A proof of the falsity of the axiom of choice.

Johan Noldus*

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Abstract

We observe two things in this paper : namely that the Banach Tarski paradox is false and that the correct part of the proof leads to a violation of the axiomof choice.

1 Proof.

The standard argument behind the Banach Tarski paradox goes as follows; one constructs two rotations a, b around an angle $r2\pi$ with r irrational around the x and z axis respectively. One considers the free group F_2 constructed by a, b which is split into five disjoint parts $S(a), S(a^{-1}), S(b), S(b^{-1}), e$ where $S(a)$ contains all irreducible words starting with the letter a. Clearly, $S(a) \sim S(b)$ geometrically and equally so when inverses are taken. The *axiom of choice* allows one to substract a set M containing one representant of each F_2 orbit on the two sphere. Consider the sets

$$A = S(a)M, B = S(a^{-1})M, C = S(b)M, D = S(b^{-1})M, M$$

and attribute the numbers $\lambda, \mu, \lambda, \mu, \lambda, \kappa \geq 0$ to them which represent volumes which are rotationally invariant. Given that $bD = A \cup B \cup D \cup M$ we obtain that $\kappa = \lambda = \mu = 0$ and therefore, since $S^2 = M \cup C \cup bD = M \cup A \cup aB$ and, given that M occurs twice, we have no two metric unit spheres. However, it does show that for rotationally invariant measures, this hypothetical partition only carries the trivial zero one. Therefore, we have either that the axiom of choice is wrong or it is a wrong idea that all imaginable measures allowed from a metric point of view should be allowed for.

Notice further that $b^2D = S^2$ so that actually A, B, C, M should be empty given that b is continuous. Hence, M does not exist which proves the falsity of the axiom of choice.

*email: johan.noldus@gmail.com, Relativity group, departement of mathematical analysis, University of Gent, Belgium.