A new concept for simultaneity in TSR (v5, 2018-02-14)

Per Hokstad

phokstad@gmail.com

Abstract. We introduce a new approach to simultaneity in the theory of special relativity (TSR), considering two reference frames (RFs) moving relative to each other at constant speed, *v.* The two clocks at the origins play a crucial role. We synchronize these at time 0, and due to symmetry we conclude that identical readings of these two 'basic clocks' provides a sense of simultaneity. The second element of the approach is a modified version of the Lorentz transformation. We combine the two (clock, space) observations of each RF at a specific position, to give a vector of two orthogonal components. The absolute value of this time vector provides a measure for 'distance in time', and the vector proves useful in the discussion of various types of simultaneity. We can also present the time vector as a complex variable, and there is a relation to the Minkowski distance.

Key words: Time dilation, simultaneity, symmetry, Lorentz transformation, time vector, Minkowski distance, travelling twin.

1 Introduction

The concept of *simultaneity* becomes crucial when inertial reference frames (RFs) are moving relative to each other. Of course, we have the 'basic simultaneity'; *i.e.* simultaneity of events occurring at the same instant *and* same location, but these are rather the same event, just seen in the perspective of two different RFs. For events at a distance, we can essentially observe simultaneity from the 'perspective' of a certain RF. When the synchronized clocks of a specific RF show the same readings, we have simultaneous events *in the perspective of* this RF.

In general, one may define simultaneity by use of light rays; *e.g.* see standard textbooks, Einstein (2004), Giulini (2005), Mermin (2005), but due to the relativity of simultaneity, also see Debs and Redhead (1996), there is no unique definition of distant simultaneity. Here we present another approach - based on Hokstad (2016, 2018) - having two main elements:

- There is a clock at the origins of each RF. We synchronized these 'basic clocks' at time zero when they are at the same location, (a 'point of initiation'). Due to symmetry, they remain synchronized, and we utilise the 'basic clocks' to define a distant simultaneity in this symmetric situation.
- When we consider just one space coordinate (x), the two RFs will together have four (clock, space) observations at a specific location. The Lorentz transformation (LT) provides a relation between these. Here we combine these four variables in a new way and obtain two state vectors, related by an orthogonal transformation; (being a version of the LT.

We specify the two-dimensional state vector as a 'time vector'; depending on a 'basic clock' reading at the current location, and the distance from the RF's own basic clock (measured as the time required of a light signal). The absolute value of this time vector specifies one type of simultaneity, and the vector proves useful when investigating simultaneity 'at a distance', when there is an inherent symmetry. We can give the time vector as a complex variable, and include a comment on the travelling twin paradox.

2 Foundation

We here present some assumptions, and give both the standard and a new version of the LT.

2.1 Basic assumptions and notation

We start out with a reference frame (RF), K_0, where the position along the x-axis is denoted x_0. At virtually any position there are synchronized clocks with clock reading denoted, t_0. We will simply refer to (x_0, t_0) as an event. Further, there is a reference frame, K_v (with the same orientation), moving along the x-axis of K_0 at velocity v. On K_v we have

x_v = The position on K_v, being identical to the location x_0 at a time t_0 on K_0

t_v = Clock reading at position x_v on K_v, when x_v corresponds to x_0, and the clock on K_0 reads t_0.

Observers (observational equipment) on both of these two RFs agree on these four observations. Further,

- There is a complete *symmetry* between the two RFs K_0 and K_v; these being identical in all respects.
- The clock at $x_v = 0$ and the clock at $x_0 = 0$ will when $t_v = t_0 = 0$ be at the same location, and they are then synchronized. We refer to this as the 'point of initiation', and these clocks as 'basic clocks'.
- We may choose the *perspective* of any RF. Events with the same clock reading, t at various positions, x on this RF, will be simultaneous events in the perspective of this frame.

2.2 The Lorentz transformation (LT) and time dilation

The LT represents the fundament for our discussions. In the above notation the LT takes the form

$$t_v = \frac{t_0-(v/c^2)x_0}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$x_v = \frac{x_0-vt_0}{\sqrt{1-(v/c)^2}} \tag{2}$$

Two special cases are of particular interest. Recall that the first clock comparison is carried out at the origins $x_v = x_0 = 0$ when $t_v = t_0 = 0$. Now repeating some essential (and hopefully well-known) arguments given in Hokstad (2016, 2018), we specify two choices for the second comparison of clock readings.

First we compare the clock located at $x_v = 0$ on K_v with the passing clocks on K_0, showing time t_0. This clock on K_0 must according to (2) have position $x_0 = vt_0$, and (1) gives the relation between the two clock readings at this position. So for $x_v = 0$, (*i.e.* follow the basic clock at the origin of K_v) we get:

$$t_v = t_0\sqrt{1-(v/c)^2} = \sqrt{t_0^2-(x_0/c)^2} \tag{3}$$

This equals the standard 'time dilation formula'. Secondly, we can compare the clock located at $x_0 = 0$ on K_0 with a passing clock on K_v (at position $x_v = -vt_v$). Thus, for $x_0 = 0$, *i.e.* following the basic clock at the origin of K_0, we get the following relation

$$t_0 = t_v\sqrt{1-(v/c)^2} = \sqrt{t_v^2-(x_v/c)^2} \tag{4}$$

The relations, (3), (4) are apparently contradictory; *eq.* (3) tells that the clock on K_v goes slower, and (4) tells that the clock on K_0 goes slower; *cf.* the Dingle's question, (McCausland 2008, 2012). Thus, the time dilation is not a feature of the RF, but follows from which single clock we choose to follow when we perform the second clock comparisons. Therefore, we prefer to formulate the time dilation formulas (3), (4) in compact form as

$$t^{BC} = t^{MC}\sqrt{1-(v/c)^2} \tag{5}$$

(representing what we could refer to as the 'essential LT'). Here we have introduced the notation

t^{BC} = The clock reading of a basic clock (BC), *i.e.* clock located at the origin of a RF[1].

t^{MC} = The clock reading at the same location but on the other RF; *i.e.* the clock reading on a RF using multiple clocks (MC) for clock comparisons with the basic clock.

Therefore, as we see from (3), (4), both of the RFs can apply a BC for a certain clock comparison, and then conclude that 'time goes slower' on the RF using BC in this case. However, the same RF would apply MC for a clock comparison with a BC on the other RF; and we would then conclude that 'time

[1] We have previously used t^{SC} (where SC = Single Clock) to denote this clock reading

goes slower' on this other RF. It is the observational principle, *i.e.* choice of clocks for the clock comparisons that matters. We give a more thorough discussion of (5) in Hokstad (2018). This is actually a well-known result. According to Petkov (2012) already Minkowski referred to proper time and coordinate time, corresponding the above two concepts of time. However, the underlying duality has perhaps not received the attention it deserves in the standard literature.

2.3 An alternative formulation of the Lorentz transformation (LT)

We now proceed to replace v in the LT, (1), (2) with an angle, θ_v, given by

$$\sin\theta_v = v/c \tag{6}$$

implying that

$$\cos\theta_v = \sqrt{1-(v/c)^2}.$$

Now the LT, (1), (2) can be formulated as:

$$t_v \cdot \cos\theta_v = t_0 - (x_0/c)\cdot\sin\theta_v \tag{7}$$

$$x_v/c\cdot\cos\theta_v = x_0/c - t_0\cdot\sin\theta_v \tag{8}$$

As we restrict to consider one space coordinate the LT involves four state variables, t_0, x_0, t_v, x_v. If we specify any two of these four variables, the other two will be given by the LT. The standard version of the LT, (1), (2) gives (t_v, x_v) expressed by (t_0, x_0), or *vice versa*. But similarly, we could reformulate the LT to give a relation between (t_0, t_v) and (x_0, x_v). And – as a third possibility – we can formulate the LT as a relation between (t_0, x_v) and (t_v, x_0). In the present work we follow up on this third possibility. First, by combining (7) and (8), we can replace (8) by

$$x_v/c = (x_0/c)\cdot\cos\theta_v - t_v\sin\theta_v \tag{9}$$

To give the resulting modified version of the LT we introduce the matrix

$$C_v = \begin{bmatrix} \cos\theta_v & \sin\theta_v \\ -\sin\theta_v & \cos\theta_v \end{bmatrix} \tag{10}$$

This is an orthogonal matrix as

$$C_v^{-1} = C_v^T = C_{-v} = \begin{bmatrix} \cos\theta_v & -\sin\theta_v \\ \sin\theta_v & \cos\theta_v \end{bmatrix}$$

Now (7) and (9) give a new version of the LT, which we in matrix form can write[2]

$$\begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} = C_v \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} \tag{11}$$

Now also introduce the two 'time vectors' related to our two RFs, K_0 and K_v

$$\vec{t} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} \tag{12}$$

$$\vec{t'} = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} \tag{13}$$

and then we write the relation (11) as

$$\vec{t'} = C_v\vec{t} \tag{14}$$

We will denote this the orthogonal version of the Lorentz transformation. A nice feature of this formulation is that it represents a rotation, θ_v, of the (t_v, x_0/c) plane, (with the components t_v and x_0/c being orthogonal). So also the vector $\vec{t} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix}$ will be given by $\vec{t'} = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix}$, using the same rotation in opposite direction, *i.e.* we replace -v by v, (and applying $C_{-v} = C_v^{-1}$ here rather than C_v. We note that

[2] Again observe some change in the notation, as compared to previous versions

he vectors, $\vec{t}$ and $\vec{t'}$ provide identical information, as they actually define the same event. We will see that it becomes rather natural to link $\vec{t}$ to the RF, K_0, and $\vec{t'}$ to the RF, K_v.

Both components of the vectors, $\vec{t}$ and $\vec{t'}$ represent time. The first component equals the clock reading of one of the RFs. The second component equals the position of the other RF for the event in question, divided by c; so it equals the time for a light flash to go from the origin to this position..

3 Simultaneity and the time vector

Within the framework of TSR it seems most common to verify simultaneity across reference frames by use of light rays. This differs from the approach presented here.

3.1 Concepts of simultaneity

Within a single reference frame simultaneity is easily established by the synchronization of clocks, *e.g.* using light rays, for instance see Einstein (1924), Giulini (2005), Mermin (2005). Further, two moving RFs will specify an event differently. However, this is rather the same event; but with different (time, space) parameters in the different RFs. We refer to this as *basic simultaneity.*

However, for moving reference frames there is within the TSR no unique definition of simultaneity at a distance. Rather, we refer to *relativity of simultaneity*, *e.g.* see the discussion in Debs and Redhead (1996). They in particular argue for the *conventionality of simultaneity.* That is, when establishing simultaneity at a distance by the use of light signals, the definition of simultaneity is essentially a matter on convention; any time in a certain interval can be seen as simultaneous with a specified distant event.

We would in this respect comment that even if there are several possible definitions for simultaneity at a distance, this does not mean that all are equally valid. If, for instance, we want to model a symmetric situation, there should also be a certain symmetry with respect to simultaneity.

When the events occur at different locations one could refer to the rather weak concept of *simultaneity by perspective.* One can say that events with the same clock reading (t) measured on a specific RF are simultaneous in the perspective of this frame. As we know, this simultaneity depends on the chosen RF.

However, in Hokstad (2018) we found it useful to apply an auxiliary reference frame as a tool to obtain simultaneity at a distance. We simply postulate an auxiliary RF with origin always located at the midpoint between our two main RFs. Further, we utilized the symmetry of this model, so that simultaneous clock readings at the auxiliary RF implies a certain simultaneity at a distance for the two main RFs. In particular, we strongly argued that this approach provides logical and consistent solutions to the travelling twin example.

In the present paper we will pursue a slightly different approach, based on the same symmetry. In particular we utilize the discussions of Chapter 2. First we point out that an essential requirement for the use of the LT is that we start out with three sets of synchronizations.

1. All clocks on the first RF, K_0;
2. All clocks on the second RF, K_v;
3. The two clocks at the origins of K_0 and K_v at time 0, this represent *basic simultaneity*, and we refer to these as the *basic clocks* (BCs).

Usually one will here implicitly assume that all clocks on K_0 *remain* synchronized; as also do the clocks on K_v. We will now further argue that also the two BCs at the origins of K_0 and K_v - being synchronized at time 0 - will remain synchronized. They are moving away from each other at speed, v, but in our model of complete symmetry, there is no way to claim that one of the two clocks goes faster than the other. We have the standard phrase ‘moving clock goes slower’, but that is when the ‘moving clock’ is compared with passing clocks, and not with the other clock at the origin; see discussion of Section 2.2.

So our claim is that when the two ‘basic clocks’ at the origins of the two RFs show the same time, this corresponds to simultaneous events ‘at a distance’. (We do not consider this as a new assumption, rather to be a consequence of our assumption of symmetry between the RFs.) This actually leads to a rather strong form of simultaneity, as all observers can agree on this. So this is the basis for our discussions in the next sections.

So our argument regarding simultaneity is closely linked to arguments concerning symmetry. Further, we calculate the claimed magnitude of the difference in ageing by use of the LT, which truly exhibits symmetry. However, the experimental set-up obviously matters, *cf.* discussions of Hokstad (2018).

3.2 Moving clocks and simultaneity

We will now apply the general result of Section 2.3 for two specific positions of the RFs, K_0 and K_v. Position A equals the origin of K_0, *i.e.* location $x_0 = 0$. Position B is at the other origin, *i.e.* $x_v = 0$ of K_v. In other words, A and B are the positions of the 'basic clocks' at the origins of K_0 and K_v, respectively. Further we consider the events that time equal to $t^0 \cdot \sqrt{1-(v/c)^2}$ on both these 'basic clocks'. Thus, we consider two events, that are not simultaneous, neither in the perspective of K_0, nor of K_v. However, they would be simultaneous in a certain auxiliary RF, *cf.* Hokstad (2018); demonstrating a strong sense of symmetry. The remaining parameters are easily specified as (3) and (4) apply here. We summarize the parameter values at these positions as follows (*cf.* Fig. 1):

Position A:

$$x_0 = 0, \qquad t_0 = t^0\sqrt{1-(v/c)^2} \quad (\text{on } K_0)$$
$$x_v = -v\,t^0, \qquad t_v = t^0 \quad (\text{on } K_v)$$

Position B:

$$x_0 = vt_0 \qquad t_0 = t^0 \quad (\text{on } K_0)$$
$$x_v = 0, \qquad t_v = t^0\sqrt{1-(v/c)^2} \quad (\text{on } K_v)$$

The corresponding time vectors, as introduced in Section 2.3 become:

Position A:

$$\vec{t} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t^0 \tag{15}$$

$$\vec{t'} = C_v\vec{t} = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ -v/c \end{pmatrix} t^0 = \begin{pmatrix} \cos\theta_v \\ -\sin\theta_v \end{pmatrix} t^0 \tag{16}$$

Position B:

$$\vec{t} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ v/c \end{pmatrix} t^0 = \begin{pmatrix} \cos\theta_v \\ \sin\theta_v \end{pmatrix} t^0 \tag{17}$$

$$\vec{t'} = C_v\vec{t} = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t^0 \tag{18}$$

We observe that the two time vectors (17), (18) are essentially identical to the two time vectors (15), (16); (just interchanging their order). There is one minor asymmetry, as the position, $x_v = -v\,t^0$ in *eq.* (16), whilst the position $x_v = v\,t^0$ (without the minus sign) in *eq.* (17). However, this difference appears since K_0 and K_v here have the same orientation. Therefore, the clock at the origin of K_0 moves along the negative axis of K_v, (thus, $x_v < 0$ at A); while the origin of K_v moves along the positive axis of K_0, (thus $x_v > 0$ at B), but actually there is a complete symmetry in the results of positions A and B.

This symmetry is our main argument for simultaneity at a distance (at A and B). Following the argument of Section 3.1, the event $(x_0,\ t_0)$ = (0, $t^0\sqrt{1-(v/c)^2}$) at A and the event $(x_v,\ t_v)$ = (0, $t^0\sqrt{1-(v/c)^2}$) at B are simultaneous at a distance, even if they are related to two different RFs. They both provide the clock readings of the basic clocks at these positions, (the origins of K_0 and K_v, respectively).

Further, the two (identical) events, $(x_0,\ t_0)$ and $(x_v,\ t_v)$ at Position A represent 'basic simultaneity', ('same location, same time'). The same is valid for these two events at Position B. In total, we will conclude that all four events illustrated in Fig. 1 are simultaneous (at least in some sense).

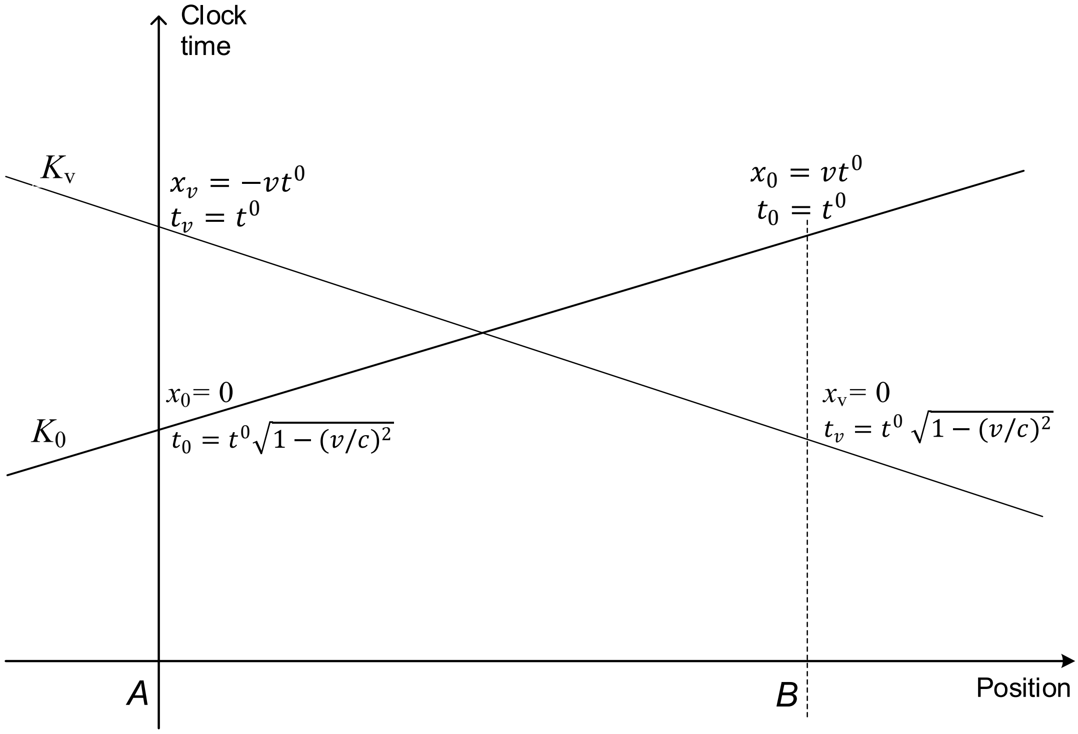

Figure 1 Positions A (at origin of K_0) and B (at origin of K_v) at time $t^0\sqrt{1-(v/c)^2}$ at both origins.

We further argue that there are reasons to associate these four simultaneous events by the time vectors, (15) - (18), rather than the apparently more 'natural' choice of using say $(t_0,\ x_0/c)$ and$(t_v, x_v/c)$. The main reasons for our choice of time vector are that

- The vectors, $\vec{t}$ and $\vec{t'}$ of (15) - (18) all have the same absolute value, t^0, which seems a sensible feature for times of simultaneity.
- The *orthogonal* transformation C_v plays a crucial role in relating the four events of positions A and B. It relates the two events at the same position (either A or B), thus, representing 'basic' simultaneity. Further, it relates the two events being on the same RF (either K_0 and K_v), but at different positions.
- The two components of the time vector, relates to the two BC readings relevant for the event.
- The time vectors, $\vec{t}$ at A, and $\vec{t'}$ at B, specifying the 'simultaneity as a distance', are identical, as both are equal to $\vec{t} = \binom{1}{0}\, t^0$. (However, also a traditional choice, like $(t_0,\ x_0/c)$ and $(t_v, x_v/c)$ satisfies such an equality.)

Now also observe that we can write all the four time vectors, (15)-(18), expressed by one single vector

$$\vec{t}(t^0, w) = \begin{pmatrix} \sqrt{1-(w/c)^2} \\ w/c \end{pmatrix} t^0 = \begin{pmatrix} \cos\theta_w \\ \sin\theta_w \end{pmatrix} t^0 \tag{19}$$

This vector will equal either $\vec{t}$ or $\vec{t'}$ of (15) - (18) by inserting either $w = 0$, $w = v$, or $w = -v$. Here the values, $\pm\, v$, correspond to the distance between A and B, which equals $\mathrm{x} = \mathrm{v}t^0$.

A further illustration of our choice of time vectors is given in Fig. 2. this shows the time vector, $\vec{t} = \vec{t}(t^0, w)$ in the coordinate system, $(t_v, x_0/c)$, both at Positions A, (w=0), and Position B, ($w = v$). Also the $(t_0, x_v/c)$ coordinate system is given (equals $(t_v, x_0/c)$, rotated an angle θ_v). Of course, the rotation from Position A to B is equal to the rotation performed by the matrix, C_v.

We mention that both components of this time vector has a rather simple interpretation. Both at A and B, the first component is the clock reading of the 'basic clock' at the position in question. The second component equals the position (divided by c) of the RF, (here K_0). This is the time required to for the light to go from the origin of K_0 to the current position (A or B). So, both represent 'distance in time'.

The vectors, $\vec{t'} = \vec{t}\,(t^0, w)$, for w = -v and w=0, equal the $\vec{t}$ - vectors in the rotated coordinate system, $(t_0, x_v/c)$. These $\vec{t'}$- vectors are not inserted in Fig. 2, but are easily identified and located, using (16) and (18).

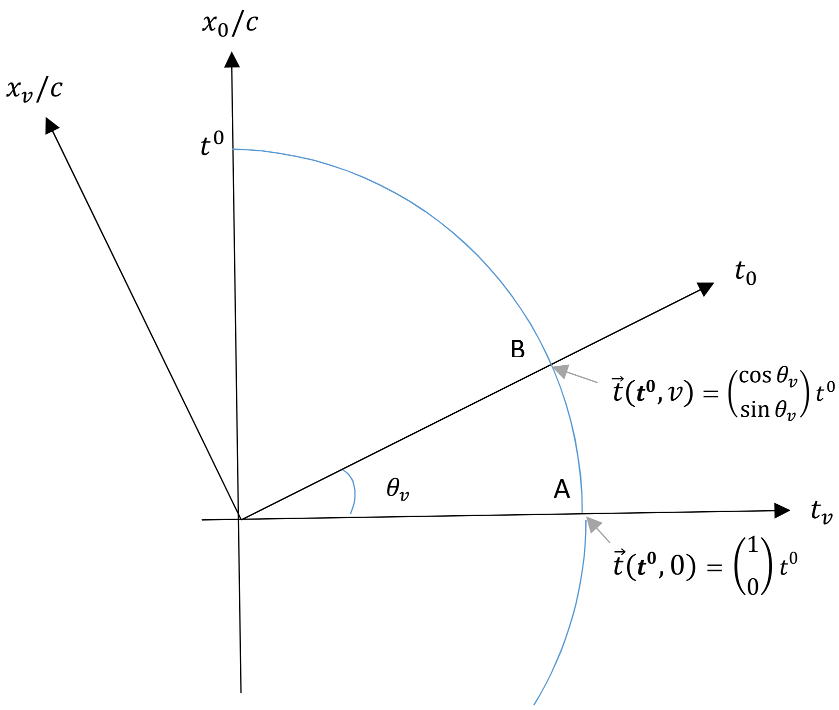

Figure 2 Time vectors, $\vec{t}$ in coordinate system $(t_v, x_0/c)$ at the positions A and B for RFs, K_0 and K_v in the symmetric case when $t_v = t_0\ (= t^0)$. We can also read the same two vectors in the rotated coordinate system, $(t_0, x_v/c)$.

3.3 The time vector

The result for the time vector, $\vec{t}$ of the previous section were derived from an analysis of two specific RFs, K_0 and K_v, moving relative to each other at speed, v. However, as indicated in (19) it suggests that we can introduce a more generic time vector of a specific RF, say K.

On an arbitrary RF, K, we choose a time (clock reading), $t = t^0$, and specify a position, $x = w \cdot t^0$. So letting $w = x/t^0$ we define the time vector for the event (t^0, x) as

$$\vec{t}(t^0, w) = \begin{pmatrix}\sqrt{1-(w/c)^2} \\ w/c\end{pmatrix} t^0$$

We have the following alternative expressions for this vector

$$\vec{t}(t^0, w) = \begin{pmatrix}\sqrt{1-(w/c)^2} \\ w/c\end{pmatrix} t^0 = \begin{pmatrix}t^0\sqrt{1-(w/c)^2} \\ x/c\end{pmatrix} = \begin{pmatrix}\sqrt{(t^0)^2-(x/c)^2} \\ x/c\end{pmatrix} \tag{20}$$

Here we can interpret w as the velocity relative to K of a (possibly imagined) RF, K_w. Thus, (20) is the generalization of the vector, $\vec{t}$, given in (15) and (17) at locations x = 0 and x = v t^0.

Now the vectors (20) correspond to points on the semicircle with radius, t^0 in the (t, x/c) space; see Fig. 3. In summary, for a specific clock time, t^0 and position, $x = wt^0$ on K we interpret the two components of the time vector (20) as follows:

- The first component, $t^0\sqrt{1-(w/c)^2}$ equals the clock reading of the basic clock (BC) located at the origin of the (possibly imagined) RF, K_w; which by now have arrived at the position x = v t^0 on K. We could call this the 'basic time' ('basic clock reading') at this position.

- The second component equals the distance, x from the origin (and thus from the basic clock) of K itself to the given position, measured as the time $t^0 \cdot w/c = x/c$ required for a light flash to go to this distance.

Both components refer to a distance from the 'point of initiation', ($x = t = 0$). So by introducing the vector (20) we split the clock reading ('clock time'), t^0 at position, x = v t^0 on K in two components:

1. $t^0 \cdot \sqrt{1-(w/c)^2}$; the clock reading of the (imagined) BC currently at this position
2. $t^0 \cdot w/c$; time required for a light flash to go from the RF's own BC (at its origin) to the current position.

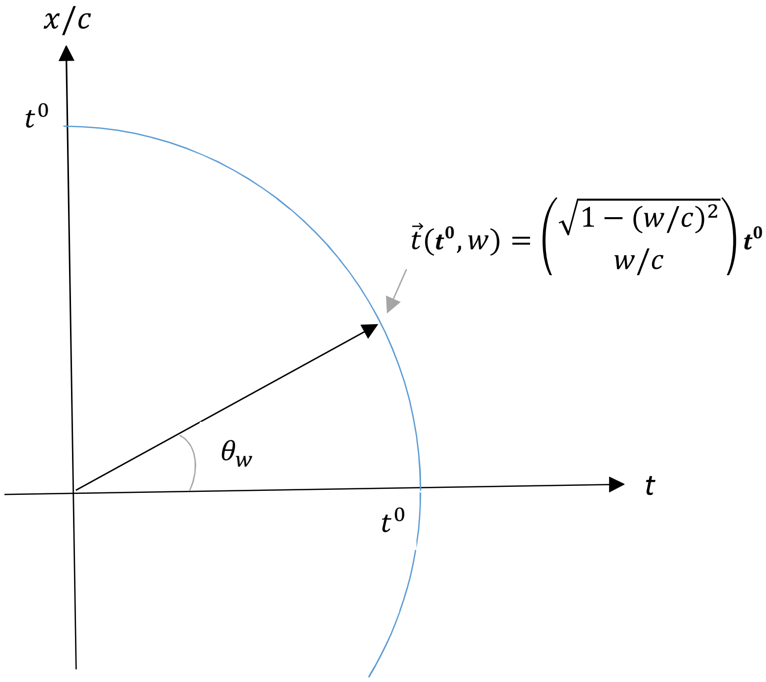

Figure 3 Time vector, $\vec{t}(t^0, w) = \begin{pmatrix} \sqrt{1-(w/c)^2} \\ w/c \end{pmatrix} t^0$ for a specific clock time t^0 at the position, $x = wt^0$; (here sin θ_w= w/c).

Only for $x = w = 0$, the BC of the first component refers to the BC at K itself; and in that case the second component equals zero. At all other positions, the first component refers to an 'imagined BC'. Thus, we implicitly assume that we initially synchronized the basic clocks at K and K_w.

We note that the absolute value of our time vector is independent of w:

$$\left| \vec{t}(t^0, w) \right| = t^0 \qquad (21)$$

and equals the clock reading at any position, $x = wt^0$ on K. This can be used to define a week form for simultaneity. Here time vectors having the same absolute value (21) represents 'simultaneity in the perspective of' the RF K. Events on K having time vectors with the same absolute value, means that they have same clock time on K and thus are simultaneous on K.

Further, we have realized (Section3.2) that when two RFs are moving relative to each other at speed, v, the (orthogonal) LT represents a rotation of our time vector (not changing its absolute value). Thus, also the 'basic simultaneity', (same event described by two different RFs) implies that the two time vectors have same absolute value. Thus, two time vectors of same absolute value, related by an orthogonal transformation, are either 'simultaneous in the perspective of' an RF if the vectors are related to the same RF, or they demonstrate 'basic simultaneity', if they are related to different RFs. (Recall here that we are referring to two RFs with a common 'point of initiation'.)

However, additional requirements seems needed to define 'simultaneity at a distance', *cf.* the two positions A and B of Section 3.2. In that case, one of the time vectors at each location equals $\binom{1}{0}\, t^0$, and the other is an orthogonal transformation (rotation) of this.

In total, it seems the given definition of a time vector has some attractive features, and in particular seems suitable for specifying various forms of simultaneity.

We observe that there is a link between the present approach and Minkowski's approach to space-time. As stated in Petkov (2012), Minkowski refers to the time of the basic clock, (*cf.* $\sqrt{(t^0)^2 - (x/c)^2}$ of (20)), as 'proper time, and t^0 as 'coordinate time'. Now Minkowski (1909) introduced a four-dimensional space-time, and defined space-time distance as $\sqrt{c^2t^2 - x^2 - y^2 - z^2}$ in his four-dimensional space. So even if we extended our approach to handle three space coordinates, the suggested measures of distance differ. Our distance (from the point of initiation) simply equals the clock time, $t = t^0$.

3.4 Time formulated as a complex variable

We can of course write the time vector for the event (t^0, x) as a complex variable. Now in in polar form, we can write the vector $\vec{t}(t^0, w)$ in (21) as

$$\mathbf{t}(t^0, w) = t^0 e^{i\theta_w}, \quad (w = x/t^0) \tag{22}$$

The magnitude, t^0 still equals the clock reading, and $w = x/t^0$ can be interpreted as a velocity relative to *K* of an (imagined) 'basic clock', now having arrived at the position, *x*. The argument, $\theta_w \in (-\pi/2,\ \pi/2)$, is given by

$$\sin \theta_w = w/c.$$

When $\theta_w = 0$, (and $w = 0$) this BC is located on *K* itself, and the time variable becomes a real number.

Both the real part, $\mathrm{Re}(\mathbf{t}(t^0, w)) = t^0\sqrt{1 - (w/c)^2} = t^0 \cos \theta_w$, and the imaginary part, $\mathrm{Im}(\mathbf{t}(t^0, w)) = t^0(w/c) = t^0 \sin \theta_w = x/c$ are defined above.

Finally, we can generalize (22) to hold for a three-dimensional space, with coordinates (*x, y, z*). We then define *w* by $w = \sqrt{x^2 + y^2 + z^2}\ /t^0$. Thus, *w* essentially specifies a position (distance from origin) at time t^0 on the RF *K*, and we still have $\sin \theta_w = w/c$, using the new definition of *w*.

4 The travelling twin

The travelling twin paradox is frequently discussed, *e.g.* see Schuler and Robert (2014). As stated for instance in Mermin (2005) this illustrates that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together. We discussed the example at length in Hokstad (2018), and now just give a short comment related to symmetry which is rather fundamental in this thought experiment, *cf.* Debs *et al* (1996).

We start out with two synchronized clocks at the origins of two reference frames; the RF of the earth, and the RF of the rocket of the travelling twin. This is exactly the situation described in Ch. 3. In our symmetry argument, we concluded that identical readings of these two basic clocks implies simultaneity. Thus, by the arrival of the travelling twin at the star it is fair to say that both twins have aged the same amount of years. Both twins observe two clock readings, their own clock, and the clock on the other twin´s passing RF; and both observe their own clock to be slower. We also refer to the illustration in Fig. 1; the lower parts of positions A and B correspond to (the clocks possessed by) the two twins.

Now the actual clock readings by the reunion of the twins will depend of the further experimental set-up. However, if the travelling twin immediately start his return, and the earthbound twin remains on the earth, we agree with the standard argument, and obtain the same result as *e.g.* Mermin (2005); *cf.* Hokstad (2018).

5 Conclusions

The Lorentz transformation (LT) relates the (clock, space) observations of one reference frame (RF) with the (clock, space) observations at the same location on another RF. In the present work we define a state vector, which combines the clock reading of one RF with the space coordinate of the other RF. Thus, we obtain two vectors, related by an *orthogonal* version of the LT. We can interpret both components of these vectors as aspects of time and refer to them as time vectors, but they also involve location.

The absolute value of our time vector equals the clock time at the relevant position, and it is seen as a measure for the overall distance in time from the 'point of initiation', *i.e.* time 0 when the origins of the two RFs had a common location. This vector provides a means to define certain forms of simultaneity. Time vectors on the same RF, having same absolute value specify events that are simultaneous 'in the perspective of this RF. Further, if the orthogonal time vectors specify events on different RFs, they are related by the orthogonal LT, and therefore specify 'basic simultaneity'.

However, the main simultaneity result presented here is based on one fundamental claim: Due to the inherent symmetry of the two given RFs we have that the two clocks at the origin of the two RFs at time 0 will remain 'synchronized'. We refer to these as 'basic clocks', and identical clock readings of these two symmetric clocks will represent simultaneity at a distance. The suggested time vector is suitable to identify also this form of simultaneity.

Considering the time vector for any event (t , x) of an actual RF, we introduce an 'imagined' RF with its basic clock currently located at the chosen position, x. The first component of the time vector equals the clock reading of this (imagined) 'basic clock'. The second component equals the distance to the 'basic clock' of the actual RF, measured as the time required for a light signal to go this distance. We might also give the time vector, given as a complex variable. The approach has similarities with the time-space of Minkowski.

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