Refutation of higher-order logic as bivalent

From: en.wikipedia.org/wiki/Higher-order_logic

"First-order logic quantifies only variables that range over individuals; second-order logic, in addition, also quantifies over sets; third-order logic also quantifies over sets of sets, and so on. For example, the second-order sentence

$$\forall P ((0 \in P \land \forall i (i \in P \to i + 1 \in P)) \to \forall n (n \in P)) \quad (1.1)$$

expresses the principle of mathematical induction. Higher-order logic is the union of first-, second-, third-, … , *n*th-order logic; i.e., higher-order logic admits quantification over sets that are nested arbitrarily deeply."

We evaluate higher-order logic based on the principle of mathematical induction.

We assume the Meth8/VŁ4 apparatus and method.

LET: p q r P i n; # necessity, all, ∀; % possibility, one or some; + Or; - Not Or; & And; > Imply, →; < Not Imply, less than, ∈; 1 (%p>#p); 0 ((%p>#p)-(%p>#p)) .

The designated proof value is T; F contradiction; C falsity; N truth. The 16-valued truth tables are row-major, horizontally.

Eq. 1.1 is a higher-order logic expression where the entire formula is universally quantified on one set (P) over universally quantified variables (i, n).

Meth8/VŁ4 treats sets and variables as variables. Therefore Eq. 1.1 is rendered as:

(#p & (((((%p>#p)-(%p>#p))<p) & (#q&((q<p)>((q+(%q>#q))<p))))>(#r&(r<p))))
= (p=p) ; FNFN FNFN FNFN FNFN (1.2)

Because Eq. 1.2 as rendered is *not* tautologous, the quantification over quantification is not bivalent.

We alleviate this constraining condition by distributing the quantified expression over nested expressions. At each nested level, the quantification is explicitly distributed for clarity.

(((#p&((%p>#p)-(%p>#p)))<(#p&p))&(#p&((#q&(q<p))>(#q&((q+(%q>#q))<p))))) >
(#p&(#r&(r>p))) ; TTTT TTTT TTTT TTTT (2.2)

Eq. 2.2 as rendered is tautologous. The truth tables of the main antecedent, consequent, and result are:

(((#p&((%p>#p)-(%p>#p)))<(#p&p))&(#p&((#q&(q<p))>(#q&((q+(%q>#q))<p))))) ;
FFFF FFFF FFFF FFFF (2.2.1)
(#p&(#r&(r>p))) ; FFFF FNFN FFFF FNFN (2.2.2)
> TTTT TTTT TTTT TTTT (2.2)

As an experiment we may simplify the quantification further so as to avoid any artifacts by expressing quantified variables directly:

((((%p>#p)-(%p>#p))<#p)&((#q<#p)>((#q+(%q>#q))<#p))) > (#r<#p) ;
NNNN NNNN NNNN NNNN (3.2)

Eq. 3.2 as rendered is *not* tautologous, although true with a value of N. Hence with absolute quantification, the induction principle as stated is *not* tautologous.

We conclude that higher-order logic is not bivalent and that nested quantification is better expressed as explicitly distributed.