Title: Prime numbers and composite numbers congruent to 1,4,7,2,5,8 (mod 9)
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Abstract: This paper develops a modified an old and well-known expression for calculating and obtaining all prime numbers greater than three and composite numbers divisible by numbers greater than three. This paper develops formulas to break down the prime numbers and the composite numbers in their reductions, these formulas based on equalities allow to regroup them according to congruence characteristics.

Keywords: Prime numbers, composite numbers, congruence.

Introduction

The study of the prime numbers is wonderful, I have discovered a brilliant expression that contains all the prime numbers greater than 3.

The Prime numbers and composite numbers expressed in the form $(6 * n \pm 1)$ have the characteristic of having 6 different reductions, the reductions are obtained by adding their digits. These 6 reductions are 1,4,7,2,5,8, this paper develops the correct formulas to obtain prime numbers and composite numbers for each reduction.

Methods

The way to solve the exercises will be looking for the pattern of the composite numbers that have the same congruences, then it will be very easy to find the prime numbers. Three reductions are within the sequence $(6 * n + 1)$ and the other three reductions within the sequence $(6 * n - 1)$. These reductions are repeated every 18 numbers. The data supplied are sufficient for obtaining a new formula.

Definition

There are 6 types of reductions for prime numbers greater than 3 and for composite numbers divisible by numbers greater than 3.
These 6 reductions are divided into two groups, on the one hand $A = 1,4,7$ and on the other $B = 2,5,8$.
These are each associated to the expression $A = (6 * n + 1)$ and $B = (6 * n - 1)$.
The reductions are equivalent with the congruences with (mod 9)

A) A simple way to know which reduction has a number is as follows.

The reductions are obtained by adding the digits of a number. Also if we divide the numbers by 9 we obtain in their decimals the value of their reduction. The reductions are equivalent to the rest in the division.

Example

A) 13= 1+3=**4**	13/9= 1,4444....

B) 67=6+7=13=1+3=**4**	67/9= 7,4444.....
C) 29=2+9=11=1+1=2	29/9=3,22222.......

B) Each reduction is congruent to:

Reduction 1≡ $\mathbf{1}\ (\mathbf{mod}\ \mathbf{9})$ we will develop it in Theorem 1
Reduction 2≡ $\mathbf{2}\ (\mathbf{mod}\ \mathbf{9})$ we will develop it in Theorem 2
Reduction 4≡ $\mathbf{4}\ (\mathbf{mod}\ \mathbf{9})$ we will develop it in Theorem 3
Reduction 5≡ $\mathbf{5}\ (\mathbf{mod}\ \mathbf{9})$ we will develop it in Theorem 4
Reduction 7≡ $\mathbf{7}\ (\mathbf{mod}\ \mathbf{9})$ we will develop it in Theorem 5
Reduction 8≡ $\mathbf{8}\ (\mathbf{mod}\ \mathbf{9})$ we will develop it in Theorem 6

Theorem 1

At point A we establish the original sequence $\beta = (6 * n \pm 1)$ on which we will try to calculate composite numbers and prime numbers.
In point B we will apply the subsequence $N_1 = (18 * n + 19)$ for the calculation of numbers with reduction 1.
At point C, I demonstrate and apply the expression $\beta * (\delta + 18 * z)$ by means of equality, which allows the calculation and obtaining only compound numbers with reduction 1.
At point D, I demonstrate how composite numbers are distributed.
At point E, I demonstrate and apply this same expression $\beta * (\delta + 18 * z)$ to cancel the composite numbers by inequality and thus obtain only prime numbers greater than 3 with reduction 1.

A) Sequence β

This known expression shows in a disordered way, without following any possible pattern, prime numbers greater than 3 and composite numbers divisible by numbers greater than 3.

$$\beta = (6 * n \pm 1)$$

β_a=$(6 * n + 1) =$ 7,13,19,25,31,37,43,49,55,61,67,73,79,85,........
β_b= $(6 * n - 1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83, ...

B) Formula for numbers with reduction 1

Numbers with reduction 1 that are within the sequence β_a=$(6 * n + 1)$

$$\mathbf{numbers} \equiv \mathbf{1}\ (\mathbf{mod}\ \mathbf{9}).$$
$$N_1 = (18 * n + 19)$$

N_1= 19, 37, 55, 73, 91, 109, 127, 145, 163, 181, 199, 217, 235, 253, 271, 289, 307, 325,……
n≥ 0
z≥ 0

C) Formula for composite number with reduction 1

The application for the calculation of all the composite numbers with reduction 1 is linked to the form $(18 * n + 19)$. So that this formula of results only compounds we must condition it.
My contribution will be in equating the values of the previous formula with the expression $\beta * (\delta + 18 * z)$. The discovery of this expression allows obtaining only the numbers composed with reduction 1. In this way the prime numbers with reduction are discarded 1.

Nc_1= **Composite numbers** $\equiv$ **1 (mod 9).**

$$Nc_1 = (18 * n + 19)_{=\beta*(\delta+18*z)}$$

n≥ 0
z≥ 0

$\beta = (6 * n \pm 1)$ = 5,7,11,13,17,19,23,25,29,31,35,37,......... *continue infinitely*

δ has 6 variants

These 6 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

δ =17,7,11,19,23,13

11	13	23	7	17	19

Demonstration

$Nc_1 = (18 * n + 19)$
$=\beta_1*(11+18*z)$
$=\beta_2*(13+18*z)$
$=\beta_3*(23+18*z)$
$=\beta_4*(7+18*z)$
$=\beta_5*(17+18*z)$
$=\beta_6*(19+18*z)$
$=\beta_7*(11+18*z)$
$=\beta_8*(13+18*z)$
$=\beta_9*(23+18*z)$
$=\beta_{10}*(7+18*z)$
$=\beta_{11}*(17+18*z)$
$=\beta_{12}*(19+18*z)$
continue infinitely

$= Nc_1 = (18 * n + 19)$
$=5*(11+18*z)$
$=7*(13+18*z)$
$=11*(23+18*z)$
$=13*(7+18*z)$
$=17*(17+18*z)$
$=19*(19+18*z)$
$=23*(11+18*z)$
$=25*(13+18*z)$
$=29*(23+18*z)$
$=31*(7+18*z)$
$=35*(17+18*z)$
$=37*(19+18*z)$
continue infinitely

We can add more β numbers and expand the formula infinitely.

We solve the previous example when Z=0, Z=1, Z=2,.............

therefore it is

$$
\begin{aligned}
Nc_{\mathbf{1}} \quad & =55,145,235,....\\
& =91,217,343,....\\
& =253,451,649,....\\
& =91,325,559,....\\
& =289,595,901,....\\
& =361,703,1045,...\\
& =253,667,1081,.....\\
& =325,775,1225 ....\\
& =667,1189,1711,...\\
& =217,775,1333,...\\
& =595,1225,1855,......\\
& =703,1369,2035,...\\
& continue \quad infinetely
\end{aligned}
$$

D) Distances between composite numbers with reduction 1.

The distance between composite numbers with reduction 1 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_1}$= 18* $\boldsymbol{\beta}$

$\boldsymbol{D_1}$= Distance between composite number (Reduction 1).

Example

A. $\beta = 5; \quad D_1 = 18 * 5 = 90$
B. $\beta = 7; \quad D_1 = 18 * 7 = 126$
C. $\beta = 11; \quad D_1 = 18 * 11 = 198$
D. $\beta = 13; \quad D_1 = 18 * 13 = 234$

E) Formula for Prime numbers with reduction 1

To calculate the prime numbers we will use the same formula as for the calculation of the composite numbers although we will use the inequality symbol. By using this symbol we are canceling all the numbers composed of the expression $(18 * n + 19)$, so only the prime numbers will remain.

$\boldsymbol{P_1}$= Prime numbers ≡ 1 (mod 9).

Formula Prime numbers

$$P_{\mathbf{4}} = (18 * n + 19)_{\neq \beta * (\delta + 18 * z)}$$

n≥ 0
z≥ 0

Demonstration

$$P_1 = (18*n+19) \quad \begin{matrix} \neq \beta_1*(11+18*z) \\ \neq \beta_2*(13+18*z) \\ \neq \beta_3*(23+18*z) \\ \neq \beta_4*(7+18*z) \\ \neq \beta_5*(17+18*z) \\ \neq \beta_6*(19+18*z) \\ \neq \beta_7*(11+18*z) \\ \neq \beta_8*(13+18*z) \\ \neq \beta_9*(23+18*z) \\ \neq \beta_{10}*(7+18*z) \\ \neq \beta_{11}*(17+18*z) \\ \neq \beta_{12}*(19+18*z) \\ continue \;\; infinitely \end{matrix}$$

$$= P_1 = (18*n+19) \quad \begin{matrix} \neq 5*(11+18*z) \\ \neq 7*(13+18*z) \\ \neq 11*(23+18*z) \\ \neq 13*(7+18*z) \\ \neq 17*(17+18*z) \\ \neq 19*(19+18*z) \\ \neq 23*(11+18*z) \\ \neq 25*(13+18*z) \\ \neq 29*(23+18*z) \\ \neq 31*(7+18*z) \\ \neq 35*(17+18*z) \\ \neq 37*(19+18*z) \\ continue \;\; infinitely \end{matrix}$$

We solve the previous example when Z=0, Z=1, Z=2,.............

therefore it is

$$P_1 = (18*n+19) \quad \begin{matrix} \neq 55,145,235,.... \\ \neq 91,217,343,.... \\ \neq 253,451,649,.... \\ \neq 91,325,559,.... \\ \neq 289,595,901,.... \\ \neq 361,703,1045,... \\ \neq 253,667,1081,...... \\ \neq 325,775,1225,.... \\ \neq 667,1189,1711,... \\ \neq 217,775,1333... \\ \neq 595,1225,1855,...... \\ \neq 703,1369,2035,.... \\ continue \;\; infinetely \end{matrix}$$

We get the following prime numbers

P_1=19, 37, 73, 109, 127, 163, 181, 199, 271, 307, 379, 397, 433, 487, 523, 541, 577, 613, 631, 739, 757, 811, 829, 883, 919, 937, 991, 1009, 1063, 1117, 1153, 1171, 1279, 1297, 1423, 1459, 1531, 1549, 1567, 1621, 1657, 1693, 1747, 1783, 1801, 1873, 1999,........
All the Prime numbers are reduced to 1.

Reference A061237 **(The On-line Enciclopedia of integers sequences)**

Graphics tables 1

In the graph we can see how the numbers with reduction 1 are systematically ordered every 18 numbers.

Reduction 1							
	1						
	7						
	13	1					
	19	2	1				
	25	3	2	1			
1	31	4	3	2	1		
2	37	5	4	3	2	1	
3	43	6	5	4	3	2	1

		4	49	7	6	5	4	3	2
		5	55	1	7	6	5	4	3
	1	1	61	2	8	7	6	5	4
	2	2	67	3	9	8	7	6	5
	3	3	73	4	10	9	8	7	6
	4	4	79	5	11	10	9	8	7
	5	5	85	6	12	11	10	9	8
1	6	1	91	7	13	12	11	10	9
2	7	2	97	1	1	13	12	11	10
3	8	3	103	2	2	14	13	12	11
4	9	4	109	3	3	15	14	13	12
5	10	5	115	4	4	16	15	14	13
6	11	1	121	5	5	17	16	15	14
7	1	2	127	6	6	18	17	16	15
8	2	3	133	7	7	19	18	17	16
9	3	4	139	1	8	1	19	18	17
10	4	5	145	2	9	2	20	19	18
11	5	1	151	3	10	3	21	20	19

Theorem 2

At point A we establish the original sequence $\beta = (6 * n \pm 1)$ on which we will try to calculate composite numbers and prime numbers.
In point B we will apply the subsequence $N_2 = (18 * n + 11)$ for the calculation of numbers with reduction 2.
At point C, I demonstrate and apply the expression $\beta * (\delta + 18 * z)$ by means of equality, which allows the calculation and obtaining only composite numbers with reduction 2.
At point D, I demonstrate how composite numbers are distributed.
At point E, I demonstrate and apply this same expression $\beta * (\delta + 18 * z)$ to cancel the composite numbers by inequality and thus obtain only prime numbers greater than 3 with reduction 2.

A) Sequence β

This known expression shows in a disordered way, without following any possible pattern, prime numbers greater than 3 and composite numbers divisible by numbers greater than 3.

$$\beta = (6 * n \pm 1)$$

β_a=$(6 * n + 1) =$ 7,13,19,25,31,37,43,49,55,61,67,73,79,85,........
β_b= $(6 * n - 1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83, ...

B) Formula for numbers with reduction 2

Numbers with reduction 4 that are within the sequence β_b=$(6 * n - 1)$

$$\textbf{numbers} \equiv 2 \ (\textbf{mod } 9).$$
$$N_2 = (18 * n + 11)$$

n≥ 0
N_2= 11,29,47,65,83,101,119,137,155,173,191,209,227,245,263,……

C) Formula for composite number with reduction 2

The application for the calculation of all the composite numbers with reduction 2 is linked to the form $(18 * n + 11)$. So that this formula of results only compounds we must condition it.
My contribution will be in equating the values of the previous formula with the expression
$\beta * (\delta + 18 * z)$. The discovery of this expression allows obtaining only the numbers composed with reduction 2. In this way the prime numbers with reduction are discarded 2.

Nc_2= $\boldsymbol{Composite}$ $\textbf{numbers} \equiv 2 \ (\textbf{mod } 9)$.

$$Nc_2 = (18 * n + 11)_{=\beta*(\delta+18*z)}$$

n≥ 0
z≥ 0
$\beta = (6 * n \pm 1)$ = 5,7,11,13,17,19,23,25,29,31,35,37,……… $continue\ infinitely$

δ has 6 variants

These 6 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

δ = 13,17,19,23,7,11

13	17	19	23	7	11

Demonstration

$Nc_2 = (18 * n + 11)$
$=\beta_1*(13+18*z)$
$=\beta_2*(17+18*z)$
$=\beta_3*(19+18*z)$
$=\beta_4*(23+18*z)$
$=\beta_5*(7+18*z)$
$=\beta_6*(11+18*z)$
$=\beta_7*(13+18*z)$
$=\beta_8*(17+18*z)$
$=\beta_9*(19+18*z)$
$=\beta_{10}*(23+18*z)$
$=\beta_{11}*(7+18*z)$
$=\beta_{12}*(11+18*z)$
$continue\ \ infinitely$

$= Nc_2 = (18 * n + 11)$
$=5*(13+18*z)$
$=7*(17+18*z)$
$=11*(19+18*z)$
$=13*(23+18*z)$
$=17*(7+18*z)$
$=19*(11+18*z)$
$=23*(13+18*z)$
$=25*(17+18*z)$
$=29*(19+18*z)$
$=31*(23+18*z)$
$=35*(7+18*z)$
$=37*(11+18*z)$
$continue\ \ infinitely$

We can add more β numbers and expand the formula infinitely.

We solve the previous example when Z=0, Z=1, Z=2,………….

therefore it is

$$Nc_{\mathbf{2}} = (18 * n + 11)$$

$=65,155,245,.....$
$=119,245,371,....$
$=209,407,605,....$
$=299,533,767,....$
$=119,425,731,....$
$=209,551,893,....$
$=299,713,1127,....$
$=425,875,1325,....$
$=551,1073,1595,....$
$=713,1271,1829,....$
$=245,875,1505,....$
$=407,1073,1739,.....$
continue infinetely

D) Distances between composite numbers with reduction 2.

The distance between composite numbers with reduction 4 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_2}$= **18*** $\boldsymbol{\beta}$

$\boldsymbol{D_2}$**=** Distance between composite number (Reduction 2).

Example

E. $\beta = 5; \quad D_2 = 18 * 5 = 90$
F. $\beta = 7; \quad D_2 = 18 * 7 = 126$
G. $\beta = 11; \quad D_2 = 18 * 11 = 198$
H. $\beta = 13; \quad D_2 = 18 * 13 = 234$

E) Formula for Prime numbers with reduction 2

To calculate the prime numbers we will use the same formula as for the calculation of the composite numbers although we will use the inequality symbol. By using this symbol we are canceling all the numbers composed of the expression $(18 * n + 11)$, so only the prime numbers will remain.

P_2= Prime numbers ≡ 2 (mod 9).

Formula Prime numbers

$$P_{\mathbf{2}} = (18 * n + 11)_{\neq \beta * (\delta + 18 * z)}$$

n≥ 0
z≥ 0

Demonstration

$$P_2 = (18*n+11) \quad \begin{array}{c} \neq\beta_1*(13+18*z) \\ \neq\beta_2*(17+18*z) \\ \neq\beta_3*(19+18*z) \\ \neq\beta_4*(23+18*z) \\ \neq\beta_5*(7+18*z) \\ \neq\beta_6*(11+18*z) \\ \neq\beta_7*(13+18*z) \\ \neq\beta_8*(17+18*z) \\ \neq\beta_9*(19+18*z) \\ \neq\beta_{10}*(23+18*z) \\ \neq\beta_{11}*(7+18*z) \\ \neq\beta_{12}*(11+18*z) \\ continue\ \ infinitely \end{array} \; = \; P_2 = (18*n+11) \quad \begin{array}{c} \neq 5*(13+18*z) \\ \neq 7*(17+18*z) \\ \neq 11*(19+18*z) \\ \neq 13*(23+18*z) \\ \neq 17*(7+18*z) \\ \neq 19*(11+18*z) \\ \neq 23*(13+18*z) \\ \neq 25*(17+18*z) \\ \neq 29*(19+18*z) \\ \neq 31*(23+18*z) \\ \neq 35*(7+18*z) \\ \neq 37*(11+18*z) \\ continue\ \ infinitely \end{array}$$

We solve the previous example when Z=0, Z=1, Z=2,............

therefore it is

$$P_2 = (18*n+11) \quad \begin{array}{c} \neq 65,155,245,..... \\ \neq 119,245,371,.... \\ \neq 209,407,605,.... \\ \neq 299,533,767,.... \\ \neq 119,425,731,.... \\ \neq 209,551,893,.... \\ \neq 299,713,1127,.... \\ \neq 425,875,1325,.... \\ \neq 551,1073,1595,.... \\ \neq 713,1271,1829,.... \\ \neq 245,875,1505,.... \\ \neq 407,1073,1739,..... \\ continue\ \ infinetely \end{array}$$

We get the following prime numbers

P_2=11, 29, 47, 83, 101, 137, 173, 191, 227, 263, 281, 317, 353, 389, 443, 461, 479, 569, 587, 641, 659, 677, 821, 839, 857, 911, 929, 947, 983, 1019, 1091, 1109, 1163, 1181, 1217, 1289, 1307, 1361, 1433, 1451, 1487, 1523, 1559, 1613, 1667, 1721, 1811, 1847,........
All the Prime numbers are reduced to 2.

Reference A061238 **(The On-line Enciclopedia of integers sequences)**

Graphics tables 2

In the graph we can see how the numbers with reduction 2 are systematically ordered every 18 numbers.

Reduction 2

					5
				1	11
			1	2	17
		1	2	3	23
	1	2	3	4	29
1	2	3	4	5	35

1	2	3	4	5	1	41	1			
2	3	4	5	6	2	47	2			
3	4	5	6	7	3	53	3			
4	5	6	7	8	4	59	4			
5	6	7	8	9	5	65	5			
6	7	8	9	10	1	71	6	1		
7	8	9	10	11	2	77	7	2		
8	9	10	11	1	3	83	1	3		
9	10	11	12	2	4	89	2	4		
10	11	12	13	3	5	95	3	5		
11	12	13	14	4	1	101	4	6	1	
12	13	14	15	5	2	107	5	7	2	
13	14	15	16	6	3	113	6	8	3	
14	15	16	17	7	4	119	7	9	4	
15	16	17	1	8	5	125	1	10	5	
16	17	18	2	9	1	131	2	11	6	1
17	18	19	3	10	2	137	3	12	7	2
18	19	20	4	11	3	143	4	13	8	3
19	20	21	5	1	4	149	5	1	9	4
20	21	22	6	2	5	155	6	2	10	5

Theorem 1

At point A we establish the original sequence $\beta = (6 * n \pm 1)$ on which we will try to calculate composite numbers and prime numbers.
In point B we will apply the subsequence $N_4 = (18 * n + 13)$ for the calculation of numbers with reduction 4.
At point C, I demonstrate and apply the expression $\beta * (\delta + 18 * z)$ by means of equality, which allows the calculation and obtaining only composite numbers with reduction 4.
At point D, I demonstrate how composite numbers are distributed.
At point E, I demonstrate and apply this same expression $\beta * (\delta + 18 * z)$ to cancel the composite numbers by inequality and thus obtain only prime numbers greater than 3 with reduction 4.

A) Sequence β

This known expression shows in a disordered way, without following any possible pattern, prime numbers greater than 3 and composite numbers divisible by numbers greater than 3.

$$\beta = (6 * n \pm 1)$$

β_a=$(6 * n + 1) =$ 7,13,19,25,31,37,43,49,55,61,67,73,79,85,........
β_b= $(6 * n - 1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83, ...

B) Formula for numbers with reduction 4

Numbers with reduction 4 that are within the sequence β_a=$(6*n+1)$

$$\textbf{numbers} \equiv \textbf{4 (mod 9)}.$$
$$N_4 = (18*n+13)$$

n≥ 0

N_4= 13, 31, 49, 67, 85, 103, 121, 139, 157, 175, 193, 211, 229, 247, 265, 283, 301, 319, 337,….

C) Formula for composite number with reduction 4

The application for the calculation of all the composite numbers with reduction 4 is linked to the form $(18 * n + 13)$. So that this formula of results only compounds we must condition it.
My contribution will be in equating the values of the previous formula with the expression
$\beta * (\delta + 18 * z)$. The discovery of this expression allows obtaining only the numbers composed with reduction 4. In this way the prime numbers with reduction are discarded 4.

$Nc_{\mathbf{4}}$= $\boldsymbol{Composite}$ $\textbf{numbers} \equiv \textbf{4 (mod 9)}.$

$$Nc_{\mathbf{4}} = (18*n+13)_{=\beta*(\delta+18*z)}$$

n≥ 0
z≥ 0

$\beta = (6*n\pm1)$ = 5,7,11,13,17,19,23,25,29,31,35,37,……… $continue\ infinitely$

δ has 6 variants

These 6 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

δ =17,7,11,19,23,13

17	7	11	19	23	13

Demonstration

$$Nc_4 = (18*n+13) \quad \begin{array}{c} =\beta_1*(17+18*z) \\ =\beta_2*(7+18*z) \\ =\beta_3*(11+18*z) \\ =\beta_4*(19+18*z) \\ =\beta_5*(23+18*z) \\ =\beta_6*(13+18*z) \\ =\beta_7*(17+18*z) \\ =\beta_8*(7+18*z) \\ =\beta_9*(11+18*z) \\ =\beta_{10}*(19+18*z) \\ =\beta_{11}*(23+18*z) \\ =\beta_{12}*(13+18*z) \\ continue\ \ infinitely \end{array} = Nc_4 = (18*n+13) \quad \begin{array}{c} =5*(17+18*z) \\ =7*(7+18*z) \\ =11*(11+18*z) \\ =13*(19+18*z) \\ =17*(23+18*z) \\ =19*(13+18*z) \\ =23*(17+18*z) \\ =25*(7+18*z) \\ =29*(11+18*z) \\ =31*(19+18*z) \\ =35*(23+18*z) \\ =37*(13+18*z) \\ continue\ \ infinitely \end{array}$$

We can add more β numbers and expand the formula infinitely.

We solve the previous example when Z=0, Z=1, Z=2,............

therefore it is

$$Nc_4 = (18*n+13) \quad \begin{array}{c} =85,175,265,.... \\ =49,175,301,.... \\ =121,319,517,.... \\ =247,481,715,.... \\ =391,697,1003,.... \\ =247,589,931,... \\ =391,805,1219,...... \\ =175,625,1075,.... \\ =319,841,1363,... \\ =589,1147,1705,... \\ =805,1435,2065,...... \\ =481,1147,1813,... \\ continue\ \ infinetely \end{array}$$

D) Distances between composite numbers with reduction 4.

The distance between composite numbers with reduction 4 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_4}$= 18* $\boldsymbol{\beta}$

$\boldsymbol{D_4}$= Distance between composite number (Reduction 4).

Example

I. $\beta = 5; \quad D_4 = 18*5 = 90$
J. $\beta = 7; \quad D_4 = 18*7 = 126$
K. $\beta = 11; \quad D_4 = 18*11 = 198$
L. $\beta = 13; \quad D_4 = 18*13 = 234$

E) Formula for Prime numbers with reduction 4

To calculate the prime numbers we will use the same formula as for the calculation of the composite numbers although we will use the inequality symbol. By using this symbol we are canceling all the numbers composed of the expression $(18*n+13)$, so only the prime numbers will remain.

$\boldsymbol{P_4}$ **= Prime numbers ≡ 4 (mod 9).**

Formula Prime numbers

$$P_4 = (18*n+13)_{\neq\beta*(\delta+18*z)}$$

n≥ 0
z≥ 0

Demonstration

$$P_4 = (18*n+13) \begin{matrix} \neq\beta_1*(17+18*z) \\ \neq\beta_2*(7+18*z) \\ \neq\beta_3*(11+18*z) \\ \neq\beta_4*(19+18*z) \\ \neq\beta_5*(23+18*z) \\ \neq\beta_6*(13+18*z) \\ \neq\beta_7*(17+18*z) \\ \neq\beta_8*(7+18*z) \\ \neq\beta_9*(11+18*z) \\ \neq\beta_{10}*(19+18*z) \\ \neq\beta_{11}*(23+18*z) \\ \neq\beta_{12}*(13+18*z) \\ continue\ \ infinitely \end{matrix} = P_4 = (18*n+13) \begin{matrix} \neq 5*(17+18*z) \\ \neq 7*(7+18*z) \\ \neq 11*(11+18*z) \\ \neq 13*(19+18*z) \\ \neq 17*(23+18*z) \\ \neq 19*(13+18*z) \\ \neq 23*(17+18*z) \\ \neq 25*(7+18*z) \\ \neq 29*(11+18*z) \\ \neq 31*(19+18*z) \\ \neq 35*(23+18*z) \\ \neq 37*(13+18*z) \\ continue\ \ infinitely \end{matrix}$$

We solve the previous example when Z=0, Z=1, Z=2,............

therefore it is

$$P_4 = (18*n+13) \begin{matrix} \neq 85,175,265,.... \\ \neq 49,175,301,.... \\ \neq 121,319,517,.... \\ \neq 247,481,715,.... \\ \neq 391,697,1003,.... \\ \neq 247,589,931,... \\ \neq 391,805,1219,...... \\ \neq 175,625,1075,.... \\ \neq 319,841,1363,... \\ \neq 589,1147,1705,... \\ \neq 805,1435,2065,...... \\ \neq 481,1147,1813,... \\ continue\ \ infinetely \end{matrix}$$

<u>We get the following prime numbers</u>

P_4=13, 31, 67, 103, 139, 157, 193, 211, 229, 283, 337, 373, 409, 463, 499, 571, 607, 643, 661, 733, 751, 769, 787, 823, 859, 877, 967, 1021, 1039, 1093, 1129, 1201, 1237, 1291, 1327, 1381, 1399, 1453, 1471, 1489, 1543, 1579, 1597, 1669, 1723, 1741, 1759,..........
All the Prime numbers are reduced to 4.

Reference A061239 (The On-line Enciclopedia of integers sequences)

Graphics tables 3

In the graph we can see how the numbers with reduction 4 are systematically ordered every 18 numbers.

					Reduction 4							
					1							
					7							
					13	1						
					19	2	1					
					25	3	2	1				
				1	31	4	3	2	1			
				2	37	5	4	3	2	1		
				3	43	6	5	4	3	2	1	
				4	49	7	6	5	4	3	2	1
				5	55	1	7	6	5	4	3	2
			1	1	61	2	8	7	6	5	4	3
			2	2	67	3	9	8	7	6	5	4
			3	3	73	4	10	9	8	7	6	5
			4	4	79	5	11	10	9	8	7	6
			5	5	85	6	12	11	10	9	8	7
		1	6	1	91	7	13	12	11	10	9	8
		2	7	2	97	1	1	13	12	11	10	9
		3	8	3	103	2	2	14	13	12	11	10
		4	9	4	109	3	3	15	14	13	12	11
		5	10	5	115	4	4	16	15	14	13	12
	1	6	11	1	121	5	5	17	16	15	14	13
	2	7	1	2	127	6	6	18	17	16	15	14
	3	8	2	3	133	7	7	19	18	17	16	15
	4	9	3	4	139	1	8	1	19	18	17	16
	5	10	4	5	145	2	9	2	20	19	18	17
1	6	11	5	1	151	3	10	3	21	20	19	18
2	7	12	6	2	157	4	11	4	22	21	20	19

Theorem 4

At point A we establish the original sequence $\beta = (6 * n \pm 1)$ on which we will try to calculate composite numbers and prime numbers.
In point B we will apply the subsequence $N_5 = (18 * n + 5)$ for the calculation of numbers with reduction 5.
At point C, I demonstrate and apply the expression $\beta * (\delta + 18 * z)$ by means of equality, which allows the calculation and obtaining only compound numbers with reduction 5.
At point D, I demonstrate how composite numbers are distributed.
At point E, I demonstrate and apply this same expression $\beta * (\delta + 18 * z)$ to cancel the composite numbers by inequality and thus obtain only prime numbers greater than 3 with reduction 5.

A) Sequence β

This known expression shows in a disordered way, without following any possible pattern, prime numbers greater than 3 and composite numbers divisible by numbers greater than 3.

$$\beta = (6 * n \pm 1)$$

β_a =$(6 * n + 1)$ = 7,13,19,25,31,37,43,49,55,61,67,73,79,85,........

β_b = $(6 * n - 1)$ = 5,11,17,23,29,35,41,47,53,59,65,71,77,83, ...

B) Formula for numbers with reduction 5

Numbers with reduction 5 that are within the sequence β_b=$(6 * n - 1)$

$$\textbf{numbers} \equiv \textbf{5 (mod 9)}.$$

$$N_5 = (18 * n + 5)$$

n≥ 0

N_5 = 5, 23, 41, 59, 77, 95, 113, 131, 149, 167, 185, 203, 221, 239, 257, 275, 293, 311, 329,.....

C) Formula for composite number with reduction 5

The application for the calculation of all the composite numbers with reduction 5 is linked to the form $(18 * n + 5)$. So that this formula of results only compounds we must condition it.

My contribution will be in equating the values of the previous formula with the expression $\beta * (\delta + 18 * z)$. The discovery of this expression allows obtaining only the numbers composed with reduction 5. In this way the prime numbers with reduction are discarded 5.

Nc_5= $\boldsymbol{Composite}$ **numbers** $\equiv$ **5 (mod 9)**.

$$Nc_5 = (18 * n + 5)_{=\beta *(\delta + 18 * z)}$$

n≥ 0

z≥ 0

$\beta = (6 * n \pm 1)$ = 5,7,11,13,17,19,23,25,29,31,35,37,......... *continue infinitely*

δ has 6 variants

These 6 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

δ =19,11,7,17,13,23

19	11	7	17	13	23

Demonstration

$Nc_5 = (18*n+5)$ $=\beta_1*(19+18*z)$ $= Nc_5 = (18*n+5)$ $=5*(19+18*z)$

$Nc_5 = (18*n+5)$		$= Nc_5 = (18*n+5)$	
	$=\beta_1*(19+18*z)$		$=5*(19+18*z)$
	$=\beta_2*(11+18*z)$		$=7*(11+18*z)$
	$=\beta_3*(7+18*z)$		$=11*(7+18*z)$
	$=\beta_4*(17+18*z)$		$=13*(17+18*z)$
	$=\beta_5*(13+18*z)$		$=17*(13+18*z)$
	$=\beta_6*(23+18*z)$		$=19*(23+18*z)$
	$=\beta_7*(19+18*z)$		$=23*(19+18*z)$
	$=\beta_8*(11+18*z)$		$=25*(11+18*z)$
	$=\beta_9*(7+18*z)$		$=29*(7+18*z)$
	$=\beta_{10}*(17+18*z)$		$=31*(17+18*z)$
	$=\beta_{11}*(13+18*z)$		$=35*(13+18*z)$
	$=\beta_{12}*(23+18*z)$		$=37*(23+18*z)$
	continue infinitely		*continue infinitely*

We can add more β numbers and expand the formula infinitely.

We solve the previous example when Z=0, Z=1, Z=2,............

therefore it is

$$
\begin{aligned}
Nc_5 \quad & =95,185,275,.... \\
& =77,203,329,.... \\
& =77,275,473,.... \\
& =221,455,689,.... \\
& =221,527,833,.... \\
& =437,779,1121,.... \\
& =437,851,1265,...... \\
& =275,725,1175,.... \\
& =203,725,1247,... \\
& =527,1085,1643,... \\
& =455,1085,1715,...... \\
& =851,1517,2183,... \\
& \textit{continue infinetely}
\end{aligned}
$$

D) Distances between composite numbers with reduction 5.

The distance between composite numbers with reduction 5 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_5}$**= 18*** $\boldsymbol{\beta}$

$\boldsymbol{D_5}$**=** Distance between composite number (Reduction 5).

Example

M. $\beta = 5; \quad D_5 = 18*5 = 90$
N. $\beta = 7; \quad D_5 = 18*7 = 126$
O. $\beta = 11; \quad D_5 = 18*11 = 198$
P. $\beta = 13; \quad D_5 = 18*13 = 234$

E) Formula for Prime numbers with reduction 5

To calculate the prime numbers we will use the same formula as for the calculation of the composite numbers although we will use the inequality symbol. By using this symbol we are canceling all the numbers composed of the expression $(18 * n + 5)$, so only the prime numbers will remain.

P_5= Prime numbers ≡ 5 (mod 9).

Formula Prime numbers

$$P_5 = (18 * n + 5)_{\neq \beta * (\delta + 18 * z)}$$

n≥ 0
z≥ 0

Demonstration

$$P_5 = (18 * n + 5) \begin{matrix} \neq \beta_1 * (19+18*z) \\ \neq \beta_2 * (11+18*z) \\ \neq \beta_3 * (7+18*z) \\ \neq \beta_4 * (17+18*z) \\ \neq \beta_5 * (13+18*z) \\ \neq \beta_6 * (23+18*z) \\ \neq \beta_7 * (19+18*z) \\ \neq \beta_8 * (11+18*z) \\ \neq \beta_9 * (7+18*z) \\ \neq \beta_{10} * (17+18*z) \\ \neq \beta_{11} * (13+18*z) \\ \neq \beta_{12} * (23+18*z) \\ continue \quad infinitely \end{matrix} = P_5 = (18 * n + 5) \begin{matrix} \neq 5 * (19+18*z) \\ \neq 7 * (11+18*z) \\ \neq 11 * (7+18*z) \\ \neq 13 * (17+18*z) \\ \neq 17 * (13+18*z) \\ \neq 19 * (23+18*z) \\ \neq 23 * (19+18*z) \\ \neq 25 * (11+18*z) \\ \neq 29 * (7+18*z) \\ \neq 31 * (17+18*z) \\ \neq 35 * (13+18*z) \\ \neq 37 * (23+18*z) \\ continue \quad infinitely \end{matrix}$$

We solve the previous example when Z=0, Z=1, Z=2,............

therefore it is

$$P_5 = (18 * n + 5) \begin{matrix} \neq 95,185,275,.... \\ \neq 77,203,329,.... \\ \neq 77,275,473,.... \\ \neq 221,455,689,.... \\ \neq 221,527,833,.... \\ \neq 437,779,1121,... \\ \neq 437,851,1265,...... \\ \neq 275,725,1175,.... \\ \neq 203,725,1247,... \\ \neq 527,1085,1643,... \\ \neq 455,1085,1715,...... \\ \neq 851,1517,2183,... \\ continue \quad infinetely \end{matrix}$$

We get the following prime numbers

P_5=5, 23, 41, 59, 113, 131, 149, 167, 239, 257, 293, 311, 347, 383, 401, 419, 491, 509, 563, 599, 617, 653, 743, 761, 797, 887, 941, 977, 1013, 1031, 1049, 1103, 1193, 1229, 1283, 1301, 1319, 1373, 1409, 1427, 1481, 1499, 1553, 1571, 1607, 1697, 1733, 1787,........
All the Prime numbers are reduced to 5.

Reference A061240 **(The On-line Enciclopedia of integers sequences)**

Graphics tables 4

In the graph we can see how the numbers with reduction 5 are systematically ordered every 18 numbers.

Reduction 5

						5					
					1	11					
				1	2	17					
			1	2	3	23					
		1	2	3	4	29					
	1	2	3	4	5	35					
1	2	3	4	5	1	41	1				
2	3	4	5	6	2	47	2				
3	4	5	6	7	3	53	3				
4	5	6	7	8	4	59	4				
5	6	7	8	9	5	65	5				
6	7	8	9	10	1	71	6	1			
7	8	9	10	11	2	77	7	2			
8	9	10	11	1	3	83	1	3			
9	10	11	12	2	4	89	2	4			
10	11	12	13	3	5	95	3	5			
11	12	13	14	4	1	101	4	6	1		
12	13	14	15	5	2	107	5	7	2		
13	14	15	16	6	3	113	6	8	3		
14	15	16	17	7	4	119	7	9	4		
15	16	17	1	8	5	125	1	10	5		
16	17	18	2	9	1	131	2	11	6	1	
17	18	19	3	10	2	137	3	12	7	2	
18	19	20	4	11	3	143	4	13	8	3	
19	20	21	5	1	4	149	5	1	9	4	
20	21	22	6	2	5	155	6	2	10	5	
21	22	23	7	3	1	161	7	3	11	6	1

Theorem 5

At point A we establish the original sequence $\beta = (6 * n \pm 1)$ on which we will try to calculate composite numbers and prime numbers.
In point B we will apply the subsequence $N_7 = (18 * n + 7)$ for the calculation of numbers with reduction 7.
At point C, I demonstrate and apply the expression $\beta * (\delta + 18 * z)$ by means of equality, which allows the calculation and obtaining only compound numbers with reduction 7.
At point D, I demonstrate how composite numbers are distributed.
At point E, I demonstrate and apply this same expression $\beta * (\delta + 18 * z)$ to cancel the composite numbers by inequality and thus obtain only prime numbers greater than 3 with reduction 7.

A) Sequence β

This known expression shows in a disordered way, without following any possible pattern, prime numbers greater than 3 and composite numbers divisible by numbers greater than 3.

$$\beta = (6 * n \pm 1)$$

β_a=$(6 * n + 1) =$ 7,13,19,25,31,37,43,49,55,61,67,73,79,85,........

β_b= $(6 * n - 1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83, ...

B) Formula for numbers with reduction 7

Numbers with reduction 4 that are within the sequence β_a=$(6 * n + 1)$

$$\textbf{numbers} \equiv \textbf{7 (mod 9)}.$$
$$N_7 = (18 * n + 7)$$

n≥ 0

N_7 = 7,25,43,61,79,97,115,133,151,169,187,205,223,241,259,277,……

C) Formula for composite number with reduction 7

The application for the calculation of all the composite numbers with reduction 7 is linked to the form $(18 * n + 7)$. So that this formula of results only compounds we must condition it.
My contribution will be in equating the values of the previous formula with the expression
$\beta * (\delta + 18 * z)$. The discovery of this expression allows obtaining only the numbers composed with reduction 7. In this way the prime numbers with reduction are discarded 7.
.

Nc_7= $\textbf{\textit{Composite}}$ $\textbf{numbers} \equiv \textbf{7 (mod 9)}.$

$$Nc_7 = (18 * n + 7)_{=\beta *(\delta + 18 * z)}$$

n≥ 0
z≥ 0

$\beta = (6 * n \pm 1)$ = 5,7,11,13,17,19,23,25,29,31,35,37,……… $continue\ infinitely$

δ has 6 variants

These 6 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

δ = 23,19,17,13,11,7

23	19	17	13	11	7

Demonstration

$Nc_7 = (18*n+7)$ $=\beta_1*(23+18*z)$
$=\beta_2*(19+18*z)$
$=\beta_3*(17+18*z)$
$=\beta_4*(13+18*z)$
$=\beta_5*(11+18*z)$
$=\beta_6*(7+18*z)$
$=\beta_7*(23+18*z)$
$=\beta_8*(19+18*z)$
$=\beta_9*(17+18*z)$
$=\beta_{10}*(13+18*z)$
$=\beta_{11}*(11+18*z)$
$=\beta_{12}*(7+18*z)$
$continue \quad infinitely$

$= Nc_7 = (18*n+7)$ $=5*(23+18*z)$
$=7*(19+18*z)$
$=11*(17+18*z)$
$=13*(13+18*z)$
$=17*(11+18*z)$
$=19*(7+18*z)$
$=23*(23+18*z)$
$=25*(19+18*z)$
$=29*(17+18*z)$
$=31*(13+18*z)$
$=35*(11+18*z)$
$=37*(7+18*z)$
$continue \quad infinitely$

We can add more β numbers and expand the formula infinitely.

We solve the previous example when Z=0, Z=1, Z=2,............

therefore it is

$Nc_7 = (18*n+7)$ =115,205,295,....
=133,259,385,..
=187,385,583,....
=169,403,637,..
=187,493,799,...
=133,475,817,.....
=529,943,1357,...
=475,925,1375,...
=493,1015,1537,...
=403,961,1519,....
=385,1015,1645,....
=259,925,1591,.....
$continue \quad infinetely$

D) Distances between composite numbers with reduction 7.

The distance between composite numbers with reduction 4 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_7}$**= 18*** $\boldsymbol{\beta}$

$\boldsymbol{D_7}$**=** Distance between composite number (Reduction 7).

Example

Q. $\beta = 5;$ $D_7 = 18*5 = 90$
R. $\beta = 7;$ $D_7 = 18*7 = 126$
S. $\beta = 11;$ $D_7 = 18*11 = 198$
T. $\beta = 13;$ $D_7 = 18*13 = 234$

E) Formula for Prime numbers with reduction 7

To calculate the prime numbers we will use the same formula as for the calculation of the composite numbers although we will use the inequality symbol. By using this symbol we are canceling all the numbers composed of the expression $(18*n+7)$, so only the prime numbers will remain.

$\boldsymbol{P_7}$= Prime numbers ≡ 7 (mod 9).

Formula Prime numbers

$$P_7 = (18*n+7)_{\neq\beta*(\delta+18*z)}$$

n≥ 0
z≥ 0

Demonstration

$$P_7 = (18*n+7) \begin{array}{c} \neq\beta_1*(23+18*z) \\ \neq\beta_2*(19+18*z) \\ \neq\beta_3*(17+18*z) \\ \neq\beta_4*(13+18*z) \\ \neq\beta_5*(11+18*z) \\ \neq\beta_6*(7+18*z) \\ \neq\beta_7*(23+18*z) \\ \neq\beta_8*(19+18*z) \\ \neq\beta_9*(17+18*z) \\ \neq\beta_{10}*(13+18*z) \\ \neq\beta_{11}*(11+18*z) \\ \neq\beta_{12}*(7+18*z) \\ continue\ \ infinitely \end{array} = P_7 = (18*n+7) \begin{array}{c} \neq 5*(23+18*z) \\ \neq 7*(19+18*z) \\ \neq 11*(17+18*z) \\ \neq 13*(13+18*z) \\ \neq 17*(11+18*z) \\ \neq 19*(7+18*z) \\ \neq 23*(23+18*z) \\ \neq 25*(19+18*z) \\ \neq 29*(17+18*z) \\ \neq 31*(13+18*z) \\ \neq 35*(11+18*z) \\ \neq 37*(7+18*z) \\ continue\ \ infinitely \end{array}$$

We solve the previous example when Z=0, Z=1, Z=2,............

therefore it is

$$P_7 = (18*n+7) \begin{array}{c} \neq 115,205,295,.... \\ \neq 133,259,385,.. \\ \neq 187,385,583,.... \\ \neq 169,403,637,.. \\ \neq 187,493,799,... \\ \neq 133,475,817,..... \\ \neq 529,943,1357,... \\ \neq 475,925,1375,... \\ \neq 493,1015,1537,... \\ \neq 403,961,1519,.... \\ \neq 385,1015,1645,.... \\ \neq 259,925,1591,..... \\ continue\ \ infinetely \end{array}$$

We get the following prime numbers

P_7=7, 43, 61, 79, 97, 151, 223, 241, 277, 313, 331, 349, 367, 421, 439, 457, 547, 601, 619, 673, 691, 709, 727, 853, 907, 997, 1033, 1051, 1069, 1087, 1123, 1213, 1231, 1249, 1303, 1321, 1429, 1447, 1483, 1609, 1627, 1663, 1699, 1753, 1789, 1861, 1879, 1933,………
All the Prime numbers are reduced to 7.

Reference [A061241](A061241) **(The On-line Enciclopedia of integers sequences)**

Graphics tables 5

In the graph we can see how the numbers with reduction 7 are systematically ordered every 18 numbers.

Reduction 7

			1					
			7					
			13	1				
			19	2	1			
			25	3	2	1		
		1	31	4	3	2	1	
		2	37	5	4	3	2	1
		3	43	6	5	4	3	2
		4	49	7	6	5	4	3
		5	55	1	7	6	5	4
	1	1	61	2	8	7	6	5
	2	2	67	3	9	8	7	6
	3	3	73	4	10	9	8	7
	4	4	79	5	11	10	9	8
	5	5	85	6	12	11	10	9
1	6	1	91	7	13	12	11	10
2	7	2	97	1	1	13	12	11
3	8	3	103	2	2	14	13	12
4	9	4	109	3	3	15	14	13
5	10	5	115	4	4	16	15	14
6	11	1	121	5	5	17	16	15
7	1	2	127	6	6	18	17	16
8	2	3	133	7	7	19	18	17
9	3	4	139	1	8	1	19	18
10	4	5	145	2	9	2	20	19
11	5	1	151	3	10	3	21	20

Theorem 6

At point A we establish the original sequence $\beta = (6 * n \pm 1)$ on which we will try to calculate composite numbers and prime numbers.
In point B we will apply the subsequence $N_8 = (18 * n + 17)$ for the calculation of numbers with reduction 8.
At point C, I demonstrate and apply the expression $\beta * (\delta + 18 * z)$ by means of equality, which allows the calculation and obtaining only compound numbers with reduction 8.
At point D, I demonstrate how composite numbers are distributed.
At point E, I demonstrate and apply this same expression $\beta * (\delta + 18 * z)$ to cancel the composite numbers by inequality and thus obtain only prime numbers greater than 3 with reduction 8.

A) Sequence β

This known expression shows in a disordered way, without following any possible pattern, prime numbers greater than 3 and composite numbers divisible by numbers greater than 3.

$$\beta = (6 * n \pm 1)$$

$\beta_a = (6 * n + 1) =$ 7,13,19,25,31,37,43,49,55,61,67,73,79,85,……..

$\beta_b = (6*n-1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83, ...

B) Formula for numbers with reduction 8

Numbers with reduction 8 that are within the sequence β_b=$(6*n-1)$

$$\textbf{numbers} \equiv \textbf{8 (mod 9)}.$$
$$N_8 = (18*n+17)$$

n≥ 0

N_8= 17, 35, 53, 71, 89, 107, 125, 143, 161, 179, 197, 215, 233, 251, 269, 287, 305, 323,.....

C) Formula for composite number with reduction 8

The application for the calculation of all the composite numbers with reduction 8 is linked to the form $(18 * n + 17)$. So that this formula of results only compounds we must condition it.
My contribution will be in equating the values of the previous formula with the expression
$\beta * (\delta + 18 * z)$. The discovery of this expression allows obtaining only the numbers composed with reduction 8. In this way the prime numbers with reduction are discarded 8.

Nc_8= $\boldsymbol{Composite}$ **numbers** $\equiv$ **8** (**mod 9**).

$$Nc_{\mathbf{8}} = (18*n+17)_{=\beta*(\delta+18*z)}$$

n≥ 0
z≥ 0
$\beta = (6*n \pm 1)$ = 5,7,11,13,17,19,23,25,29,31,35,37,......... $continue\ infinitely$

δ has 6 variants

These 6 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

δ =7,23,13,11,19,17

7	23	13	11	19	17

Demonstration

$$Nc_{\mathbf{8}} = (18 * n + 17) \quad \begin{matrix} =\beta_1*(7+18*z) \\ =\beta_2*(23+18*z) \\ =\beta_3*(13+18*z) \\ =\beta_4*(11+18*z) \\ =\beta_5*(19+18*z) \\ =\beta_6*(17+18*z) \\ =\beta_7*(7+18*z) \\ =\beta_8*(23+18*z) \\ =\beta_9*(13+18*z) \\ =\beta_{10}*(11+18*z) \\ =\beta_{11}*(19+18*z) \\ =\beta_{12}*(17+18*z) \\ continue \ \ infinitely \end{matrix} \quad = Nc_{\mathbf{8}} = (18 * n + 17) \quad \begin{matrix} =5*(7+18*z) \\ =7*(23+18*z) \\ =11*(13+18*z) \\ =13*(11+18*z) \\ =17*(19+18*z) \\ =19*(17+18*z) \\ =23*(7+18*z) \\ =25*(23+18*z) \\ =29*(13+18*z) \\ =31*(11+18*z) \\ =35*(19+18*z) \\ =37*(17+18*z) \\ continue \ \ infinitely \end{matrix}$$

We can add more β numbers and expand the formula infinitely.

We solve the previous example when Z=0, Z=1, Z=2,............

therefore it is

$$Nc_{\mathbf{8}} = (18 * n + 17) \quad \begin{matrix} =35,125,215,.... \\ =161,287,413,.... \\ =143,341,539,.... \\ =143,377,611,.... \\ =323,629,935,.... \\ =323,665,1007,.... \\ =161,575,989,... \\ =575,1025,1475,... \\ =377,899,1421,... \\ =341,899,1457,.... \\ =665,1295,1925,.... \\ =629,1295,1961,.... \\ continue \ \ infinetely \end{matrix}$$

D) Distances between composite numbers with reduction 8.

The distance between composite numbers with reduction 4 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_8}$**= 18*** $\boldsymbol{\beta}$

$\boldsymbol{D_8}$**=** Distance between composite number (Reduction 8).

Example

U. $\beta = 5;\quad D_8 = 18 * 5 = 90$
V. $\beta = 7;\quad D_8 = 18 * 7 = 126$
W. $\beta = 11;\quad D_8 = 18 * 11 = 198$
X. $\beta = 13;\quad D_8 = 18 * 13 = 234$

E) Formula for Prime numbers with reduction 8

To calculate the prime numbers we will use the same formula as for the calculation of the composite numbers although we will use the inequality symbol. By using this symbol we are canceling all the numbers composed of the expression $(18 * n + 17)$, so only the prime numbers will remain.

P_8= $\textit{Prime numbers} \equiv 8\ (mod\ 9)$.

Formula Prime numbers

$$P_4 = (18 * n + 17)_{\neq \beta *(\delta + 18*z)}$$

n≥ 0
z≥ 0

Demonstration

$$P_8 = (18*n+17) \begin{array}{c} \neq\beta_1*(7+18*z) \\ \neq\beta_2*(23+18*z) \\ \neq\beta_3*(13+18*z) \\ \neq\beta_4*(11+18*z) \\ \neq\beta_5*(19+18*z) \\ \neq\beta_6*(17+18*z) \\ \neq\beta_7*(7+18*z) \\ \neq\beta_8*(23+18*z) \\ \neq\beta_9*(13+18*z) \\ \neq\beta_{10}*(11+18*z) \\ \neq\beta_{11}*(19+18*z) \\ \neq\beta_{12}*(17+18*z) \\ continue\ \ infinitely \end{array} = P_8 = (18*n+17) \begin{array}{c} \neq 5*(7+18*z) \\ \neq 7*(23+18*z) \\ \neq 11*(13+18*z) \\ \neq 13*(11+18*z) \\ \neq 17*(19+18*z) \\ \neq 19*(17+18*z) \\ \neq 23*(7+18*z) \\ \neq 25*(23+18*z) \\ \neq 29*(13+18*z) \\ \neq 31*(11+18*z) \\ \neq 35*(19+18*z) \\ \neq 37*(17+18*z) \\ continue\ \ infinitely \end{array}$$

We solve the previous example when Z=0, Z=1, Z=2,.............

therefore it is

$$P_8 = (18*n+17) \begin{array}{c} \neq 35,125,215,.... \\ \neq 161,287,413,... \\ \neq 143,341,539,... \\ \neq 143,377,611,... \\ \neq 323,629,935,... \\ \neq 323,665,1007,.... \\ \neq 161,575,989,... \\ \neq 575,1025,1475,... \\ \neq 377,899,1421,... \\ \neq 341,899,1457,.... \\ \neq 665,1295,1925,.... \\ \neq 629,1295,1961,.... \\ continue\ \ infinetely \end{array}$$

We get the following prime numbers

P_8=17, 53, 71, 89, 107, 179, 197, 233, 251, 269, 359, 431, 449, 467, 503, 521, 557, 593, 647, 683, 701, 719, 773, 809, 827, 863, 881, 953, 971, 1061, 1097, 1151, 1187, 1223, 1259, 1277, 1367, 1439, 1493, 1511, 1583, 1601, 1619, 1637, 1709, 1871, 1889, 1907,...........

All the Prime numbers are reduced to 8.

Reference A061242 (The On-line Enciclopedia of integers sequences)

Graphics tables 6

In the graph we can see how the numbers with reduction 8 are systematically ordered every 18 numbers.

Reduction 8

						5				
					1	11				
				1	2	17				
			1	2	3	23				
		1	2	3	4	29				
	1	2	3	4	5	35				
1	2	3	4	5	1	41	1			
2	3	4	5	6	2	47	2			
3	4	5	6	7	3	53	3			
4	5	6	7	8	4	59	4			
5	6	7	8	9	5	65	5			
6	7	8	9	10	1	71	6	1		
7	8	9	10	11	2	77	7	2		
8	9	10	11	1	3	83	1	3		
9	10	11	12	2	4	89	2	4		
10	11	12	13	3	5	95	3	5		
11	12	13	14	4	1	101	4	6	1	
12	13	14	15	5	2	107	5	7	2	
13	14	15	16	6	3	113	6	8	3	
14	15	16	17	7	4	119	7	9	4	
15	16	17	1	8	5	125	1	10	5	
16	17	18	2	9	1	131	2	11	6	1
17	18	19	3	10	2	137	3	12	7	2
18	19	20	4	11	3	143	4	13	8	3
19	20	21	5	1	4	149	5	1	9	4
20	21	22	6	2	5	155	6	2	10	5

Conclusion

The order of the prime numbers and composite numbers is done by combining the β numbers.
These formulas are simple and easy although extensive, and infinity.
These formulas allow to obtain in a simple way the prime numbers greater than three congruent to (1,4,7,2,5,8) Mod 9. Also the composite numbers divisible by numbers greater than three congruent to (1,4,7,2,5,8) Mod 9.
The prime numbers 7,11,13,17,19 and 23 are the key to the formula to understand how these numbers are distributed. These numbers are ordered systematically in all their forms.

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