A Novel Closed Form for Truncated Distributions with Integer Parameters & Hyper-Normal Probabilistic-Weighted Correlation

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Ranged Normal Probability

$$Let\ \mathcal{R} := [j, k] = \{x \in \mathbb{Z} | j < k - 1\}$$

The probability distribution formulation is valid for all integer rangers of at least 3 and assumes at least a difference of two between the k and j. It would be very difficult to extend beyond that range as it would affect the constants in the Gaussian formula here:

$$\int_{j}^{k} \left(\varpi(x|j,k) = \mathfrak{a}_{k-j} e^{-\frac{\left(x - \frac{k+j}{2}\right)^2}{0.5(k-j)-0.5}}\right) dx = 1$$

When solving for $\mathfrak{a}_{k-j}$, as that is the well-researched component of the Gaussian function that controls the amplitude and thus the area under the curve, we find:

$$\mathfrak{a}(r) = \frac{\mathfrak{h}(r)}{\operatorname{erf}(\sqrt{\mathfrak{u}(r) + 3^{\mathfrak{u}(r)}})}$$

While $\sqrt{\mathfrak{u}(r) + 3^{\mathfrak{u}(r)}}$ may appear to be a very odd form, manual evaluation found solutions of $\operatorname{erf}(1.5)\ and\ \operatorname{erf}(2.25)\ for\ r = 3\ and\ 9$. It made logical sense that the answer had to be to the power of n. The solution is remarkably clean in my mind.

$$\mathfrak{h}(r) = \sqrt{\frac{2}{\pi(r-1)}} | r\ mod\ 2 = 0 \ \vee \ \sqrt{\frac{1}{\pi(\frac{r}{2} - \frac{1}{2})}} \quad \mathfrak{u}(r) = \frac{\ln(3)\mathfrak{b}(r) - \mathfrak{c}(r)\mathcal{W}(\ln(3)\mathfrak{d}(r)3^{\frac{\mathfrak{g}(r)}{\mathfrak{c}(r)}})}{\ln(3)\mathfrak{c}(r)}$$

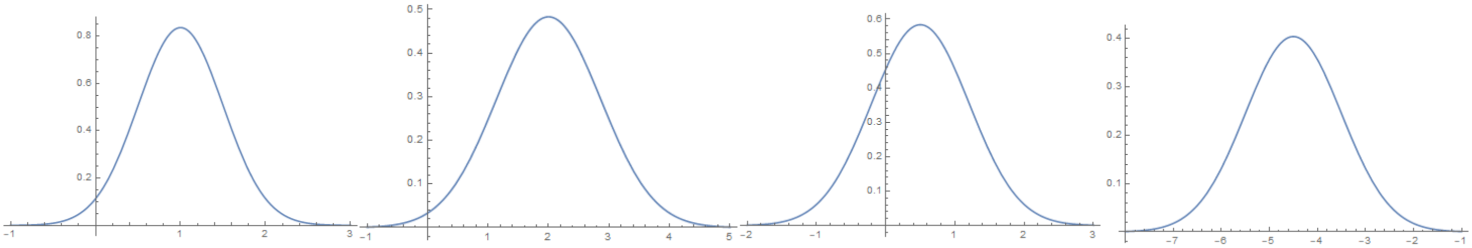
Where $\mathcal{W}$ is the Product Log function.

$$\mathfrak{b}(r) = (2^{r-1} | r < 10 \vee \frac{r^2}{2}) \ | r\ mod\ 2 = 0 \ \vee \ r^2$$

$$\mathfrak{c}(r) = (r-1) |\ r\ mod\ 2 = 0 \ \vee 2r - 2$$

$$\mathfrak{d}(r) = \left(9\left(3^{\frac{r-2}{2}-1}\right) \middle| r > 2 \ \wedge r\ mod\ 2 = 0\right) \vee 9 | r = 2 \vee 3^{\frac{r}{2}+\frac{1}{2}}$$

$$\mathfrak{g}(r) = \left(\left(\frac{r}{2} + 1\right) \middle| r > 2 \ \wedge r\ mod\ 2 = 0\right) \vee 0 | r = 2 \vee 1$$



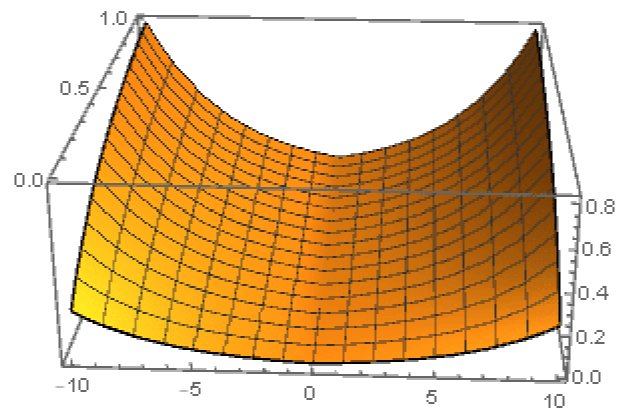
EXAMPLE PLOTS OF THE FINAL FUNCTION

(LEFT TO RIGHT) J=0, K = 2, J=0, K = 4 , J= -1, K= 2, AND J = -7, K = -2

This formulation provides a x spread of a fixed range, -10 to 10, and a normal distribution across those points with a guaranteed area of 1 under the curve providing a true probability distribution.

HYPER-NORMAL WEIGHTING

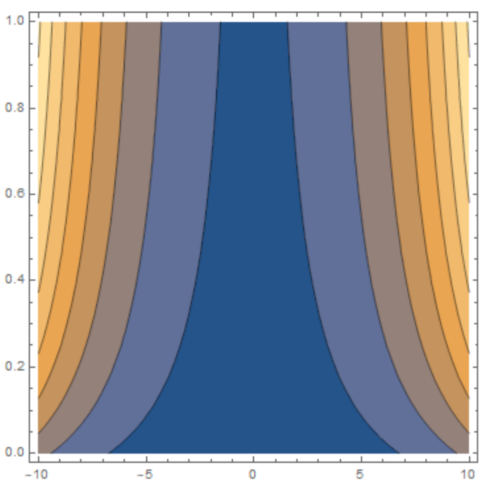
I constructed a zeta function which produces values [0,1] for a weighted combination of probability and score. It is what I call a 3D Sigmoid Function and its plot is on the right.



$$\zeta(x,\varpi) = \frac{\left(\frac{4}{\pi}\right)^{|x|}\left(\sqrt[3]{\pi}\right)^{-\frac{1}{\varpi+0.25}}}{\left(\frac{4}{\pi}\right)^{-|x|}\left(\frac{2\pi}{7}\right)^{-\frac{1}{\varpi+0.25}^{2}}+10}$$

$$\varsigma(j,k) = \frac{\sum_{x=j}^{k} x\,\zeta\big(x,\varpi(x|j,k)\big)}{\sum_{x=j}^{k}\zeta\big(x,\varpi(x|j,k)\big)}$$

The sigma function produces a weighted average of the score and the zeta weight for that score by summing the multiple of the score and weight and dividing that sum by the sum of weights. This takes into account both the normal probability distribution and creates hyper-normal behavior due to the behavior of zeta which favors heavier weights in a sigmoid fashion.



<- Zeta Contour

Sigma Contour ->

The zeta function will generate a heavier weight to values more disjoint from the origin, beginning to accentuate around +/- 5. Since the probabilities will be more distributed on wider ranges the resultant value of sigma will approach the expectation of the underlying distribution.