Number p^2-q^2 where p and q primes needs very few iterations of "reverse and add" to reach a palindrome

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Abstract. In this paper I make the following observation: the number n = p^2 – q^2, where p and q are primes, needs very few iterations of "reverse and add" to reach a palindrome. For instance, taking q = 563 and p = 104723, it can be seen that only 3 iterations are needed to reach a palindrome: n = 104723^2 – 563^6 = 10966589760 and we have: 10966589760 + 6798566901 = 17765156661; 17765156661 + 16665156771 = 34430313432 and 34430313432 + 23431303443 = 57861616875, a palindromic number. So, relying on this, I conjecture that there exist an infinity of n, even considering q and p successive, that need just one such iteration to reach a palindrome (see sequence A015976 in OEIS for these numbers) and I also conjecture that there is no a difference between two squares of primes to be a Lychrel number.

Conjecture I:

There exist an infinity of numbers n = p^2 – q^2, where q and p successive primes, that need just one iteration of "reverse and add" to reach a palindrome (see sequence A015976 in OEIS for these numbers).

The sequence of palindromes obtained from these numbers n:

: 77 (5^2 – 3^2 = 16 and 16 + 61 = 77);
: 66 (7^2 – 5^2 = 24 and 24 + 42 = 66);
: 99 (11^2 – 7^2 = 72 and 72 + 27 = 99);
: 141 (17^2 – 13^2 = 120 and 120 + 21 = 141);
: 99 (19^2 – 17^2 = 72 and 72 + 27 = 99);
: 525 (29^2 – 23^2 = 312 and 312 + 213 = 525);
: 141 (31^2 – 29^2 = 120 and 120 + 21 = 141);
: 525 (41^2 – 37^2 = 312 and 312 + 213 = 525);
: 606 (53^2 – 47^2 = 600 and 600 + 6 = 606);
: 282 (61^2 – 59^2 = 240 and 240 + 42 = 282);
: 3333 (89^2 – 83^2 = 1032 and 1032 + 2301 = 3333);
: 888 (107^2 – 103^2 = 840 and 840 + 48 = 888);
: 666 (109^2 – 107^2 = 432 and 432 + 234 = 666);
: 3993 (127^2 – 113^2 = 3360 and 3360 + 633 = 3993);
: 3333 (131^2 – 127^2 = 1032 and 1032 + 2301 = 3333);
: 9669 (137^2 – 131^2 = 1608 and 1608 + 8061 = 9669);
(...)

On the fact that the value of these differences is sometimes the same, note that, according to A069482 in OEIS [prime(n + 1)^2 – prime (n)^2], it is conjectured that "There is no upper bound on the number of repetitions that will occur for some a(n) values, because the number of possible ways of producing a value of a(n) grows with increasing n, despite decreasing prime density".

Conjecture II:

There is no a difference between two squares of primes to be a Lychrel number.

An heuristic argument in the favor of this supposition is the fact that these differences seem to need very few iterations of "reverse and add" to reach a palindrome; for instance:

: for n = 104723^2 – 563^2 = 10966589760 we have: 10966589760 + 6798566901 = 17765156661; 17765156661 + 16665156771 = 34430313432; 34430313432 + 23431303443 = 57861616875.