

**Refutation of Cantor's diagonal argument** © Copyright 2017 by Colin James III All rights reserved.

From: en.wikipedia.org/wiki/Cantor%27s\_diagonal\_argument

"A generalized form of the diagonal argument was used by Cantor to prove Cantor's theorem: for every set  $S$ , the power set of  $S$ —that is, the set of all subsets of  $S$  (here written as  $\mathbf{P}(S)$ )—has a larger cardinality than  $S$  itself. This proof proceeds as follows: Let  $f$  be any function from  $S$  to  $\mathbf{P}(S)$ . It suffices to prove  $f$  cannot be surjective. That means that some member  $T$  of  $\mathbf{P}(S)$ , i.e. some subset of  $S$ , is not in the image of  $f$ . As a candidate consider the set:

$$T = \{ s \in S: s \notin f(s) \}. \tag{0.1}$$

For every  $s$  in  $S$ , either  $s$  is in  $T$  or not. If  $s$  is in  $T$ , then by definition of  $T$ ,  $s$  is not in  $f(s)$ , so  $T$  is not equal to  $f(s)$ . [1.1]

On the other hand, if  $s$  is not in  $T$ , then by definition of  $T$ ,  $s$  is in  $f(s)$ , so again  $T$  is not equal to  $f(s)$  ... [2.1]

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VL4. Meth8 allows to mix four logical values with four analytical values. The designated *proof* value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	$p=p$	T	Tautology	proof	11	3
2	$p@p$	F	Contradiction	absurdum	00	0
3	$\%p\>\#p$	N	Non-contingency	truth	01	1
4	$\%p<\#p$	C	Contingency	falsity	10	2

LET:  $\sim$  Not;  $+$  Or;  $\&$  And;  $\setminus$  Not and;  $>$  Imply;  $<$  Not imply,  $\in$ ;  $=$  Equivalent to;  $@$  Not equivalent to;  $\#$  all, every;  $\%$  some, each;  $pqr$  fTSs;  $s \notin f(s) \sim (s > f(s))$

Results are the repeating proof table(s) of 16-values in row major horizontally.

$$q = ((s < r) > \sim (s < (p \& s))) ; \quad \text{F F T T} \quad \text{F F T T} \quad \text{T F F T} \quad \text{F F T T} \tag{0.2}$$

$$(q = ((s < r) > \sim (s < (p \& s)))) > ((\#s < r) > ((s < q) > \sim (s > (p \& s)))) > (q @ (p \& s)) ; \quad \text{T T T T} \quad \text{T T T T} \quad \text{F T T F} \quad \text{T T T F} \tag{1.2}$$

$$(q = ((s < r) > \sim (s < (p \& s)))) > ((\sim (\#s < r) > ((s < q) > (s > (p \& s)))) > (q @ (p \& s))) ; \quad \text{T T T T} \quad \text{T T T T} \quad \text{C T T F} \quad \text{T T T F} \tag{2.2}$$

Because Eqs. 1.2 and 2.2 result in the same consequent, they are rewritten to remove respective common terms and set as an equivalence according to Eqs. [1.1] and [2.1].

$$((\#s < r) > ((s < q) > \sim (s > (p \& s)))) = (\sim (\#s < r) > ((s < q) > (s > (p \& s)))) ; \quad \text{T T T T} \quad \text{T T T T} \quad \text{N C T T} \quad \text{F T T T} \tag{3.2}$$

Eqs. 1.2 and 2.2 as rendered are *not* tautologous. Hence Cantor's diagonal argument is not supported.