Three sequences of palindromes obtained from squares of primes

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Abstract. In this paper I make the following two conjectures: (I) There exist an infinity of squares of primes p^2 such that (p^2 + 4*196) + R(p^2 + 4*196), where R(n) is the number obtained reversing the digits of n, is a palindromic number; note that I wrote 4*196 instead 784 because 196 is a number known to be related with palindromes: is the first Lychrel number, which gives the name to the "196-algorithm"; (II) For every square of odd prime p^2 there exist an infinity of primes q such that the number (p^2 + 16*q^2) + R(p^2 + 16*q^2) is a palindrome. The three sequences (presumed infinite by the conjectures above) mentioned in title of the paper are: (1) Palindromes of the form (p^2 + 4*196) + R(p^2 + 4*196), where p^2 is a square of prime; (2) Palindromes of the form (p^2 + 16*q^2) + R(p^2 + 16*q^2), where p^2 is a square of prime and q the least prime for which is obtained such a palindrome; (3) Palindromes of the form (13^2 + 16*q^2) + R(13^2 + 16*q^2), where q is prime.

Conjecture I:

There exist an infinity of squares of primes p^2 such that (p^2 + 4*196) + R(p^2 + 4*196), where R(n) is the number obtained reversing the digits of n, is a palindromic number.

Note: I wrote 4*196 instead 784 because 196 is a number known to be related with palindromes: is the first Lychrel number (a Lychrel number is a natural number that cannot form a palindrome through the iterative process of repeatedly reversing its digits and adding the resulting numbers, process sometimes called the 196-algorithm, 196 being the smallest such number – see the sequence A023108 in OEIS).

Note: it may seem contradictory that a Lychrel number (196) can help to obtain both palindromes and Lychrel numbers (because, if you take Lychrel primes - sequence A135316 in OEIS - you see that 8 from the first 38 Lychrel primes can be written as p + k*196, where p is also a Lychrel prime: 887 = 691 + 196; 4349 = 1997 + 12*196; 8179 = 4259 + 20*196; 8269 = 1997 + 32*196; 8719 = 4799 + 20*196; 10883 = 691 + 52*196); 12763 = 3943 + 45*196; 13597 = 11833 + 9*196).

Conjecture II:

For every square of odd prime p^2 there exist an infinity of primes q such that the number (p^2 + 16*q^2) + R(p^2 + 16*q^2) is a palindrome.

Three sequences of palindromes
(presumed infinite by the two conjectures above):

Sequence 1:

Palindromes of the form (p^2 + 4*196) + R(p^2 + 4*196), where p^2 is a square of prime:

: 4774 (17^2 + 4*196 = 1073; 1073 + 3701 = 4774);
: 6556 (19^2 + 4*196 = 1145; 1145 + 5411 = 6556);
: 4444 (23^2 + 4*196 = 1313; 1313 + 3131 = 4444);
: 6886 (29^2 + 4*196 = 1625; 1625 + 5261 = 6886);
: 5665 (37^2 + 4*196 = 2153; 2153 + 3512 = 5665);
: 5995 (43^2 + 4*196 = 2633; 2633 + 3362 = 5995);
: 9889 (59^2 + 4*196 = 4265; 4265 + 5624 = 9889);
: 9559 (61^2 + 4*196 = 4505; 4505 + 5054 = 9559);
: 8998 (67^2 + 4*196 = 5273; 5273 + 3725 = 8998);
: 9229 (73^2 + 4*196 = 6113; 6113 + 3116 = 9229);
(...)

Note that palindromes were obtained for ten from the first twenty odd primes!

Sequence 2:

Palindromes of the form (p^2 + 16*q^2) + R(p^2 + 16*q^2), where p^2 is a square of odd prime and q the least prime for which is obtained such a palindrome:

: 5885 (3^2 + 16*13^2 = 2713; 2713 + 3172 = 5885);
: 949 (5^2 + 16*5^2 = 425; 425 + 524 = 949);
: 67876 (7^2 + 16*31^2 = 15425; 15425 + 52451 = 67876);
: 646 (11^2 + 16*5^2 = 521; 521 + 125 = 646);
: 626 (13^2 + 16*3^2 = 313; 313 + 313 = 626);
: 767 (17^2 + 16*3^2 = 433; 433 + 334 = 767);
: 6556 (19^2 + 16*7^2 = 5411; 5411 + 1145 = 6556);
: 4444 (23^2 + 16*7^2 = 1313; 1313 + 3131 = 4444);
: 2662 (29^2 + 16*5^2 = 1241; 1241 + 1421 = 2662);
: 6116 (31^2 + 16*3^2 = 1105; 1105 + 5011 = 6116);
: 4664 (37^2 + 16*3^2 = 1513; 1513 + 3151 = 4664);
: 3883 (41^2 + 16*5^2 = 2081; 2081 + 1802 = 3883);
(...)

Note that for the first twelve odd primes p is obtained a palindrome for a prime q less than or equal to 31!

Sequence 3:

Palindromes of the form (13^2 + 16*q^2) + R(13^2 + 16*q^2), where q is prime:

: 626 (13^2 + 16*3^2 = 313; 313 + 313 = 626);
: 7117 (13^2 + 16*11^2 = 5012; 5012 + 2105 = 7117);
: 59095 (13^2 + 16*37^2 = 22073; 22073 + 37022 = 59095);
: 76267 (13^2 + 16*53^2 = 45113; 45113 + 31154 = 76267);
: 620026 (33^2 + 16*79^2 = 100025; 100025 + 520001 = 620026);
: 467764 (13^2 + 16*97^2 = 150713; 150713 + 317051 = 467764);
(...)