Resolution to the Banach-Tarski Paradox

This experiment logically tests the Banach-Tarski Paradox as an equivalence and an implication.

At en.wikipedia.org/wiki/Banach%E2%80%93Tarski_paradox , we find after "[s]ome details fleshed out", Step 3:

$$S^2 = \ldots = (E - D) \cup (S^2 - E) = S^2 - D \tag{1.1}$$

We assume the Meth8 apparatus using VŁ4, where the designated proof value is T tautology and F contradiction. The 16-value truth table is presented row major and horizontally.

LET: s S^2; q E; p D; = Equivalent to; ∪ + Or; ⊃ > Imply; - Not Or; & And

s = (((q-p)+(s-q)) = (s-p)) ; `FTTF FTTF FTTT FTTT` (1.2)

Because Eq. 1.2 is not tautologous, we weaken the argument for the equivalent to connective =, with replacement by the connective > Imply.

s > (((q-p)+(s-q)) > (s-p)) ; `TTTT TTTT FTTT FTTT` (1.3)

Eq. 1.3 is the equivalent to writing Eq 1.1 in the text symbols as:

$$S^2 \supset (E - D) \cup (S^2 - E) \supset S^2 - D. \tag{1.4}$$

While Eq. 1.3 is relatively less contradictory than Eq.1.2, it remains that both Eq. 1.1 and Eq. 1.4 in the text symbols remain as not tautologous.

This means the Banach-Tarski Paradox, as rendered, is not a paradox, not a theorem, and non-tautologous.

What follows is that the Von Neumann Paradox on the Euclidean plane is also suspicious as a paradox and possibly not a paradox.

Refutation of axiom of choice via refutation of the Gödel-Löb Axiom

This example replicates the proof for provability logic of the Gödel-Löb axiom GL as $\Box(\Box p \rightarrow p) \rightarrow \Box p$. If p is "*choice*", this transcribes in words to: "The necessity of *choice*, as always implying *a choice*, implies always *a choice*."

The axiom transcribes to #(#p>p)>#p for test input to Meth8 with output in Tab. 6. Model 2.2 is validated as one of five models. Hence by VŁ4 the Gödel-Löb axiom is suspect.

For the GL axiom to be validated in five of five models, the expression is rewritten as $\Box(\Box p \rightarrow p) \leftrightarrow (p \vee \neg p)$, in words: "The necessity of *choice*, as always implying *a choice*, is equivalent to always *a choice* or *no choice*."

A simpler rendition of a validated GL-type axiom is either $\Box(\Box \neg p \rightarrow p) \leftrightarrow \Box p$ or $\Box(\Box p \rightarrow \neg p) \leftrightarrow \Box \neg p$ as respectively in words: "The necessity of *no choice*, as always implying *a choice*, is equivalent to always *a choice*."; or "The necessity of *choice*, as always implying *no choice*, is equivalent to always *no choice*."

If GL fails, then so also does Zermelo-Fraenkel set theory and axiom of choice (ZFC) as the basis of modern mathematics.

Table 6

Test input as processed is: #(#p>p)>#p				
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2
#p				
FNFN	UEUE	UUUU	UIUI	UPUP
p				
FTFT	UEUE	UEUE	UEUE	UEUE
#(#p>p)				
NNNN	EEEE	UUUU	IIII	PPPP
#p				
FNFN	UEUE	UUUU	UIUI	UPUP
#(#p>p)>#p ; not validated tautologous				
CTCT	UEUE	EEEE	PEPE	IEIE
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2