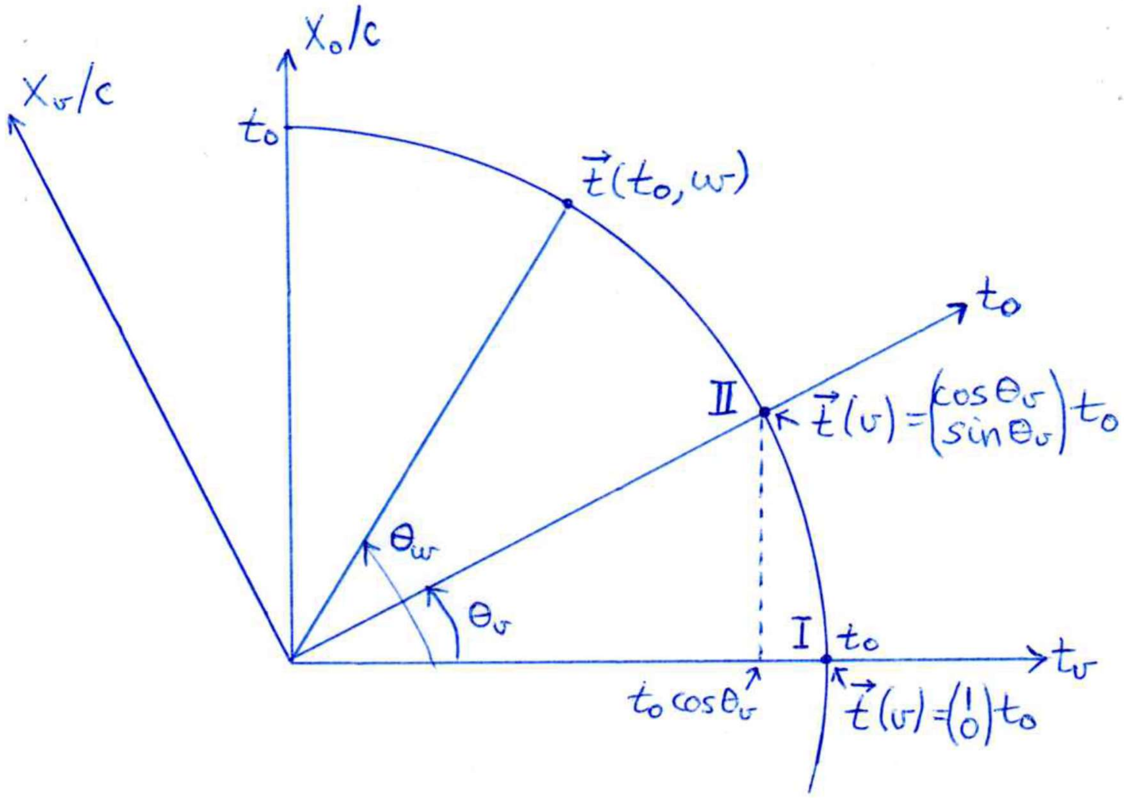

Figure 1 Time vectors, $\vec{t}(v)$ with coordinates $(t_v, x_0/c)$ at the Positions I and II for K_0 and K_v in the symmetric case when $t_v = t_0$. From the figure we can also read the same two vectors in the rotated coordinate system, $(t_0, x_v/c)$. Further, the figure gives the general time vector, $\vec{t}(t_0, w) = \begin{pmatrix} \sqrt{1-(w/c)^2} \\ w/c \end{pmatrix} t_0$ at an arbitrary position $x_0 = wt_0$ on K_0.