Title: Formula for Prime numbers and composite numbers.
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Abstract: This paper develops a modified an old and well-known expression for calculating and obtaining all prime numbers greater than three and composite numbers divisible by numbers greater than 3. The key for this formula to work correctly is in the equalities and inequalities. These equalities and inequalities are created from the uncovering of the patterns of the composite numbers. The composite numbers follow very clear and determining patterns, making it possible to find them through a formula.

Keywords: Prime numbers, composite numbers.

Introduction

The study of the prime numbers is wonderful, But to understand them, first study the composite numbers, in the absence of an expression that involves all of them I have investigated and I have discovered a brilliant expression that contains all the prime numbers greater than 3 and all composite number that are not divisible by 2 and by 3. This expression comes from investigating first how they are distributed the composite numbers, this allowed me to explore its order and understand its mechanism. The expression of the prime numbers is its result.
The expression to obtain the prime numbers is similar to how we use the sieve of Eratosthenes.

Methods

We can obtain composite numbers that are not divisible by 2 and by 3 under the expressions $(6 * n + 1)$ and $(6 * n - 1)$, This paper shows through equalities which are the composite numbers.
The prime numbers greater than three can be expressed under the expressions $(6 * n + 1)$ and $(6 * n - 1)$, This paper shows through inequalities which are the prime numbers.

Theorem 1

A) Table of Order number in columns A, B

This table is used to calculate which are the divisors of a number and its multiples.
We apply the results of it in the table on the next page. (Table2)

Sequence numbers
$\beta(a) = (6 * n + 1) = 7,13,19,25,31$ etc.
n>0

$\beta(a)$ - L= Order	Location	β (a)
	1	1
	2	5
7-3=**4**	3	**7**
	4	11
13-5=**8**	5	**13**

Sequence numbers
$\beta(b) = (6 * n - 1) = 5,11,17,23,29,35,$ etc.
n>0

$\beta(b)$- L= Order	Location	$\beta(b)$
5-1=**4**	1	5
	2	7
11-3=**8**	3	11
	4	13
17-5=**12**	5	17

	6	17		6	19
19-7=**12**	7	**19**	23-7=**16**	7	23
	8	23		8	25
25-9=**16**	9	25	29-9=**20**	9	29
	10	29		10	31
31-11=**20**	11	**31**	35-11=**24**	11	35
	12	35		12	37
37-13=**24**	13	37	41-13=**28**	13	41
	14	41		14	43
43-15=**28**	15	43	47-15=**32**	15	47
	16	47			

Table 1

In yellow Prime numbers, in red Composite numbers. The location of the numbers is always odd.
As we can see in both columns we obtain the same results, order numbers that start with the number 4 and add 4 with the next one until infinity.

Theorem 2

A) Composite numbers construction chart Multiples and location of composite numbers in columns A, B

In this table we can see on the left the numbers in column A, on the right those in column B.
The black boxes indicate that this number is a multiple in column A, the blue boxes indicate that this number is a multiple in column B.

Numbers in column A of the form $(6 * n + 1)$
Numbers in column B of the form $(6 * n - 1)$

Examples

A) The 91 is located in the column A (black), in its row appear the black boxes 13 and 7, well the 91 has these divisors.
B) 119 is located in column B (blue), in its row appear the black boxes 17 and 7, well the 119 has these divisors.
C) The 115 is located in column A (black), in its row appear the black boxes 5 and 23, well the 115 has these divisors.
D) 143 is located in column B (blue), row black squares 11 and 13 appear, because 119 has these divisors.

B) The Order number

The Order number is used to calculate how the Prime number divisor interacts with the opposite column. These graphical tables allow to observe and understand the mechanism by which the composite numbers are ordered and consequently the prime numbers.

Sequence of each column.

$A = (6*n+1)$
$B = (6*n+5)$ or $(6*n-1)_{n>0}$
$C = (6*n+2)$
$D = (6*n+3)$
$E = (6*n+4)$
$F = (6*n+6)$

$n \geq 0$

The Order number (left) — The Order number (right)

20	16	12	8	4	A	C	D	E	B	F	4	8	12	16	20	24
31	25	19	13	7	Prime numbers						5	11	17	23	29	35
					1	2	3	4	5	6						
					7	8	9	10	11	12	1					
				1	13	14	15	16	17	18	2	1				
			1	2	19	20	21	22	23	24	3	2	1			
		1	2	3	25	26	27	28	29	30	4_{5}	3	2	1		
	1	2	3	4_{7}	31	32	33	34	35	36	5	4	3	2	1	
1	2	3	4	5	37	38	39	40	41	42	1	5	4	3	2	1
2	3	4	5	6	43	44	45	46	47	48	2	6	5	4	3	2
3	4	5	6	7	49	50	51	52	53	54	3	7	6	5	4	3
4	5	6	7	1	55	56	57	58	59	60	4_{5}	8_{11}	7	6	5	4
5	6	7	8_{13}	2	61	62	63	64	65	66	5	9	8	7	6	5
6	7	8	9	3	67	68	69	70	71	72	1	10	9	8	7	6
7	8	9	10	4_{7}	73	74	75	76	77	78	2	11	10	9	8	7
8	9	10	11	5	79	80	81	82	83	84	3	1	11	10	9	8
9	10	11	12	6	85	86	87	88	89	90	4_{5}	2	12_{17}	11	10	9
10	11	12_{19}	13	7	91	92	93	94	95	96	5	3	13	12	11	10
11	12	13	1	1	97	98	99	100	101	102	1	4	14	13	12	11
12	13	14	2	2	103	104	105	106	107	108	2	5	15	14	13	12
13	14	15	3	3	109	110	111	112	113	114	3	6	16	15	14	13
14	15	16	4	4_{7}	115	116	117	118	119	120	4_{5}	7	17	16_{23}	15	14
15	16_{25}	17	5	5	121	122	123	124	125	126	5	8_{11}	1	17	16	15
16	17	18	6	6	127	128	129	130	131	132	1	9	2	18	17	16
17	18	19	7	7	133	134	135	136	137	138	2	10	3	19	18	17
18	19	1	8_{13}	1	139	140	141	142	143	144	3	11	4	20	19	18
19	20	2	9	2	145	146	147	148	149	150	4_{5}	1	5	21	20_{29}	19
20_{31}	21	3	10	3	151	152	153	154	155	156	5	2	6	22	21	20
21	22	4	11	4_{7}	157	158	159	160	161	162	1	3	7	23	22	21
22	23	5	12	5	163	164	165	166	167	168	2	4	8	1	23	22
23	24	6	13	6	169	170	171	172	173	174	3	5	9	2	24	23
24	25	7	1	7	175	176	177	178	179	180	4_{5}	6	10	3	25	24_{35}
25	1	8	2	1	181	182	183	184	185	186	5	7	11	4	26	25
26	2	9	3	2	187	188	189	190	191	192	1	8_{11}	12_{17}	5	27	26
27	3	10	4	3	193	194	195	196	197	198	2	9	13	6	28	27
28	4	11	5	4_{7}	199	200	201	202	203	204	3	10	14	7	29	28

Table 2

The number that accompanies the 4, 8, 12, 16 etc., (the order number) in small is the divisor Prime.

On the left of blue we can see how the order numbers are ordered every 4 numbers from 4. The same happens on the right in black the numbers ordered every 4 numbers from 4.

The black boxes on the left are arranged every 7 boxes between a divider and another, the blue boxes on the right are ordered every 7 boxes. The blue boxes on the left are arranged every 5 boxes between a divider and another, the black boxes on the right are ordered every 5 boxes.

To this table we can complete it with more numbers and divisors to know how the composite numbers are constructed in columns A, B. Columns C, E, F are composite, since they are divisible by 2 and columns D, F are composite since they are divisible by 3.

Theorem 3

A) Formula to obtain composite numbers

This formula allows us to obtain all composite numbers divisible by numbers greater than 3. This formula will be prepared to look for equalities. The formula is born from observing how the β divisor numbers behave and how they generate composite numbers in a systematic and mechanical way. This formula works in a very similar way to the Sieve of Eratosthenes, first look for multiples of 5, then those of 7, those of 11, although it also looks for multiples of composite numbers such as 25, 35 or 49. This formula works from the β sequence.

$$= \beta * (6 \pm 1 + 6 * z)$$

$\beta = (6 * n \pm 1) = 5,7,11,13,17,19,23,25,29,31,35,37,41,43,47,49,\ldots..$
$n > 0$
$Nc = Composite\ number$
$Z \geq 0$

Formula 1: composite number within the sequence β_b

$$Nc_b = Composite\ number \text{ of the form } (6 * n - 1) = \beta_a * (6+1+6*z)$$
$$= \beta_b * (6-1+6*z)$$

$\beta_a = (6 * n - 1)$ = 5,11,17,23,29,35,41,.....
$\beta_b = (6 * n + 1)$ = 7,13,19,25,31,37,43,....

Example

$Nc_b = (6 * n - 1)$ $=5*(7+6*z)$ $=$ $(6 * n - 1)$ $=35,65,95,125,\ldots\ldots$ $=$
$=7*(5+6*z)$ $=35,77,119,161,203,\ldots\ldots.$
$=11*(7+6*z)$ $=77,143,209,275,\ldots..$
$=13*(5+6*z)$ $=65,143,221,299,\ldots$
$=17*(7+6*z)$ $=119,221,323,425,\ldots$
$=19*(5+6*z)$ $=95,209,323,437,\ldots\ldots.$
$=23*(7+6*z)$ $=161,299,437,\ldots..$
infinitely continuous — infinitely continuous

Formula 2: composite number within the sequence β_a

$$Nc_a = Composite\ number \text{ of the form } (6 * n + 1) = \beta_b * (6-1+6*z)$$
$$= \beta_a * (6+1+6*z)$$

$\beta_a = (6 * n + 1)$ = 7,13,19,25,31,37,43,....
$\beta_b = (6 * n - 1)$ = 5,11,17,23,29,35,41,.....

Example

$Nc_a = (6*n+1)$ $=5*(5+6*z)$ $=$ $(6*n+1)$ $=25,55,85,115,....$ $=$

$=5*(5+6*z)$	=25,55,85,115,....
$=7*(7+6*z)$	=49,91,133,175,.........
$=11*(5+6*z)$	=55,121,187,253,......
$=13*(7+6*z)$	=91,169,247,325,....
$=17*(5+6*z)$	=85,187,289,391,......
$=19*(7+6*z)$	=133,247,361,.........
$=23*(5+6*z)$	=115,253,391,......
infinitely continuous	infinitely continuous

Five examples of individual composite numbers.

$$Nc = \beta*(6 \pm 1 + 6*z)$$

Nc 5= $\beta*(5+6*z) =$ $5*(5+6*z) =$ 25,55,85,115,145,175,205,235,265,...........
Nc 5= $\beta*(7+6*z) =$ $5*(7+6*z) =$ 35,65,95,125,155,185,215,245, ...
The distance between each number is ***β*** *6= 5*6=30
In Nc (a) begins in ***β*** *5= 5*5=25
In Nc (b) begins in ***β*** *7= 5*7=35

Nc 7 = $\beta*(5+6*z) =$ $7*(5+6*z) =$ 35,77,119,161,203,245,287,329,371,...........
Nc 7= $\beta*(7+6*z) =$ $7*(7+6*z) =$ 49,91,133,175,217,259,301,343, ...
The distance between each number is ***β*** *6= 7*6=42
In Nc (a) begins in ***β*** *7= 7*7=49
In Nc (b) begins in ***β*** *5= 7*5=35

Nc 11= $\beta*(5+6*z) =$ $11*(5+6*z) =$ 55,121,187,253,319 ,................
Nc 11= $\beta*(7+6*z) =$ $11*(7+6*z) =$ 77,143,209,275,341, ...
The distance between each number is ***β*** *6= 11*6=66
In Nc (a) begins in ***β*** *5= 11*5=55
In Nc (b) begins in ***β*** *7= 11*7=77

Nc 13= $\beta*(5+6*z) =$ $13*(5+6*z) =$ 65, 143, 221, 299, ...
Nc 13= $\beta*(7+6*z) =$ $13*(7+6*z) =$ 91,169, 247, ...
The distance between each number is ***β*** *6= 13*6=78
In Nc (a) begins in ***β*** *7= 13*7=91
In Nc (b) begins in ***β*** *5= 13*5=65

Nc 17= $\beta*(5+6*z) =$ $17*(5+6*z) =$ 85, 187, 289, ...
Nc 17= $\beta*(7+6*z) =$ $17*(7+6*z) =$ 119, 221,323, ...
The distance between each number is ***β*** *6= 17*6=102
In Nc (a) begins in ***β*** *5= 17*5=85
In Nc (b) begins in ***β*** *7= 17*7=119

We can keep adding more numbers ***β*** and expand the formula.

Theorem 4

A) Formula for obtaining Prime numbers

The formula for obtaining prime numbers is divided into two sequences, $(6*n+1)$ and $(6*n-1)$. These sequences are well known, my contribution will be in the equalities and inequalities to find the numbers we are looking for. This formula is obtained from the patterns of the composite numbers.

$$\neq \beta*(6 \pm 1 + 6*z)$$

P= Prime numbers >3

$n => 0$
$Z =\geq 0$

Formula 1: Prime numbers within the sequence $\beta_a = 7,13,19,25,31,37,43,49,55,..$

$$P_a = (6*n+1)_{\substack{\neq\beta_b*(5+6*z)\\ \neq\beta_a*(7+6*z)}}$$

β_a= (6*n+1) = 7,13,19,25,31,37,43,....
β_b= (6*n-1) = 5,11,17,23,29,35,41,.....
$n > 0$

Example

$$P_a = (6*n+1)_{\substack{\neq5*(5+6*z)\\ \neq7*(7+6*z)\\ \neq11*(5+6*z)\\ \neq13*(7+6*z)\\ \neq17*(5+6*z)\\ \neq19*(7+6*z)\\ \neq23*(5+6*z)\\ \text{infinitely continuous}}} = (6*n+1)_{\substack{\neq25,55,85,115,....\\ \neq49,91,133,175,.........\\ \neq55,121,187,253,......\\ \neq91,169,247,325,....\\ \neq85,187,289,391,......\\ \neq133,247,361,.........\\ \neq115,253,391,......\\ \text{infinitely continuous}}} =$$

P_a = 7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109, 127, 139, 151, 157, 163,181, 193, 199, 211,223,229, 241, 271, 277, 283, 307, 313, 331, 337, 349, 367,.........

Reference
A002476 (The On-line Enciclopedia of integers sequences)

Formula 2: Prime numbers within the sequence $\beta_b = 5,11,17,23,29,35,41,47,53,59,65,..$

$$P_b = (6*n-1)_{\substack{\neq\beta_a*(7+6*z)\\ \neq\beta_b*(5+6*z)}}$$

β_b= (6*n-1) = 5,11,17,23,29,35,41,.....
β_a= (6*n+1) = 7,13,19,25,31,37,43,....
$n > 0$

Example

$$P_b = (6*n-1)_{\substack{\neq5*(7+6*z)\\ \neq7*(5+6*z)\\ \neq11*(7+6*z)\\ \neq13*(5+6*z)\\ \neq17*(7+6*z)\\ \neq19*(5+6*z)\\ \neq23*(7+6*z)\\ \text{infinitely continuous}}} = (6*n-1)_{\substack{\neq35,65,95,125,......\\ \neq35,77,119,161,203,.......\\ \neq77,143,209,275,.....\\ \neq65,143,221,299,...\\ \neq119,221,323,425,...\\ \neq95,209,323,437,.......\\ \neq161,299,437,.....\\ \text{infinitely continuous}}} =$$

P_b = 5, 11, 17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113, 131, 137, 149, 167, 173, 179, 191, 197,227, 233, 239, 251, 257, 263, 269, 281, 293, ...

Reference
A007528 (The On-line Enciclopedia of integers sequences)

Formula 3
Integrated formula

Prime numbers within the sequence $\beta = (6 * n \pm 1) = 5,7,11,13,17,19,23,25,29,31,35, \dots \dots$

$$P = (6 * n \pm 1)$$

$\neq \beta_{b1} * (5 + 6 * z)$
$\neq \beta_{b1} * (7 + 6 * z)$
$\neq \beta_{a1} * (5 + 6 * z)$
$\neq \beta_{a1} * (7 + 6 * z)$
$\neq \beta_{b2} * (5 + 6 * z)$
$\neq \beta_{b2} * (7 + 6 * z)$
$\neq \beta_{a2} * (5 + 6 * z)$
$\neq \beta_{a2} * (7 + 6 * z)$

Continuously infinitely repeating the series

$\beta_{b1} = 5,\ \beta_{b2} = 11,\ \beta_{b3} = 17, \dots \dots \text{ of the form } (6 * n - 1)$
$\beta_{a1} = 7,\ \beta_{a2} = 13,\ \beta_{a3} = 19, \dots \dots \text{ of the form } (6 * n + 1)$

$P = (6 * n \pm 1)$

$\neq 5*(5+6*z)$
$\neq 5*(7+6*z)$
$\neq 7*(5+6*z)$
$\neq 7*(7+6*z)$
$\neq 11*(5+6*z)$
$\neq 11*(7+6*z)$
$\neq 13*(5+6*z)$
$\neq 13*(7+6*z)$
$\neq 17*(5+6*z)$
$\neq 17*(7+6*z)$
$\neq 19*(5+6*z)$
$\neq 19*(7+6*z)$
$\neq 23*(5+6*z)$
$\neq 23*(7+6*z)$
infinitely continuous

$=$ $(6 * n \pm 1)$

$\neq$25,55,85,115,....
$\neq$35,65,95,125,......
$\neq$35,77,119,161,203,.......
$\neq$49,91,133,175,.........
$\neq$55,121,187,253,......
$\neq$77,143,209,275,.....
$\neq$65,143,221,299,...
$\neq$91,169,247,325,....
$\neq$85,187,289,391,......
$\neq$119,221,323,425,...
$\neq$95,209,323,437,.......
$\neq$133,247,361,.........
$\neq$115,253,391,......
$\neq$161,299,437,.....
infinitely continuous

$=$

Prime numbers

P = 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103,107, 109, 113, 127, 131, 137,139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, ...

Table 3

In yellow we have the prime numbers in red the composite numbers.

$\beta\ (a) = (6 * \mathrm{n} + 1)$ $\qquad$ $\beta\ (b) = (6 * \mathrm{n} - 1)$

Prime Numbers					
β (a)				β (b)	
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66

67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120
121	122	123	124	125	126
127	128	129	130	131	132
133	134	135	136	137	138
139	140	141	142	143	144
145	146	147	148	149	150

Conclusion

These formulas allow to obtain in a simple way the prime numbers greater than three. Also the composite numbers divisible by numbers greater than three.
The order of the prime numbers and composite numbers is done by combining the β numbers. By applying the condition of equality and inequality, we obtain the expected results.
These formulas are simple and easy although extensive, and infinity.
For the first time we can say that there is a formula to calculate and obtain them in a very similar way to the Sieve of Eratosthenes.

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