A simple proof of the generalized Poincare conjecture.

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Abstract

We show that any closed n dimensional topological connected and simply connected space (or piecewise linear manifold) is homeomorphic to the n sphere.

1 Proof

Any topological space $\mathcal{M}$ can be endowed with a path metric; take a point $p \in \mathcal{M}$ and consider the level surfaces $S_r = \{x|d(p, x) = r\}$. For a n dimensional topological space, those will all be homeomorphic to $n-1$ spheres for r sufficiently small. Since $\mathcal{M}$ has no boundary, there must exist a critical r_0 such that S_{r_0} is not homeomorphic to a $n-1$ sphere; out of continuity arguments, it is either (a) a point (b) pointlike contractions and point identifications. In case (a), we are done since we have obtained a topological S^n. In case (b) the contractions will create disconnected components whereas the identifications will create "tori"; every disconnected component being a closed $n-1$ dimensional topological space. In case of the "tori", this will create closed non-contractible curves; the "tori" cannot shrink to a point given that this would violate the condition that it is a topological space of dimension n. Identifying different components or unpinshing the tori (which should happen eventually) leads to non-contractible curves. Therefore, the generic situation is that of a tree graph with points blown up to a $n-1$ sphere except at the endpoints. Such topological space is clearly homeomorphic to S^n. These arguments repeat ad verbatim for piecewise linear manifolds settling all cases in that regard. Regarding smooth structures, the theorem remains valid in the same differentiability class; Donaldson's counterexample in seven dimensions, if it exists, is clearly false. QED

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