Refutation of the direct correspondence of quantum gates to reversible classical gates

Taken from:

Faugère, J-C., Horan, K., Kahrobaei, D., Kaplan, M, Kashefi, E., Perret, L. (2017). "Fast quantum algorithm for solving multivariate quadratic equations". arxiv.org/pdf/1712.07211.pdf

"2.3 Quantum Gates: The following gates are quantum gates of interest which operate on qubits, each directly corresponding to reversible classical gates. For qubits $|x\rangle$, $|y\rangle$, $|z\rangle$ the gates perform the following operations:

– CNOT (XOR, Feynman)	$CNOT|x\rangle|y\rangle = |x\rangle|x + y\rangle$	[1.1]
– Toffoli (AND)	$T|x\rangle|y\rangle|z\rangle = |x\rangle|y\rangle|z + xy\rangle$	[2.1]
– X (NOT)	$X|x\rangle = |\bar{x}\rangle = |1 + x\rangle$	[3.1]
– n-qubit Toffoli (AND)	$T_n|x_1\rangle \ldots |x_n\rangle = |x_1\rangle \ldots |x_{n-1}\rangle|x_n + (x_1 \ldots x_{n-1})\rangle$	[4.1]
– Swap	$S|x\rangle|y\rangle = |y\rangle|x\rangle$"	[5.1]

LET: p q r $|x\rangle$ $|y\rangle$ $|z\rangle$; also p q r s $|x_1\rangle$ $|x_2\rangle$ $|x_3\rangle$ $|x_4\rangle$; n 4;
~ Not; + Or; & And; = Equivalence; @ Not Equivalence, XOR

T is tautology as the designated *proof* value, with F as contradiction
The 16-valued truth tables are presented row-major and horizontally.

Using the Meth8/VŁ4 apparatus and method, we render Eqs. 1.1-5.1 as:

(p@q)=(p&(p+q)) ;	TTFF TTFF TTFF TTFF	(1.2)
((p&q)&r)=((p&q)&(r+(p&q))) ;	TTTF TTTT TTTF TTTT	(2.2)
~p=((p\p)+p) ;	TFTF TFTF TFTF TFTF	(3.2)
(((p&q)&r)&s)=((((p&p)&q)&r)&(s+(((p&p)&q)&r))) ;	TTTT TTTF TTTT TTTT	(4.2)
(p&q)=(q&p) ;	TTTT TTTT TTTT TTTT	(5.2)

Eqs. 1.2-4.2 are *not* tautologous. This means those quantum gates do not directly correspond to reversible classical gates. (Eq. 5.2 is tautologous, although trivial.)

Remark: Eqs. 2.2 and 4.2 are nearly tautologous but not, due to the single F contradiction value.

What follows is that quantum gates *cannot* map to bivalent logic.

Remark: We obtained the above conclusion in unpublished work (2008) where: the qubit was proved to be a probabilistic vector (not bivalent); and the various quantum gates were mapped to non-bivalent truth tables to show where bivalent corrections *would be*. Hence, this paper demonstrates a shorter refutation of quantum gates as reversible bivalent operators.