

Research Project Primus

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1 Theorems and Conjectures

Theorem 1.1. A natural number $n > 2$ is a prime iff $\prod_{k=1}^{n-1} k \equiv n - 1 \pmod{\sum_{k=1}^{n-1} k}$.

Theorem 1.2. Let $p \equiv 5 \pmod{6}$ be prime then , $2p + 1$ is prime iff $2p + 1 \mid 3^p - 1$.

Theorem 1.3. Let p_n be the n th prime , then

$$p_n = 1 + \sum_{k=1}^{2 \cdot (\lfloor n \ln(n) \rfloor + 1)} \left(1 - \left[\frac{1}{n} \cdot \sum_{j=2}^k \left[\frac{3 - \sum_{i=1}^j \left[\frac{\lfloor \frac{j}{i} \rfloor}{\lfloor \frac{j}{i} \rfloor} \right]}{j} \right] \right] \right)$$

Theorem 1.4. Let $P_j(x) = 2^{-j} \cdot \left((x - \sqrt{x^2 - 4})^j + (x + \sqrt{x^2 - 4})^j \right)$, where j and x are nonnegative integers . Let $N = k \cdot 2^m - 1$ such that $m > 2$, $3 \mid k$, $0 < k < 2^m$ and

$$\begin{cases} k \equiv 1 \pmod{10} \text{ with } m \equiv 2, 3 \pmod{4} \\ k \equiv 3 \pmod{10} \text{ with } m \equiv 0, 3 \pmod{4} \\ k \equiv 7 \pmod{10} \text{ with } m \equiv 1, 2 \pmod{4} \\ k \equiv 9 \pmod{10} \text{ with } m \equiv 0, 1 \pmod{4} \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(3)$, then N is prime iff $S_{m-2} \equiv 0 \pmod{N}$

Theorem 1.5. Let $P_j(x) = 2^{-j} \cdot \left((x - \sqrt{x^2 - 4})^j + (x + \sqrt{x^2 - 4})^j \right)$, where j and x are nonnegative integers . Let $N = k \cdot 2^m - 1$ such that $m > 2$, $3 \mid k$, $0 < k < 2^m$ and

$$\begin{cases} k \equiv 3 \pmod{42} \text{ with } m \equiv 0, 2 \pmod{3} \\ k \equiv 9 \pmod{42} \text{ with } m \equiv 0 \pmod{3} \\ k \equiv 15 \pmod{42} \text{ with } m \equiv 1 \pmod{3} \\ k \equiv 27 \pmod{42} \text{ with } m \equiv 1, 2 \pmod{3} \\ k \equiv 33 \pmod{42} \text{ with } m \equiv 0, 1 \pmod{3} \\ k \equiv 39 \pmod{42} \text{ with } m \equiv 2 \pmod{3} \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(5)$, then N is prime iff $S_{m-2} \equiv 0 \pmod{N}$

Theorem 1.6. Let $P_j(x) = 2^{-j} \cdot \left((x - \sqrt{x^2 - 4})^j + (x + \sqrt{x^2 - 4})^j \right)$, where j and x are nonnegative integers. Let $N = k \cdot 2^m + 1$ such that $m > 2$, $0 < k < 2^m$ and

$$\left\{ \begin{array}{l} k \equiv 1 \pmod{42} \text{ with } m \equiv 2, 4 \pmod{6} \\ k \equiv 5 \pmod{42} \text{ with } m \equiv 3 \pmod{6} \\ k \equiv 11 \pmod{42} \text{ with } m \equiv 3, 5 \pmod{6} \\ k \equiv 13 \pmod{42} \text{ with } m \equiv 4 \pmod{6} \\ k \equiv 17 \pmod{42} \text{ with } m \equiv 5 \pmod{6} \\ k \equiv 19 \pmod{42} \text{ with } m \equiv 0 \pmod{6} \\ k \equiv 23 \pmod{42} \text{ with } m \equiv 1, 3 \pmod{6} \\ k \equiv 25 \pmod{42} \text{ with } m \equiv 0, 2 \pmod{6} \\ k \equiv 29 \pmod{42} \text{ with } m \equiv 1, 5 \pmod{6} \\ k \equiv 31 \pmod{42} \text{ with } m \equiv 2 \pmod{6} \\ k \equiv 37 \pmod{42} \text{ with } m \equiv 0, 4 \pmod{6} \\ k \equiv 41 \pmod{42} \text{ with } m \equiv 1 \pmod{6} \end{array} \right.$$

Let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(5)$, then N is prime iff $S_{m-2} \equiv 0 \pmod{N}$.

Theorem 1.7. Let $P_j(x) = 2^{-j} \cdot \left((x - \sqrt{x^2 - 4})^j + (x + \sqrt{x^2 - 4})^j \right)$, where j and x are nonnegative integers. Let $N = k \cdot 2^m + 1$ such that $m > 2$, $0 < k < 2^m$ and

$$\left\{ \begin{array}{l} k \equiv 1 \pmod{6} \text{ and } k \equiv 1, 7 \pmod{10} \text{ with } m \equiv 0 \pmod{4} \\ k \equiv 5 \pmod{6} \text{ and } k \equiv 1, 3 \pmod{10} \text{ with } m \equiv 1 \pmod{4} \\ k \equiv 1 \pmod{6} \text{ and } k \equiv 3, 9 \pmod{10} \text{ with } m \equiv 2 \pmod{4} \\ k \equiv 5 \pmod{6} \text{ and } k \equiv 7, 9 \pmod{10} \text{ with } m \equiv 3 \pmod{4} \end{array} \right.$$

Let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(8)$, then N is prime iff $S_{m-2} \equiv 0 \pmod{N}$.

Theorem 1.8. Let $N = k \cdot 2^n + 1$ with $n > 1$, k is odd, $0 < k < 2^n$, $3 \mid k$ and

$$\left\{ \begin{array}{l} k \equiv 3 \pmod{30}, \quad \text{with } n \equiv 1, 2 \pmod{4} \\ k \equiv 9 \pmod{30}, \quad \text{with } n \equiv 2, 3 \pmod{4} \\ k \equiv 21 \pmod{30}, \quad \text{with } n \equiv 0, 1 \pmod{4} \\ k \equiv 27 \pmod{30}, \quad \text{with } n \equiv 0, 3 \pmod{4} \end{array} \right.$$

then N is prime iff $5^{\frac{N-1}{2}} \equiv -1 \pmod{N}$.

Theorem 1.9. Let $N = k \cdot 2^n + 1$ with $n > 1$, k is odd, $0 < k < 2^n$, $3 \mid k$ and

$$\left\{ \begin{array}{l} k \equiv 3 \pmod{42}, \quad \text{with } n \equiv 2 \pmod{3} \\ k \equiv 9 \pmod{42}, \quad \text{with } n \equiv 0, 1 \pmod{3} \\ k \equiv 15 \pmod{42}, \quad \text{with } n \equiv 1, 2 \pmod{3} \\ k \equiv 27 \pmod{42}, \quad \text{with } n \equiv 1 \pmod{3} \\ k \equiv 33 \pmod{42}, \quad \text{with } n \equiv 0 \pmod{3} \\ k \equiv 39 \pmod{42}, \quad \text{with } n \equiv 0, 2 \pmod{3} \end{array} \right.$$

then N is prime iff $7^{\frac{N-1}{2}} \equiv -1 \pmod{N}$.

Theorem 1.10. Let $N = k \cdot 2^n + 1$ with $n > 1$, k is odd, $0 < k < 2^n$, $3 \mid k$ and

$$\left\{ \begin{array}{l} k \equiv 3 \pmod{66}, \quad \text{with } n \equiv 1, 2, 6, 8, 9 \pmod{10} \\ k \equiv 9 \pmod{66}, \quad \text{with } n \equiv 0, 1, 3, 4, 8 \pmod{10} \\ k \equiv 15 \pmod{66}, \quad \text{with } n \equiv 2, 4, 5, 7, 8 \pmod{10} \\ k \equiv 21 \pmod{66}, \quad \text{with } n \equiv 1, 2, 4, 5, 9 \pmod{10} \\ k \equiv 27 \pmod{66}, \quad \text{with } n \equiv 0, 2, 3, 5, 6 \pmod{10} \\ k \equiv 39 \pmod{66}, \quad \text{with } n \equiv 0, 1, 5, 7, 8 \pmod{10} \\ k \equiv 45 \pmod{66}, \quad \text{with } n \equiv 0, 4, 6, 7, 9 \pmod{10} \\ k \equiv 51 \pmod{66}, \quad \text{with } n \equiv 0, 2, 3, 7, 9 \pmod{10} \\ k \equiv 57 \pmod{66}, \quad \text{with } n \equiv 3, 5, 6, 8, 9 \pmod{10} \\ k \equiv 63 \pmod{66}, \quad \text{with } n \equiv 1, 3, 4, 6, 7 \pmod{10} \end{array} \right.$$

then N is prime iff $11^{\frac{N-1}{2}} \equiv -1 \pmod{N}$.

Theorem 1.11. A positive integer n is prime iff $\varphi(n)! \equiv -1 \pmod{n}$

Theorem 1.12. For $m \geq 1$ number n greater than one is prime iff

$$(n^m - 1)! \equiv (n - 1)!^{\left\lceil \frac{(-1)^{m+1}}{2} \right\rceil} \cdot n^{\frac{n^m - mn + m - 1}{n-1}} \pmod{n^{\frac{n^m - mn + m + n - 2}{n-1}}}$$

Theorem 1.13. Sequence S_i is defined as $S_i = \begin{cases} 8 & \text{if } i = 0; \\ (S_{i-1}^2 - 2)^2 - 2 & \text{otherwise.} \end{cases}$ then, $F_n = 2^{2^n} + 1$, ($n \geq 2$) is a prime if and only if F_n divides $S_{2^{n-1}-1}$.

Theorem 1.14. Let $p \equiv 1 \pmod{6}$ be prime and let $5 \nmid 4p + 1$, then $4p + 1$ is prime iff $4p + 1 \mid 2^{2^p} + 1$.

Theorem 1.15. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $F_n(b) = b^{2^n} + 1$ such that $n \geq 2$ and b is even number. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_b(6)$, thus If $F_n(b)$ is prime, then $S_{2^n-1} \equiv 2 \pmod{F_n(b)}$.

Theorem 1.16. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $E_n(b) = \frac{b^{2^n} + 1}{2}$ such that $n > 1$, b is odd number greater than one. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_b(6)$, thus If $E_n(b)$ is prime, then $S_{2^n-1} \equiv 6 \pmod{E_n(b)}$.

Theorem 1.17. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$.

Let $N_p(b) = \frac{b^p + 1}{b + 1}$, where p is an odd prime and b is an odd natural number greater than one.

CASE(1). $b \equiv 1, 9 \pmod{12}$, or $b \equiv 3, 7 \pmod{12}$ and $p \equiv 1 \pmod{4}$, or $b \equiv 5 \pmod{12}$ and $p \equiv 1, 7 \pmod{12}$, or $b \equiv 11 \pmod{12}$ and $p \equiv 1, 11 \pmod{12}$.

CASE(2). $b \equiv 3, 7 \pmod{12}$ and $p \equiv 3 \pmod{4}$, or $b \equiv 5 \pmod{12}$ and $p \equiv 5, 11 \pmod{12}$, or $b \equiv 11 \pmod{12}$ and $p \equiv 5, 7 \pmod{12}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_b(4)$. Suppose $N_p(b)$ is prime, then :

- $S_{p-1} \equiv P_b(4) \pmod{N_p(b)}$ if Case(1) holds ;
- $S_{p-1} \equiv P_{b+2}(4) \pmod{N_p(b)}$ if Case(2) holds ;

Theorem 1.18. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$.

Let $M_p(a) = \frac{a^p - 1}{a - 1}$, where p is an odd prime and a is an odd natural number greater than one

CASE(1). $a \equiv 3, 11 \pmod{12}$, or $a \equiv 5, 9 \pmod{12}$ and $p \equiv 1 \pmod{4}$, or $a \equiv 7 \pmod{12}$ and $p \equiv 1, 7 \pmod{12}$, or $a \equiv 1 \pmod{12}$ and $p \equiv 1, 11 \pmod{12}$.

CASE(2). $a \equiv 5, 9 \pmod{12}$ and $p \equiv 3 \pmod{4}$, or $a \equiv 7 \pmod{12}$ and $p \equiv 5, 11 \pmod{12}$, or $a \equiv 1 \pmod{12}$ and $p \equiv 5, 7 \pmod{12}$.

Let $S_i = P_a(S_{i-1})$ with $S_0 = P_a(4)$. Suppose $M_p(a)$ is prime, then :

- $S_{p-1} \equiv P_a(4) \pmod{M_p(a)}$ if Case(1) holds ;
- $S_{p-1} \equiv P_{a-2}(4) \pmod{M_p(a)}$ if Case(2) holds ;

Conjecture 1.1. Let $b_n = b_{n-2} + \text{lcm}(n-1, b_{n-2})$ with $b_1 = 2$, $b_2 = 2$ and $n > 2$. Let $a_n = b_{n+2}/b_n - 1$, then

1. Every term of this sequence a_i is either prime or 1. 2. Every odd prime number is member of this sequence. 3. Every new prime in sequence is a next prime from the largest prime already listed.

Conjecture 1.2. Let $b_n = b_{n-1} + \text{lcm}(\lfloor \sqrt{n^3} \rfloor, b_{n-1})$ with $b_1 = 2$ and $n > 1$. Let $a_n = b_{n+1}/b_n - 1$, then

1. Every term of this sequence a_i is either prime or 1. 2. Every odd prime of the form $\lfloor \sqrt{n^3} \rfloor$ is member of this sequence. 3. Every new prime of the form $\lfloor \sqrt{n^3} \rfloor$ in sequence is a next prime from the largest prime already listed.

Conjecture 1.3. Let $b_n = b_{n-1} + \text{lcm}(\lfloor \sqrt{2} \cdot n \rfloor, b_{n-1})$ with $b_1 = 2$ and $n > 1$. Let $a_n = b_{n+1}/b_n - 1$, then

1. Every term of this sequence a_i is either prime or 1. 2. Every prime of the form $\lfloor \sqrt{2} \cdot n \rfloor$ is member of this sequence. 3. Every new prime of the form $\lfloor \sqrt{2} \cdot n \rfloor$ in sequence is a next prime from the largest prime already listed.

Conjecture 1.4. Let $b_n = b_{n-1} + \text{lcm}(\lfloor \sqrt{3} \cdot n \rfloor, b_{n-1})$ with $b_1 = 3$ and $n > 1$. Let $a_n = b_{n+1}/b_n - 1$, then

1. Every term of this sequence a_i is either prime or 1. 2. Every prime of the form $\lfloor \sqrt{3} \cdot n \rfloor$ is member of this sequence. 3. Every new prime of the form $\lfloor \sqrt{3} \cdot n \rfloor$ in sequence is a next prime from the largest prime already listed.

Conjecture 1.5. Let b and n be a natural numbers, $b \geq 2$, $n > 2$ and $n \neq 9$. Then n is prime if

and only if $\sum_{k=1}^{n-1} (b^k - 1)^{n-1} \equiv n \pmod{\frac{b^n - 1}{b - 1}}$

Conjecture 1.6. Let a , b and n be a natural numbers, $b > a > 1$, $n > 2$ and $n \notin \{4, 9, 25\}$.

Then n is prime iff $\prod_{k=1}^{n-1} (b^k - a) \equiv \frac{a^n - 1}{a - 1} \pmod{\frac{b^n - 1}{b - 1}}$

Conjecture 1.7. Let a , b and n be a natural numbers, $b > a > 0$, $n > 2$ and $n \notin \{4, 9, 25\}$.

Then n is prime iff $\prod_{k=1}^{n-1} (b^k + a) \equiv \frac{a^n + 1}{a + 1} \pmod{\frac{b^n - 1}{b - 1}}$

Conjecture 1.8. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n - 1$ such that $k > 0$, $3 \nmid k$, $k < 2^n$, $b > 0$, b is even number, $3 \nmid b$ and $n > 2$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{kb/2}(P_{b/2}(4))$, then N is prime iff $S_{n-2} \equiv 0 \pmod{N}$.

Conjecture 1.9. Let $P_j(x) = 2^{-j} \cdot \left((x - \sqrt{x^2 - 4})^j + (x + \sqrt{x^2 - 4})^j \right)$, where j and x are nonnegative integers. Let $N = k \cdot 2^m + 1$ with k odd, $0 < k < 2^m$ and $m > 2$. Let F_n be the n th Fibonacci number and let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(F_n)$, then N is prime iff there exists F_n for which $S_{m-2} \equiv 0 \pmod{N}$.

Conjecture 1.10. Let $P_j(x) = 2^{-j} \cdot \left((x - \sqrt{x^2 - 4})^j + (x + \sqrt{x^2 - 4})^j \right)$, where j and x are nonnegative integers. Let $F_m(b) = b^{2^m} + 1$ with b even, $b > 0$ and $m \geq 2$. Let F_n be the n th Fibonacci number and let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(F_n))$, then $F_m(b)$ is prime iff there exists F_n for which $S_{m-2} \equiv 0 \pmod{F_m(b)}$.

Conjecture 1.11. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = b^n - b - 1$ such that $n > 2$, $b \equiv 0, 6 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus if N is prime, then $S_{n-1} \equiv P_{(b+2)/2}(6) \pmod{N}$.

Conjecture 1.12. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = b^n - b - 1$ such that $n > 2$, $b \equiv 2, 4 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus if N is prime, then $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$.

Conjecture 1.13. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = b^n + b + 1$ such that $n > 2$, $b \equiv 0, 6 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus if N is prime, then $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$.

Conjecture 1.14. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = b^n + b + 1$ such that $n > 2$, $b \equiv 2, 4 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus if N is prime, then $S_{n-1} \equiv -P_{(b+2)/2}(6) \pmod{N}$.

Conjecture 1.15. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = b^n - b + 1$ such that $n > 3$, $b \equiv 0, 2 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus if N is prime, then $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$.

Conjecture 1.16. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = b^n - b + 1$ such that $n > 3$, $b \equiv 4, 6 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus if N is prime, then $S_{n-1} \equiv -P_{(b-2)/2}(6) \pmod{N}$.

Conjecture 1.17. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = b^n + b - 1$ such that $n > 3$, $b \equiv 0, 2 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus if N is prime, then $S_{n-1} \equiv P_{(b-2)/2}(6) \pmod{N}$.

Conjecture 1.18. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = b^n + b - 1$ such that $n > 3$, $b \equiv 4, 6 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus if N is prime, then $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$.

Conjecture 1.19. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$

Let $N = k \cdot 3^n - 2$ such that $n > 3$, $k \equiv 1, 3 \pmod{8}$ and $k > 0$. Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{3k}(6)$, thus If N is prime then $S_{n-1} \equiv P_3(6) \pmod{N}$

Conjecture 1.20. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$

Let $N = k \cdot 3^n - 2$ such that $n > 3$, $k \equiv 5, 7 \pmod{8}$ and $k > 0$. Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{3k}(6)$, thus If N is prime then $S_{n-1} \equiv P_1(6) \pmod{N}$

Conjecture 1.21. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot 3^n + 2$ such that $n > 2$, $k \equiv 1, 3 \pmod{8}$ and $k > 0$. Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{3k}(6)$, thus If N is prime then $S_{n-1} \equiv P_3(6) \pmod{N}$

Conjecture 1.22. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot 3^n + 2$ such that $n > 2$, $k \equiv 5, 7 \pmod{8}$ and $k > 0$. Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{3k}(6)$, thus If N is prime then $S_{n-1} \equiv P_1(6) \pmod{N}$

Conjecture 1.23. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n - c$ such that $b \equiv 0 \pmod{2}$, $n > bc$, $k > 0$, $c > 0$ and $c \equiv 1, 7 \pmod{8}$ Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(6))$, thus If N is prime then $S_{n-1} \equiv P_{(b/2) \cdot \lceil c/2 \rceil}(6) \pmod{N}$

Conjecture 1.24. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n - c$ such that $b \equiv 0, 4, 8 \pmod{12}$, $n > bc$, $k > 0$, $c > 0$ and $c \equiv 3, 5 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(6))$, thus If N is prime then $S_{n-1} \equiv P_{(b/2) \cdot \lfloor c/2 \rfloor}(6) \pmod{N}$

Conjecture 1.25. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n - c$ such that $b \equiv 2, 6, 10 \pmod{12}$, $n > bc$, $k > 0$, $c > 0$ and $c \equiv 3, 5 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(6))$, thus If N is prime then $S_{n-1} \equiv -P_{(b/2) \cdot \lfloor c/2 \rfloor}(6) \pmod{N}$

Conjecture 1.26. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n + c$ such that $b \equiv 0 \pmod{2}$, $n > bc$, $k > 0$, $c > 0$ and $c \equiv 1, 7 \pmod{8}$ Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(6))$, thus If N is prime then $S_{n-1} \equiv P_{(b/2) \cdot \lfloor c/2 \rfloor}(6) \pmod{N}$

Conjecture 1.27. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n + c$ such that $b \equiv 0, 4, 8 \pmod{12}$, $n > bc$, $k > 0$, $c > 0$ and $c \equiv 3, 5 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(6))$, thus If N is prime then $S_{n-1} \equiv P_{(b/2) \cdot \lceil c/2 \rceil}(6) \pmod{N}$

Conjecture 1.28. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n + c$ such that $b \equiv 2, 6, 10 \pmod{12}$, $n > bc$, $k > 0$, $c > 0$ and $c \equiv 3, 5 \pmod{8}$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(6))$, thus If N is prime then $S_{n-1} \equiv -P_{(b/2) \cdot \lceil c/2 \rceil}(6) \pmod{N}$

Conjecture 1.29. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = 2 \cdot 3^n - 1$ such that $n > 1$. Let $S_i = P_3(S_{i-1})$ with $S_0 = P_3(a)$, where $a = \begin{cases} 6, & \text{if } n \equiv 0 \pmod{2} \\ 8, & \text{if } n \equiv 1 \pmod{2} \end{cases}$ thus, N is prime iff $S_{n-1} \equiv a \pmod{N}$

Conjecture 1.30. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = 8 \cdot 3^n - 1$ such that $n > 1$. Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{12}(4)$ thus, N is prime iff $S_{n-1} \equiv 4 \pmod{N}$

Conjecture 1.31. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot 6^n - 1$ such that $n > 2$, $k > 0$, $k \equiv 2, 5 \pmod{7}$ and $k < 6^n$ Let $S_i = P_6(S_{i-1})$ with $S_0 = P_{3k}(P_3(5))$, thus N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.32. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot 6^n - 1$ such that $n > 2$, $k > 0$, $k \equiv 3, 4 \pmod{5}$ and $k < 6^n$ Let $S_i = P_6(S_{i-1})$ with $S_0 = P_{3k}(P_3(3))$, thus N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.33. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n - 1$ such that $n > 2$, $k < 2^n$ and

$$\begin{cases} k \equiv 3 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 3 \pmod{4} \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod{4} \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 1 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(18))$, then N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.34. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n - 1$ such that $n > 2$, $k < 2^n$ and

$$\begin{cases} k \equiv 9 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 1 \pmod{4} \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod{4} \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 3 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(18))$, then N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.35. Let $P_m(x) = 2^{-m} \cdot \left((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where m and x are nonnegative integers. Let $N = k \cdot b^n - 1$ such that $n > 2$, $k < 2^n$ and

$$\begin{cases} k \equiv 21 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 2, 3 \pmod{4} \\ k \equiv 21 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 1, 3 \pmod{4} \\ k \equiv 21 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 1, 2 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(3))$, then N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.36. Let F_p be the p th Fibonacci number. If p is prime, not 5, and $M \geq 2$ then $M^{F_p} \equiv M^{(p-1)(1-\frac{p}{5})/2} \pmod{\frac{M^p-1}{M-1}}$

Conjecture 1.37. Let b and n be a natural numbers , $b \geq 2$, $n > 1$ and $n \notin \{4, 8, 9\}$. Then n is prime if and only if $\sum_{k=1}^n (b^k + 1)^{n-1} \equiv n \pmod{\frac{b^n - 1}{b - 1}}$

Conjecture 1.38. If q is the smallest prime greater than $\prod_{i=1}^n C_i + 1$, where $\prod_{i=1}^n C_i$ is the product of the first n composite numbers , then $q - \prod_{i=1}^n C_i$ is prime .

Conjecture 1.39. If q is the greatest prime less than $\prod_{i=1}^n C_i - 1$, where $\prod_{i=1}^n C_i$ is the product of the first n composite numbers , then $\prod_{i=1}^n C_i - q$ is prime .

Conjecture 1.40. Let n be an odd number and $n > 1$. Let $T_n(x)$ be Chebyshev polynomial of the first kind and let $P_n(x)$ be Legendre polynomial , then n is a prime number if and only if the following congruences hold simultaneously • $T_n(3) \equiv 3 \pmod{n}$ • $P_n(3) \equiv 3 \pmod{n}$

Conjecture 1.41. Let n be a natural number greater than two . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $T_n(x)$ be Chebyshev polynomial of the first kind , then n is a prime number if and only if $T_n(x) \equiv x^n \pmod{x^r - 1, n}$.

Conjecture 1.42. Let n be a natural number greater than two and $n \neq 5$. Let $T_n(x)$ be Chebyshev polynomial of the first kind . If there exists an integer a , $1 < a < n$, such that $T_{n-1}(a) \equiv 1 \pmod{n}$ and for every prime factor q of $n - 1$, $T_{(n-1)/q}(a) \not\equiv 1 \pmod{n}$ then n is prime . If no such number a exists then n is composite .

Conjecture 1.43. Let $P_a(x) = 2^{-a} \cdot \left((x - \sqrt{x^2 - 4})^a + (x + \sqrt{x^2 - 4})^a \right)$. Let $N = k \cdot b^m \pm 1$ with b an even positive integer , $0 < k < b^m$ and $m > 2$. Let F_n be the n th Fibonacci number and let $S_i = P_b(S_{i-1})$ with $S_0 = P_{kb/2}(P_{b/2}(F_n))$, then N is prime iff there exists F_n for which $S_{m-2} \equiv 0 \pmod{N}$.

Conjecture 1.44. Let n be a natural number greater than one . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $L_n(x)$ be Lucas polynomial , then n is a prime number if and only if $L_n(x) \equiv x^n \pmod{x^r - 1, n}$.

Conjecture 1.45. Let b and n be a natural numbers , $b \geq 2$, then $\frac{b^n - 1}{b - 1} \cdot \frac{b^{\sigma(n)} - 1}{b - 1} \equiv b + 1 \pmod{\frac{b^{\varphi(n)} - 1}{b - 1}}$ for all primes and no composite with the exception of 4 and 6 .

Conjecture 1.46. Let b and n be a natural numbers , $b \geq 2$, then $\frac{b^{\varphi(n)} - 1}{b - 1} (b^{\tau(n)} - 1) + b \equiv b^{n-1} \pmod{\frac{b^n - 1}{b - 1}}$ for all primes and no composite with the exception of 4 .

Conjecture 1.47. Let p be prime number greater than three and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(2) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 11 \pmod{12}$.

Conjecture 1.48. Let p be prime number greater than two and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(3) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 7 \pmod{8}$.

Conjecture 1.49. Let p be prime number greater than three and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(5) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 5, 19, 23 \pmod{24}$

Conjecture 1.50. Let n be an odd natural number greater than one , let k be a natural number such that $k \leq n$, then n is prime if and only if :
$$\sum_{i=0}^{k-1} i^{n-1} + \sum_{j=0}^{n-k} j^{n-1} \equiv -1 \pmod{n}$$

Conjecture 1.51. Let n be a natural number greater than one and let $T_n(x)$ be Chebyshev polynomial of the first kind , then n is prime if and only if :
$$\sum_{k=0}^{n-1} 2T_{n-1}\left(\frac{k}{2}\right) \equiv -1 \pmod{n} .$$

Conjecture 1.52. Let n be a natural number greater than one and let $L_n(x)$ be Lucas polynomial , then n is prime if and only if :
$$\sum_{k=0}^{n-1} L_{n-1}(k) \equiv -1 \pmod{n} .$$

Conjecture 1.53. Let p be an odd prime number , let $R_p(3) = \frac{3^p-1}{2}$ and let $S_i = S_{i-1}^3 + 3S_{i-1}$ with $S_0 = 36$, then $R_p(3)$ is prime number iff $S_{p-1} \equiv 36 \pmod{R_p(3)}$.

Conjecture 1.54. Let p be an odd prime number greater than three , let $R_p(-3) = \frac{3^p+1}{4}$ and let $S_i = S_{i-1}^3 + 3S_{i-1}$ with $S_0 = 36$, then $R_p(-3)$ is prime number iff $S_{p-1} \equiv 36 \pmod{R_p(-3)}$.

Conjecture 1.55. Let $P_n^{(a)}(x) = \left(\frac{1}{2}\right) \cdot \left((x - \sqrt{x^2 + a})^n + (x + \sqrt{x^2 + a})^n \right)$. Given an odd integer $n (\geq 3)$ and integer a coprime to n , n is prime if and only if $P_n^{(a)}(x) \equiv x^n \pmod{n}$ holds .

Conjecture 1.56. Let n be an odd natural number greater than one . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $P_n(x)$ be Legendre polynomial , then n is a prime number if and only if $P_n(x) \equiv x^n \pmod{x^r - 1, n}$.

Conjecture 1.57. Let n be a natural number greater than one and let $F_n(x)$ be Fibonacci polynomial , then n is prime if and only if :
$$\sum_{k=0}^{n-1} F_n(k) \equiv -1 \pmod{n} .$$

Conjecture 1.58. Let a_n be the least unused prime greater than 3 such that $(a_n + a_{n-1})/2$ is prime, with $a_0 = 13$, then :

1. Every term of this sequence a_i is prime of the form $12k + 1$.
2. Every prime of the form $12k + 1$ is a member of this sequence .

Conjecture 1.59. Let m and n be a natural numbers , $m \geq 1$, $n > 2$, $n \neq 9$ and $\gcd(m, n) = 1$. Then n is prime if and only if
$$\sum_{k=1}^{n-1} (2^{mk} - 1)^{n-1} \equiv n \pmod{2^n - 1}$$

Conjecture 1.60. Let p, q, r be three consecutive prime numbers such that $p \geq 11$ and $p < q < r$, then $\frac{1}{p^2} < \frac{1}{q^2} + \frac{1}{r^2}$.

Conjecture 1.61. *Let p and q be consecutive prime numbers such that $p \geq 5$ and $p < q$, then*

$$\left[\frac{q}{p} - \frac{p}{q} \right] = 0.$$

Conjecture 1.62. *Let a, n, k be natural numbers greater than 0. If n is a prime number then*

$$\sum_{d|n} (\sigma_k(d) \cdot a^{n/d}) \equiv 2a \pmod{n}$$