Three sequences of palindromes obtained from Poulet numbers

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Abstract. In this paper I make the following two conjectures: (I) There exist an infinity of Poulet numbers P such that (P + 4*196) + R(P + 4*196), where R(n) is the number obtained reversing the digits of n, is a palindromic number; note that I wrote 4*196 instead 784 because 196 is a number known to be related with palindromes: is the first Lychrel number, which gives the name to the "196-algorithm"; (II) For every Poulet number P there exist an infinity of primes q such that the number (P + 16*q^2) + R(P + 16*q^2) is a palindrome. The three sequences (presumed infinite by the conjectures above) mentinoned in title of the paper are: (1) Palindromes of the form (P + 4*196) + R(P + 4*196), where P is a Poulet number; (2) Palindromes of the form (P + 16*q^2) + R(P + 16*q^2), where P is a Poulet number and q the least prime for which is obtained such a palindrome; (3) Palindromes of the form (1729 + 16*q^2) + R(1729 + 16*q^2), where q is prime (1729 is a well known Poulet number).

Conjecture I:

There exist an infinity of Poulet numbers P such that (P + 4*196) + R(P + 4*196), where R(n) is the number obtained reversing the digits of n, is a palindromic number.

Note: I wrote 4*196 instead 784 because 196 is a number known to be related with palindromes: is the first Lychrel number (a Lychrel number is a natural number that cannot form a palindrome through the iterative process of repeatedly reversing its digits and adding the resulting numbers, process sometimes called the 196-algorithm, 196 being the smallest such number – see the sequence A023108 in OEIS).

Note: it may seem contradictory that a Lychrel number (196) can help to obtain both palindromes and Lychrel numbers (because, if you take Lychrel primes - sequence A135316 in OEIS - you see that 8 from the first 38 Lychrel primes can be written as p + k*196, where p is also a Lychrel prime: 887 = 691 + 196; 4349 = 1997 + 12*196; 8179 = 4259 + 20*196; 8269 = 1997 + 32*196; 8719 = 4799 + 20*196; 10883 = 691 + 52*196); 12763 = 3943 + 45*196; 13597 = 11833 + 9*196).

Conjecture II:

For every Poulet number P there exist an infinity of primes q such that the number (P + 16*q^2) + R(P + 16*q^2) is a palindrome.

Three sequences of palindromes
(presumed infinite by the two conjectures above):

Sequence 1:

Palindromes of the form (P + 4*196) + R(P + 4*196), where P is a Poulet number:

: 6336 (341 + 4*196 = 1125; 1125 + 5211 = 6336);
: 6776 (561 + 4*196 = 1345; 1345 + 5431 = 6776);
: 3883 (1387 + 4*196 = 2171; 2171 + 1712 = 3883);
: 5665 (1729 + 4*196 = 2513; 2513 + 3152 = 5665);
: 8668 (2821 + 4*196 = 3605; 3605 + 5063 = 8668);
: 5665 (3277 + 4*196 = 4061; 4061 + 1604 = 5665);
(...)

Note the interesting thing that the same palindrome (5665) was obtained for two different Poulet numbers (1729 and 3277). That shows that, far from being just a topic of recreative arithmetics, palindromic numbers deserve more studies.

Sequence 2:

Palindromes of the form (P + 16*q^2) + R(P + 16*q^2), where P is a Poulet number and q the least prime for which is obtained such a palindrome:

: 888 (341 + 16*5^2 = 741; 741 + 147 = 888);
: 6776 (561 + 16*7^2 = 1345; 1345 + 5431 = 6776);
: 6446 (645 + 16*5^2 = 1045; 1045 + 5401 = 6446);
: 6556 (1105 + 16*5^2 = 1505; 1505 + 5051 = 6556);
: 2882 (1387 + 16*3^2 = 1531; 1531 + 1351 = 2882);
: 5665 (1729 + 16*7^2 = 2513; 2513 + 3152 = 5665);
: 7337 (1905 + 16*5^2 = 2305; 2305 + 5032 = 7337);
: 9889 (2047 + 16*5^2 = 2447; 2447 + 7442 = 9889);
: 5445 (2465 + 16*11^2 = 4401; 4401 + 1044 = 5445);
: 4114 (2701 + 16*5^2 = 3101; 3101 + 1013 = 4114);
: 4444 (2821 + 16*5^2 = 3221; 3221 + 1223 = 4444);
: 4664 (3277 + 16*3^2 = 3421; 3421 + 1243 = 4664);
(...)

Note that for the first twelve Poulet numbers is obtained a palindrome for a prime q less than or equal to 11!

Sequence 3:

Palindromes of the form (1729 + 16*q^2) + R(1729 + 16*q^2), where q is prime (1729 is a well known Poulet number):

: 5665 (1729 + 16*7^2 = 2513; 2513 + 3152 = 5665);
: 7777 (1729 + 16*13^2 = 4433; 4433 + 3344 = 7777);
: 9889 (1729 + 16*17^2 = 6353; 6353 + 3536 = 9889);
: 49294 (1729 + 16*23^2 = 10193; 10193 + 39101 = 49294);
: 67276 (1729 + 16*31^2 = 17105; 17105 + 50171 = 67276);
: 62626 (1729 + 16*43^2 = 31313; 31313 + 31313 = 62626);
: 686686 (1729 + 16*79^2 = 101585; 101585 + 585101 = 686686);
(...)