A new concept for simultaneity in the STR (v4, 2018-01-12)

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Abstract. We introduce a new approach to simultaneity in the special theory of relativity (STR), considering two reference frames (RFs) moving relative to each other at constant speed, *v.* The two clocks at the origins play a crucial role. We synchronize these at time 0, and refer to this event as the 'point of initiation'. Due to the symmetry we conclude that identical readings of these two 'basic clocks' represent a strong sense of simultaneity. The second element of the approach is a modified version of the Lorentz transformation. We combine the two (clock, space) observations of each RF at a specific position, giving a vector of two orthogonal components. The absolute value of this time vector provides a measure for 'distance in time' from the 'point of initiation', and events with the same 'distance' exhibits a sense of simultaneity. There is a rather close relation to the Minkowski distance. We can also present the time vector as a complex variable. The given description of simultaneity is well suited to analyse and conclude regarding the travelling twin case.

Key words: Time dilation, simultaneity, symmetry, Lorentz transformation, time vector, Minkowski distance, travelling twin.

1 Introduction

The concept of *simultaneity* becomes crucial when inertial reference frames (RFs) are moving relative to each other. Of course, we have the 'basic simultaneity'; *i.e.* simultaneity of events occurring at the same instant *and* same location, but these are rather the same event, just seen in the perspective of two different RFs. For events at a distance, we can essentially observe simultaneity from the 'perspective' of a certain RF. When the synchronized clocks of a specific RF show the same readings, we have simultaneous events *in the perspective of* this RF.

In general, one may define simultaneity by use of light rays; *e.g*. see standard textbooks, Einstein (2004), Giulini (2005), Mermin (2005), but due to the relativity of simultaneity, also see Debs and Redhead (1996), there is no unique definition of distant simultaneity. Here we present another approach - based on Hokstad (2016, 2018) - having two main elements:

- There is a clock at the origins of each RF. We synchronized these 'basic clocks' at time zero (when they are at the same location); and due to symmetry, they remain synchronized. So we utilise the 'basic clocks' to define a distant simultaneity in this symmetric situation.
- When we consider just one space coordinate (x), the two RFs will together have four (clock, space) observations at a specific location. The Lorentz transformation (LT) provides a relation between these. Here we combine these four variables in a new way and obtain two state vectors, related by an orthogonal transformation; *i.e.* a version of the LT. This provides an aid in the analysis of simultaneity.

We specify the two-dimensional state vector as a 'time vector'; depending on a 'basic clock, reading and the position (measured as the time required of a light signal). Both dimensions can be seen as elements of the 'distance in time' from the 'point of initiation'. We can then consider events with the same total distance from the 'point of initiation' to be simultaneous. We can also give the time vector as a complex variable; where the magnitude represents this 'distance' from the 'point of initiation'. In particular, this approach to simultaneity provides a tool suitable to give a solution to the travelling twin paradox.

2 Foundation

We here present some assumptions, and give both the standard and a new version of the LT.

2.1 Basic assumptions and notation

We start out with a reference frame (RF), K_0, where the position along the x-axis is denoted x_0. At virtually any position there are synchronized clocks with clock reading denoted, t_0. We will simply refer to (x_0, t_0) as an event.

Further, there is a reference frame, K_v (with the same orientation), moving along the x-axis of K_0 at velocity v. So K_0 is moving along the x-axis of K_v at velocity $-v$. On K_v we have

$x_v =$ The position on K_v, being identical to the location x_0 on K_0 at a certain instant

$t_v =$ Clock reading at position x_v on K_v, when x_v corresponds to x_0, and the clock on K_0 reads t_0.

Observers (observational equipment) on both of these two RFs agree on these four observations. Further,

- There is a complete *symmetry* between the two RFs K_0 and K_v; these being identical in all respects.
- The clock at $x_v = 0$ and the clock at $x_0 = 0$ will when $t_v = t_0 = 0$ be at the same location, and they are then synchronized. We refer to this as the 'point of initiation', and these clocks as 'basic clocks'.
- We may choose the *perspective* of any RF. Events with the same clock reading, t at various positions, x on this RF means that these events are simultaneous in the perspective of this frame.

2.2 The Lorentz transformation (LT) and time dilation

The LT represents the fundament for our discussion of time dilation. From the above notation the LT takes the form

$$t_v = \frac{t_0 - (v/c^2)x_0}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$x_v = \frac{x_0 - vt_0}{\sqrt{1-(v/c)^2}} \tag{2}$$

Two special cases are of particular interest. Recall that the first clock comparison is carried out at the origins $x_v = x_0 = 0$ when $t_v = t_0 = 0$. Now repeating arguments given in Hokstad (2016, 2018), consider two specific choices for the time and location for a second time comparison.

First we compare the clock located at $x_v = 0$ on K_v with the passing clocks on K_0, showing time t_0. This clock on K_0 must according to (2) have position $x_0 = vt_0$, and (1) gives the relation between the two clock readings at this position. In summary, for $x_v = 0$, (*i.e.* follow the basic clock at the origin of K_v) we get:

$$t_v = t_0\sqrt{1-(v/c)^2} = \sqrt{t_0^2 - (x_0/c)^2} \tag{3}$$

This equals the standard 'time dilation formula'. Secondly, we can compare the clock located at $x_0 = 0$ on K_0 with a passing clock on K_v (actually at position $x_v = -vt_v$). Thus, for $x_0 = 0$, *i.e.* following the basic clock at the origin of K_0, we get the following relation between t_0 and t_v

$$t_0 = t_v\sqrt{1-(v/c)^2} = \sqrt{t_v^2 - (x_v/c)^2} \tag{4}$$

The relations, (3), (4) are apparently contradictory; *eq.* (3) tells that the clock on K_v goes slower, and (4) tells that the clock on K_0 goes slower. Thus, the time dilation is not a feature of the RF, but follows from which single clock we choose to follow when we perform the second clock comparisons. Therefore, we prefer to formulate the time dilation formulas (3), (4) in compact form as

$$t^{BC} = t^{MC}\sqrt{1-(v/c)^2} \tag{5}$$

(representing what we could refer to as the 'essential LT'). Here we have introduced the notation

t^{BC} = The clock reading of a basic clock (BC), *i.e.* clock located at the origin of a RF[1].

t^{MC} = The clock reading at the same location but on the other RF; *i.e.* the clock reading on a RF using multiple clocks (MC) for clock comparisons with the basic clock.

Therefore, as seen from (3), (4), both of the RFs can apply a BC for a certain clock comparison, and then conclude that 'time goes slower' on this RF. However, the same RF would apply MC for the clock comparison with the basic clock on the other RF; and then we would conclude that 'time goes faster' on this RF. It is the observational principle, *i.e.* choice of clocks for the clock comparisons that matters. We give a more thorough discussion of (5) in Hokstad (2018). This is actually a well-known result. According to Petkov (2012) already Minkowski referred to proper time and coordinate time, corresponding the above two concepts of time. However, it has perhaps not received the attention it deserves in the standard literature.

2.3 An alternative formulation of the Lorentz transformation (LT)

We now proceed to replace v in the LT, (1), (2) with an angle, θ_v, given by

$$\sin\theta_v = v/c \tag{6}$$

implying that

$$\cos\theta_v = \sqrt{1-(v/c)^2}.$$

Now the LT (1), (2) can be formulated as:

$$t_v \cdot \cos\theta_v = t_0 - (x_0/c)\cdot\sin\theta_v \tag{7}$$

$$x_v/c \cdot \cos\theta_v = x_0/c - t_0\cdot\sin\theta_v \tag{8}$$

As we restrict to consider one space coordinate the LT involves four state variables, t_0, x_0, t_v and x_v. If we specify any two of these four variables, the other two will be given by the LT. The standard version of the LT gives (t_v, x_v) expressed by (t_0, x_0), or *vice versa*. But similarly, we could reformulate the LT to give a relation between (t_0, t_v) and (x_0, x_v). And – as a third possibility – we can formulate the LT as a relation between (t_0, x_v) and (t_v, x_0). In the present work we follow up on this third possibility. First, by combining (7) and (8), we can replace (8) by

$$x_v/c = (x_0/c\ \ )\cdot\cos\theta_v - t_v\sin\theta_v \tag{9}$$

To give the resulting modified version of the LT we introduce the matrix

$$C_v = \begin{bmatrix}\cos\theta_v & -\sin\theta_v\\ \sin\theta_v & \cos\theta_v\end{bmatrix} \tag{10}$$

This is an orthogonal matrix as

$$C_v^{-1} = C_v^T = C_{-v} = \begin{bmatrix}\cos\theta_v & \sin\theta_v\\ -\sin\theta_v & \cos\theta_v\end{bmatrix}$$

Now (7) and (9) give a new version of the LT, which we in matrix form can write[2]

$$\begin{pmatrix}t_0\\ x_v/c\end{pmatrix} = C_v^{-1}\begin{pmatrix}t_v\\ x_0/c\end{pmatrix} \tag{11}$$

Now we also introduce the two 'time vectors' related to our two RFs, K_0 and K_v

$$\vec{t}(v) = \begin{pmatrix}t_v\\ x_0/c\end{pmatrix} \tag{12}$$

$$\vec{t'}(v) = \begin{pmatrix}t_0\\ x_v/c\end{pmatrix} \tag{13}$$

and then we write the relation (11) as

[1] We have previously used t^{SC} (where SC = Single Clock) to denote this clock reading

[2] Again observe some change in the notation, as compared to previous versions

$$\vec{t'}(v) = C_{-v}\vec{t}(v) \tag{14}$$

We will denote the orthogonal version of the Lorentz transformation. A nice feature of this formulation is that it represents a rotation, θ_v, of the $(t_v,\ x_0/c)$ plane, (with the components t_v and x_0/c being orthogonal). So also the vector $\vec{t}(v) = \binom{t_v}{x_0/c}$ will be given by $\vec{t'}(v) = \binom{t_0}{x_v/c}$, using the same rotation in opposite direction, *i.e.* we replace -v by v, (and applying C_v here rather than $C_{-v} = C_v^{-1}$. The vectors, $\vec{t}(v)$ and $\vec{t'}(v)$ provide identical information, as they actually define the same event, (but referring to both coordinate systems). However, we will see that it becomes rather natural to link $\vec{t}(v)$ to the RF, K_0 and $\vec{t'}(v)$ to the RF, K_v.

For both time vectors, the first component equals the clock reading of a RF. The second component equals the position of the other RF for the event in question divided by c; so it equals the time for a light flash to go from the origin to this position. Therefore, both components of the vectors, $\vec{t}(v)$ and $\vec{t'}(v)$ represent time; what we could denote 'distance' in time, as measured from the 'point of initiation'.

3 Simultaneity and the time vector

Within the framework of STR it seems most common to verify simultaneity across reference frames by use of light rays. This differs from the approach presented here.

3.1 Concepts of simultaneity

Within a single reference frame simultaneity is easily established by the synchronization of clocks, *e.g.* using light rays, for instance see Einstein (1924), Giulini (2005), Mermin (2005). However, for moving reference frames there is within the STR no unique definition of simultaneity at a distance. Rather, we refer to *relativity of simultaneity*, *e.g.* see the discussion in Debs and Redhead (1996). They in particular argue for the *conventionality of simultaneity.* That is, when establishing simultaneity at a distance by the use of light signals, the definition of simultaneity is essentially a matter on convention; any time in a certain interval can be seen as simultaneous with a specified distant event.

We would in this respect comment that even if there are several possible definitions for simultaneity at a distance, this does not mean that all are equally valid. If, for instance, we want to model a symmetric situation, there should also be a certain symmetry with respect to simultaneity.

When the events occur at different locations one could refer to the rather weak concept of *simultaneity by perspective.* One can say that events with the same clock reading (t) measured on a specific RF are simultaneous in the perspective of this frame. As we know, this simultaneity depends on the chosen RF.

However, in Hokstad (2018) we found it useful to apply an auxiliary reference frame as a tool to obtain simultaneity at a distance. We simply postulate an auxiliary RF with origin always located at the midpoint between our two main RFs. Further, we utilized the symmetry of this model, so that simultaneous clock readings at the auxiliary RF implies a certain simultaneity at a distance for the two main RFs. In particular, we strongly argued that this approach provides a logical and consistent solution to the travelling twin example.

In the present paper we will pursue a slightly different approach, based on the same symmetry. In particular we utilize the discussions of Chapter 2. First we point out that an essential requirement for the use of the LT is that we start out with three sets of synchronizations.

1. All clocks on the first RF, K_0;
2. All clocks on the second RF, K_v;
3. The two clocks at the origins of K_0 and K_v at time 0, (this represent *basic simultaneity*, and we refer to these as the *basic clocks*.).

Here one will implicitly assume that all clocks on K_0 *remain* synchronized; as also do the clocks on K_v. Further, we will now argue that also the two 'basic clocks' at the origins of K_0 and K_v - being synchronized at time 0 - will remain synchronized. They are moving away from each other at speed, v, but in our model of complete symmetry, there is no way to claim that one of the two clocks goes faster than the other. We have the standard phrase 'moving clock goes slower', but that is when the 'moving clock' is compared with passing clocks, and not with the other clock at the origin; also see discussions in Hokstad (2018).

So our claim is that when the two 'basic clocks' at the origins of the two RFs show the same time, this corresponds to simultaneous events 'at a distance'. We do not consider this as a new assumption, rather to be a consequence of our assumption of symmetry between the RFs, (actually inherent in the LT). This actually leads to a rather strong form of simultaneity, as all observers can agree on this. So this is the basis for our discussions in the next sections.

So our argument regarding simultaneity is closely linked to arguments concerning symmetry. After all, we calculate the claimed magnitude of the difference in ageing by use of the Lorentz transformation, which truly exhibits symmetry. I find it hard to defend an asymmetric solution to this symmetric mathematical framework; for instance by adding some ad hoc assumptions outside the scope of the mathematical framework.

Before we leave the topic, we note that two moving RFs of course will specify an event differently. We will refer to this as *basic simultaneity,* (same instant *and* the same location). This is actually the same event; the different RFs just describing it by different (time, space) parameters, (*cf.* the LT).

3.2 Moving clocks and simultaneity

We will now apply the general result of Section 2.3 for two specific positions of the RFs. We choose the positions of the two 'basic clocks' at the origins. Position A equals the origin of K_0, *i.e.* location $x_0 = 0$. Position B is at the other origin, $x_v = 0$ of K_v. At both positions, we specify four variables:

Position A; Events (t_0^A, x_0^A) and (t_v^A, x_v^A) at the origin of K_0, *i.e.* $x_0^A = 0$, (and so $x_v^A = -vt_v^A$ on K_v). Here *eq.* (4) is valid, and we specify the four variables at this position by the two vectors (12), (13):

$$\vec{t}(v) = \begin{pmatrix} t_v^A \\ x_0^A/c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_v^A \tag{15}$$

$$\vec{t'}(v) = C_{-v}\vec{t}(v) = \begin{pmatrix} t_0^A \\ x_v^A/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ -v/c \end{pmatrix} t_v^A = \begin{pmatrix} \cos\theta_v \\ -\sin\theta_v \end{pmatrix} t_v^A \tag{16}$$

Position B; Events (t_0^B, x_0^B) and (t_v^B, x_v^B) at the origin, of $K_v, i.e. x_v^B = 0$ (and so $x_0^B = vt_0^B$). Here *eq.* (3) is valid, and we specify the four variables at this position by the two vectors

$$\vec{t}(v) = \begin{pmatrix} t_v^B \\ x_0^B/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ v/c \end{pmatrix} t_0^B = \begin{pmatrix} \cos\theta_v \\ \sin\theta_v \end{pmatrix} t_0^B \tag{17}$$

$$\vec{t'}(v) = C_{-v}\vec{t}(v) = \begin{pmatrix} t_0^B \\ x_v^B/c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_0^B \tag{18}$$

We observe that the time vectors of (15), (16) are essentially identical to the two time vectors (17), (18); (just interchanging their order). There is a minor asymmetry here, as the position, $x_v^A = -vt_v^A$ in *eq.* (16), whilst the position $x_0^B = vt_0^B$ (without the minus sign) in *eq.* (17). However, this difference appears since we here let K_0 and K_v have the *same* direction. Therefore, the clock at the origin of K_0 moves along the negative axis of K_v, (thus, $x_v^A < 0$); while the origin of K_v moves along the positive axis of K_0, (thus, $x_0^B > 0$); but actually there is a complete symmetry here.

Next consider simultaneity. The two (identical) events (t_0^A, x_0^A) and (t_v^A, x_v^A) at Position A represent 'basic simultaneity', *i.e.* they have 'same location, same time'. This is also the case for the two events (t_0^B, x_0^B) and (t_v^B, x_v^B) at Position B.

However, we will now proceed to specify the events at Positions A and B in a symmetric way, and thus obtain simultaneity for all four events follows. Thus, the two clocks at the origins will show the same time at their respective positions, that is:

$$t_0^A = t_v^B = t^0 \tag{19}$$

Then we will have the symmetric situation introduced in Section 3.1, and due to this symmetry we get that all four events are simultaneous (at least in some sense). Since also $x_0^A = x_v^B = 0$, it further follows that also $t_0^B = t_v^A = t^0$. Then all four time vectors (15) - (18) can be expressed by a common t^0, and the expressions (15) - (16) for Position A become:

$$\vec{t}(v) = \binom{1}{0} t^0 \tag{20}$$

$$\vec{t'}(v) = C_{-v}\vec{t}(v) = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ -v/c \end{pmatrix} t^0 = \begin{pmatrix} \cos\theta_v \\ -\sin\theta_v \end{pmatrix} t^0 \tag{21}$$

Similarly, the expressions (17) - (18) for Position B become:

$$\vec{t}(v) = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ v/c \end{pmatrix} t^0 \tag{22}$$

$$\vec{t'}(v) = C_{-v}\vec{t}(v) = \binom{1}{0} t^0 \tag{23}$$

We give an illustration in Fig. 1 at the end of the paper. This shows the time vector, $\vec{t}(v)$ both for Positions A (*eq.* (20)), and for Position B (*eq.* (22)) in the coordinate system, $(t_v, x_0/c)$. At both positions we can apply the same interpretation of this vector: The first component is the clock reading of the 'basic clock' at the position; the second component equals the position (divided by *c*) on the other RF (not having a BC at the location). Further, the transformed vector, $\vec{t'}(v)$, (obtained by the orthogonal LT), see (21) and (23), is actually the same vectors in the rotated coordinate system, $(t_0, x_v/c)$. However, we do not give the expressions for $\vec{t'}(v)$ in the figure. Note that the rotation from Position A to B is equal to the rotation performed by the matrix, C_v. In the following, we will choose to relate the vector, $\vec{t}(v)$ to the RF, K_0, and the transformed vector, $\vec{t'}(v)$ to the RF, K_v.

We could mention that if there is a third RF, K_u moving relative to K_v at velocity, *u*, then the resulting rotation relative to K_0 would not be θ_{u+v}. Velocities do not simply add up in the STR, and the resulting argument would rather become, $\theta_{u\oplus v}$, where we define the operator $\oplus$ by

$$u \oplus v \overset{\text{def}}{=} \frac{u+v}{1+(u\cdot v)/c^2}.$$

We should further realize that we can also apply a matrix C_w to perform a transformation of a time vector 'internally' on a given RF. This corresponds to performing a rotation of the time vector, maintaining its absolute value.

Now we should finally question whether Positions A and B, (*i.e.* the locations of the two origins) represent the only events being simultaneous in this symmetric sense with respect to K_0 and K_v. Actually, our symmetry argument will apply equally well if we considered the two locations, $x_0 = wt^0$, (rotation θ_w from position A in Fig.1) and $x_v = -wt^0$, (rotation $-\theta_w$ from position B). By making the transformation C_w of the vectors at position A, we end up with $\vec{t}(v) = \begin{pmatrix} \cos(\theta_w) \\ \sin(\theta_w) \end{pmatrix} t^0$ and $\vec{t'}(v) = \begin{pmatrix} \cos(\theta_v-\theta_w) \\ -\sin(\theta_v-\theta_w) \end{pmatrix} t^0$, respectively. By similarly making the transformation C_{-w} of the vectors at position B, we end up with the rime vectors, $\vec{t'}(v) = \begin{pmatrix} \cos(\theta_w) \\ -\sin(\theta_w) \end{pmatrix} t^0$ and $\vec{t}(v) = \begin{pmatrix} \cos(\theta_v-\theta_w) \\ \sin(\theta_v-\theta_w) \end{pmatrix} t^0$, respectively. So relative to our two RFs, K_0 and K_v the two events given by these four vectors are (for any given *w*) simultaneous in this symmetric sense. They correspond to locations being symmetric around the midpoint between the two origins (at $\theta_v/2$).

3.3 The time vector

The result for time vector, $\vec{t}(v)$ of the previous section were obviously valid for two specific RFs, K_0 and K_v, moving relative to each other at speed, *v*. However, it suggests that we can introduce a more generic time vector related to one specific RF, say *K*. At an arbitrary position, $x = w \cdot t^0$ we will in analogy with (22) define a time vector as

$$\vec{t}(\,w) = \begin{pmatrix} \sqrt{1-(w/c)^2} \\ w/c \end{pmatrix} t^0$$

Thus, we may specify any arbitrary event (t^0, x) on K; that is t^0 = clock reading at the position x. Next, letting $w = x/t$ we have various expressions for this vector, which we now rather denote $\vec{t}(t^0, w)$ to stress that it also depends on t^0 The various expressions for this vector are

$$\vec{t}(t^0, w) = \vec{t}(\,w) = \begin{pmatrix} \sqrt{1-(w/c)^2} \\ w/c \end{pmatrix} t^0 = \begin{pmatrix} t^0\sqrt{1-(w/c)^2} \\ x/c \end{pmatrix} = \begin{pmatrix} \sqrt{(t^0)^2-(x/c)^2} \\ x/c \end{pmatrix} \tag{24}$$

We can interpret this w as the velocity relative to K of a (possibly imagined) RF, K_w. Thus, this is the generalization of the vector, $\vec{t}(v)$, given in (20) and (22) for a specified velocity, v at the two Positions A and B, respectively, The vector (24) corresponds to a point on the semicircle with radius t^0; see Fig. 2. In summary, for a specific clock time, t^0 and position wt^0 on K we interpret the two components of the time vector (24) as follows:

- The first time component, $t^0\sqrt{1-(w/c)^2}$ equals the clock reading of the basic clock located at the origin of the (possibly imagined) RF, K_w; which by now have arrived at the position x on K. We could call this the 'basic time' ('basic clock reading') at this position.
- The second component is a measure for the distance, x from the origin (and thus from the basic clock) of K to the given position, measured as the time $t^0 \cdot w/c = x/c$ required for a light flash to go to this distance.

So by introducing the vector (24) we split the clock reading ('clock time'), t^0 at position, $x = wt^0$ on K in two parts:

1. $t^0\sqrt{1-(w/c)^2}$; the clock reading of the (imagined) BC at this position
2. $t^0 \cdot w/c$; time required for a light flash to come from the origin of K to this position.

Only for $x = w = 0$ we here refer to the BC at K itself. At all other positions we refer to an 'imagined BC'. Thus, we implicitly assume that we initially synchronized the basic clocks at K and K_w. Again, note that the absolute value of this time vector is independent of w:

$$\left|\vec{t}(t^0, w)\right| = t^0 \tag{25}$$

and equals the clock reading at the position, $x = wt^0$ on K. Following the discussion of the previous section we now define events to be simultaneous if their time vectors have the same absolute value (observed distance from the 'point of initiation').

This definition of simultaneity actually combines the concept the simultaneity 'in the perspective of' K and the symmetric 'simultaneity at a distance' of Section 3.2. 'Simultaneity in the perspective of' follows directly since the absolute value of the time vector simply equals the 'clock time' on K. The second claim was proved in Section 3.2, *cf.* simultaneity of Positions A and B discussed there. Actually any two position at the same distance from the midpoint between the two RFs are simultaneous by symmetry. The given simultaneity concept of course also covers the 'basic simultaneity', (same event described by two different RFs, *cf.* Section3.2 Thus, this simultaneity, illustrated in the two figures cover all three simultaneity concepts discussed in the present paper.

There is a rather close link between the present approach and Minkowski's approach to space-time. According to Petkov (2012), Minkowski refers to the time of the basic clock (*eq.* (24)), $\sqrt{(t^0)^2-(x/c)^2}$ as 'proper time, and t^0 as 'coordinate time'. Now we give our results for just one space coordinate; whilst Minkowski (1909) introduced the four-dimensional space-time. Further, Minkowski defined space-time distance as $\sqrt{c^2t^2-x^2-y^2-z^2}$ in his four-dimensional space; which differ distinctly from our measure of distance (from the point of initiation).

3.4 Time formulated as a complex variable

We can of course write the time vector for the event (t, x) as a complex variable. Now in in polar form, we can write the vector $\vec{t}(t, w)$ in (25) as

$$\mathbf{t}(t^0, w) = t^0 e^{i\theta_w}, \quad (w = x/t^0) \tag{26}$$

The magnitude, t still equals the clock reading, and $w = x/t^0$ can be interpreted as a velocity relative to K of an (imagined) 'basic' clock, now having arrived at the position, x. Then, the argument, $\theta_w \in (-\pi/2,\ \pi/2)$, is given by $\sin\theta_w = w/c$. When $\theta_w = 0$, (and $w = 0$) this basic clock is located on K_0 itself, and the time variable becomes a real number.

Both the real part, $\mathrm{Re}(\mathbf{t}(t^0, w)) = t^0\sqrt{1-(w/c)^2} = t^0\cos\theta_w$, and the imaginary part, $\mathrm{Im}(\mathbf{t}(t^0, w)) = (w/c)t^0 = t^0 \sin\theta_w = x/c$ are defined above.

Finally, we can now generalize (27) to hold for a three-dimensional space, with coordinates (x,y,z). We then define w by $w = \sqrt{x^2 + y^2 + z^2}\ /t^0$. Here, w essentially specifies a position (or distance) at time t^0 on the RF K, and we still let $\sin\theta_w = w/c$, using the new definition of w.

4 The travelling twin

As stated for instance in Mermin (2005) the travelling twin paradox illustrates that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together. We will – as we also understand the approach of Mermin (2005) – consider only the idealised, symmetric situation. Thus, we restrict to that part of the travel when TSR applies, *i.e.* no acceleration etc., and therefore assume a complete symmetry between the two twins. We discussed the example at length in Hokstad (2018), and now just give a short comment.

We start out with two synchronized clocks at the origins of two reference frames; the RF of the earth, and the RF of the rocket of the travelling twin. This is exactly the situation described in Ch. 3. In our symmetry argument, we concluded that identical readings of these two basic clocks imply simultaneity.

By the arrival of the twin to the 'star', both twins observe two clock readings, their own clock, and the clock on the other passing RF. The observations of (clock reading, space) at these two locations are identical, and give rise to identical 'time vectors', as discussed in Section 3.2. We also refer to the illustration in Fig. 1: Positions A and B correspond exactly to (the clocks possessed by) the two twins.

Ch. 10 of Mermin (2005), gives the following numerical example. The rocket has speed, $v = (3/5)c$, giving $\sqrt{1-(v/c)^2} = 4/5$. The earthbound twin measures the distance between the earth and the 'star' to equal $x_0 = 3$ light years, and it follows that in his RF, the rocket will reach the star at time, $t_0 = x_0/v = 5$ years. The clock on the rocket, however, is located at $x_v = 0$, and at the arrival at the star this clock reads $t_v = t_0 \cdot \sqrt{1-(v/c)^2} = 4$ years. We fully agree with this.

Then Mermin (2005) – as seems common in the literature - claims that the travelling twin ages 4 years during 'the same time' as the earthbound twin ages 5 years. However, we conclude differently. We claim that this argument does not handle simultaneity properly. Of course it is correct that 'in the perspective' of any of the twins, the other twin ages more slowly; but this argument is valid for both twins; *cf.* the so-called Dingle's question, McCausland (2008, 2011). As discussed in the present work, we will from symmetry reasons argue that the clock of the travelling twin, which shows 4 years (at his arrival to the star), occurs simultaneously with the event that the clock of the earthbound twin shows 4 years. Further, when the twins compare their clock with the passing clocks on the other RF, they will *both* observe a time dilation; (already *eq.* (5) provides a rather convincing argument for this). This time dilation corresponds to the length contraction they observe; actually being two sides of the same thing. In order to claim an asymmetric solution to this problem, one has to specify both the assumed asymmetry, and the model chosen to handle it. We further refer to the rather detailed discussion in Hokstad (2018).

5 Conclusions

The Lorentz transformation (LT) relates the (clock, space) observations of one reference frame (RF) with the (clock, space) observations at the same location on another RF. In the present work we define a state vector, which combines the clock reading of one RF with the space coordinate of the other RF. Thus, we obtain two vectors, related by an orthogonal version of the LT. We can interpret both components of these vectors as aspects of time and refer to them as time vectors; specifying time as a richer concept than the clock reading alone.

The vectors have orthogonal components, being related by an orthogonal transformation (rotation) in the two-dimensional time space. Thus, our approach has distinct similarities with the time-space of Minkowski (and his space-time distance). In the present paper we interpret the absolute value of our state vector as a measure of the overall distance in time from the 'point of initiation', *i.e.* time 0 when the origins of the two RFs had a common location. We thus provide a means to define (a certain) simultaneity: Events (here simply defined as pairs, (t, x) of (clock, space) observations) which have time vectors with the same absolute value are defined to be simultaneous. This definition of simultaneity at a distance generalizes the common concept of 'simultaneity 'in the perspective of' a certain RF, and also includes 'basic simultaneity' (of the same event described by two different RFs).

The main simultaneity result presented here is based on one fundamental claim. Due to the inherent symmetry of the two given RFs we have that the two clocks at the origin of the two RFs at time 0 will remain 'synchronized'. We refer to these as 'basic clocks', and identical clock readings of these two symmetric clocks will represent simultaneity at a distance.

Further, we define the time vector for any event (t , x) of a specific RF. As part of the approach we introduce an 'imagined' RF with its basic clock currently located at the chosen position, x, and then utilize the clock reading of this 'basic clock'. The time vector, given as a complex variable, has real part equal to the clock time for the 'basic clock' at the location. The imaginary part equals the distance from the 'point of initiation'- measured as the time required for a light signal to go this distance.

The given approach provides a conclusion regarding the travelling twin example under the conditions of the STR. We suggest that previous discussions has often failed to handle simultaneity properly.

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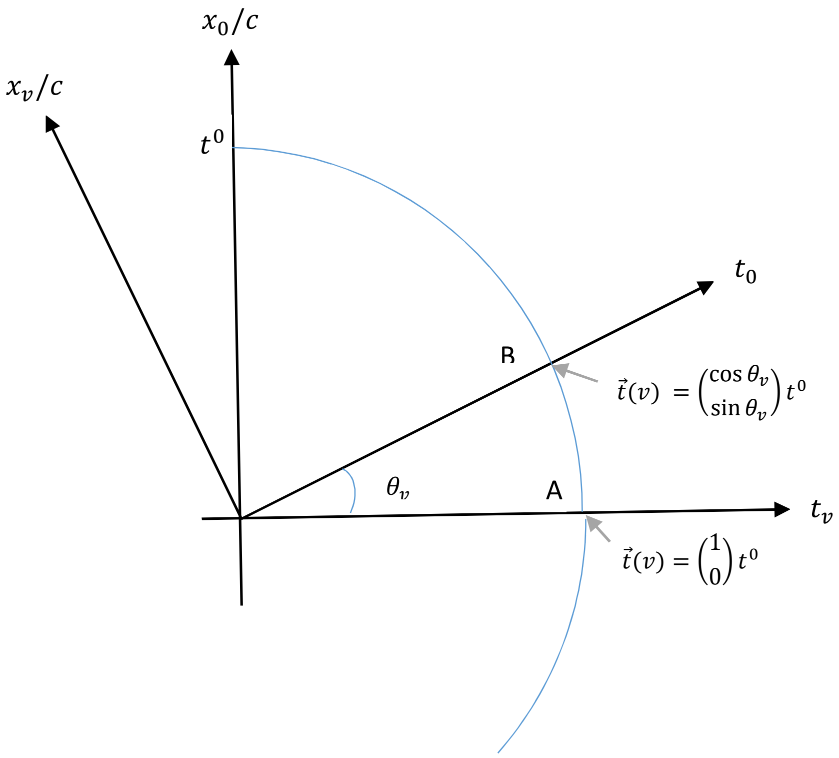

Figure 1 Time vectors, $\vec{t}(v)$ with coordinates $(t_v, x_0/c)$ at the Positions A and B for RFs, K_0 and K_v in the symmetric case when $t_v = t_0\ (= t^0)$. We can also read the same two vectors in the rotated coordinate system, $(t_0, x_v/c)$.

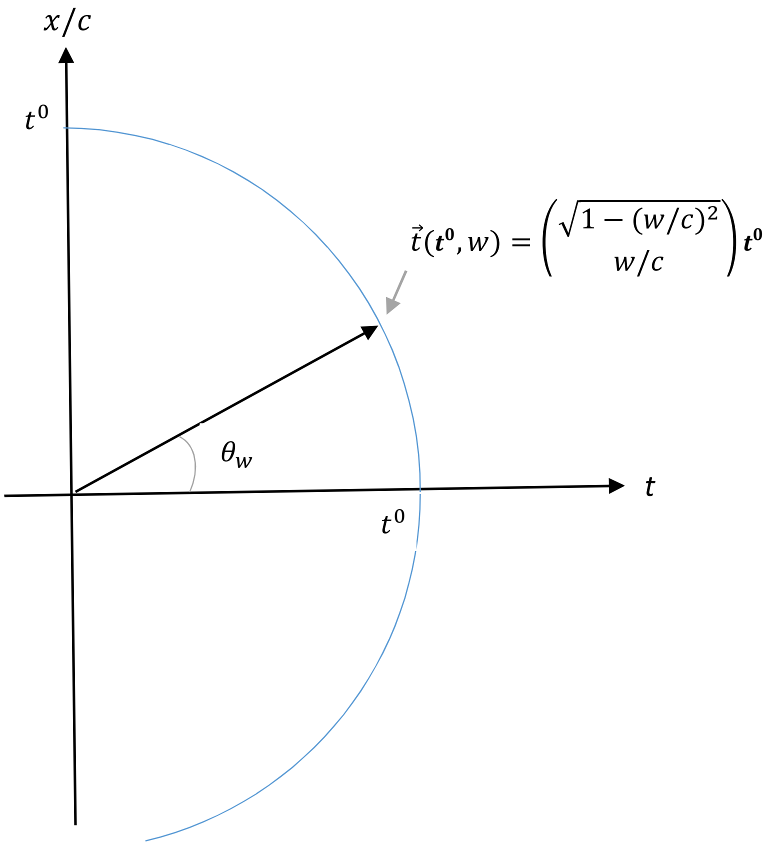

Figure 2 Time vectors, $\vec{t}(t^0, w) = \begin{pmatrix}\sqrt{1-(w/c)^2} \\ w/c\end{pmatrix} t^0$ for a specific clock time t^0 at an arbitrary position, $x = wt^0$ on a RF, K; (where $\sin\theta_w$= w/c).