The Irrationality of $\zeta(n)$: A Proof by Story

Timothy W. Jones

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NASA launches a space ship from Earth. It tells the astronauts of the ship to report back any meteorites it encounters so that the next launch can avoid crashes. In this strange universe all meteorites exist as dots on concentric circles equally spaced with the number of meteorites given by the distance to Earth. NASA further tells the astronauts that they should perpetually look in their rear view mirror to see if any new sets of meteorites has suddenly sprung into existence. If their instrumentation detects that the next ship will fly within ϵ of a meteorite, a course adjustment for the next ship will have to be made.

The first ship sets out at and immediately sees a set of $1/4$ meteorites and alters its trajectory by $1/4 + 1/16$. It perpetually sees new sets of meteorites in front of it and makes $1/4^k$ adjustments. One day, while looking in the rear view mirror it sees a new set of $1/9$ meteorites has come into existence, but it continues to make $1/4^k$ adjustments. Alarmed, one morning, Bob, one of the astronauts, determines that the current trajectory is within ϵ of a meteorite on the $1/9$ circle of meteorites. He radios in to Houston the situation. After much thought ground control determines that the next flight will use course corrections of $1/4+1/9$ and powers of these in sequence. They all hope that only $1/4^k$ and $1/9^k$ meteorites exist.

The first flight fails to report in one day and, sadly, the ground crew fears that they have collided with some $1/9$ meteorite. In this strange universe all meteorites are on these concentric circles and come in and out of existence chaotically. In fact the second ship is reporting that a $1/24$ circle of meteorites has appeared in its rear view mirror.

One day a man from another planet comes in and explains things to NASA. He tells the ground crew that all finite combinations of powers of rational numbers will fail because they all will converge to a rational number and sometime or another all ships will get too close to a meteorite and crash. She radios in to her

mother planet and talks to Bernoulli and Apery and is told that course adjustments of the type

$$z_{2n} = \sum_{k=2}^{\infty} \frac{1}{k^{2n}}$$

and

$$z_3 = \sum_{k=2}^{\infty} \frac{1}{k^3}$$

will avoid all rational numbers forever. All reflect on these course adjustments and conclude that, in general,

$$z_n = \sum_{k=2}^{\infty} \frac{1}{k^n},$$

$n \geq 2$ will also always work.