ON STATUS INDICES OF SOME GRAPHS

Sudhir R Jog[1] Shrinath L Patil[2]

[1]Department of Engineering Mathematics, K.L.S. Gogte Institute of Technology, Belagavi, Karnataka, India

[2]Department of Mathematics,S.J.P.N. Trust'sHirasugar Institute of Technology ,Nidasoshi, Karnataka, India

E-mail:sudhir@git.edu shrinathlpatil@gmail.com

***Abstract* :** Ramane,Yalnaik recently defined some molecular structural descriptors on the lines of Weiner index, Zagreb Index, etc known as status indices, harmonic status indices, status coindices and harmonic status coindices. Here we consider some graphs of fixed diameter and compute the all these parameters.

Keywords: Status of a vertex, Harmonic Status Index, Thorn graph

AMS(2010): 05C12,92E10

1. Introduction

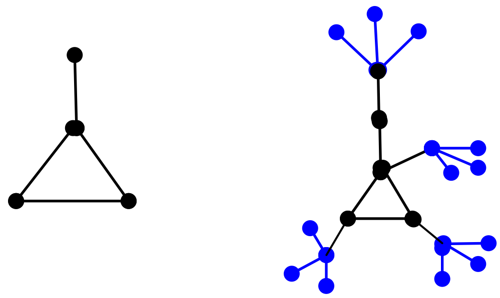
There are several molecular structural graph descriptors such as Weiner Index, Zagreb Index, Hosoya Index etc which strongly correlate studies in graph theory with chemistry. Most of these indices are based on the distance between vertices in a graph. Motivated by harmonic mean we have harmonic index of a graph defined by Fajtlowicz [5]. For more work one can refer [6]. Further motivated by the same, Ramane,Yalnaik introduced the harmonic status index of graphs [4].

In this paper we discuss the harmonic status indices and coindices of graph constructions after obtaining the status of all the vertices in it.

2. Status connectivity indices, and coindices of some graphs

In what follows, we consider a class of graphs constructed by first joining a path of length l (≥ 1) to each vertex of G and then attaching k pendent vertices to each end vertex of the path attached. Such a graph can be called l level thorn graph denoted by $G^{\wedge l\,(+k)}$.The usual thorny graph G^{+k} can be regarded as 0 level thorn graph. If $l = 1$ we get first level thorn graph $G^{\wedge 1\,(+k)}$.

Example: A graph G and a first level thorn graph $G^{\wedge 1(+3)}$ is as shown below



Definition [1]**:** The status of a vertex $u \in V(G)$ is defined as the sum of its distance from every other vertex in $V(G)$ and is denoted by $\sigma(u)$. That is,$\sigma(u) = \sum_{u \epsilon V(G)} d(u, v)$.

Definition [3] : The first status connectivity index $S_1(G)$ and second status connectivity index of a connected graph G are defined as

$S_1(G) = \sum_{uv \epsilon E(G)}[\sigma(u) + \sigma(v)]$ and $S_2(G) = \sum_{uv \epsilon E(G)}[\sigma(u)\sigma(v)]$

Similarly the first and second status connectivity coindices are defined as,

$\overline{S_1}(G) = \sum_{uv \notin E(G)}[\sigma(u) + \sigma(v)]$ and $\overline{S_2}(G) = \sum_{uv \notin E(G)}[\sigma(u)\sigma(v)]$

Definition[5]: The Harmonic index of a graph G is defined as , $H(G) = \sum_{uv \epsilon E(G)} \frac{2}{d(u)+d(v)}$

Definition[4]: The harmonic status of a connected graph G as,

$$HS(G) = \sum_{uv \in E(G)} \frac{2}{\sigma(u) + \sigma(v)}$$

Definition[4]: The harmonic status coindex of a connected graph G is defined as,

$$\overline{HS}(G) = \sum_{uv \notin E(G)} \frac{2}{\sigma(u) + \sigma(v)}$$

2.1 Status connectivity indices of zero level thorn graphs

First we evaluate the status connectivity index and coindex of 0 level thorn graphs denoted by G^{+k}. To obtain the harmonic status connectivity index and coindex of this graph we need to calculate status of each vertex and number of pairs of adjacent vertices and pairs of non adjacent vertices in $G^{\wedge 1\,(+k)}$. With respect to degree, there are two types of vertices in G^{+k} , nk pendent vertices, n vertices of degree '$r + nk$' we call them as internal.

Theorem 2.1.1: If G is a complete graph of order 'n' then the harmonic status index and coindex of thorn graph K_n^{+k} are given by,

$$HS(K_n^{+k}) = \frac{n(n-1)}{2}\frac{1}{(2nk+n-k-1)} + nk\,\frac{2}{(5nk+3n-2k-4)}$$

$$\overline{HS}\,(K_n^{+k}) = nkC_2\,\frac{1}{(3nk+2n-2k-4)} + nk(n-1)\,\frac{2}{5nk+3n-k-3}$$

Proof: The graph K_n^{+k} is of diameter 4 and there are two types of vertices in it. A set of 'nk' pendent vertices and n vertices of degree 'n+1'.Let u_i i =1,2…nk denote the pendent vertices and v_i i =1,2….n denote the vertices of degree 'n+1. Then the status of pendent vertex is

$\sigma(u_i) = 1+2(k-1) +2(n-1) +3k\,(n-1) =3nk +2n-k-3$ and the status of the internal vertex v_i , is $\sigma(v_i)$ $= 1.(n-1) +k +2k\,(n-1) =2nk +n-k-1$.

Now in K_n^{+k} there are $n(n-1)/2$ adjacent pairs of internal vertices and nk pairs of vertices forming edges formed by one internal and one external vertex. Hence by definition the harmonic status index of K_n^{+k} is

$$\mathrm{HS}(K_n^{+k}) = \frac{n(n-1)}{2}\frac{2}{2(2nk+n-k-1)} + nk\,\frac{2}{(3nk+2n-k-3+2nk+n-k-1)}$$

$$= \frac{n(n-1)}{2}\frac{1}{(2nk+n-k-1)} + nk\,\frac{2}{(5nk+3n-2k-4)}$$

Also in K_n^{+k} , $(nk)C_2$ pairs of nonadjacent pendent vertices and $nk\,(n-1)$ pairs of nonadjacent pairs of vertices formed by one pendant and one internal vertex. So that harmonic status coindex of K_n^{+k} is,

$$\overline{HS}\,(K_n^{+k}) = nkC_2\,\frac{2}{2(3nk+2n-2k-4)} + nk(n-1)\,\frac{2}{(3nk+2n-k-3+2nk+n-k-1)}$$

$$= nkC_2\,\frac{1}{(3nk+2n-2k-4)} + nk(n-1)\,\frac{2}{5nk+3n-k-3}$$

Theorem 2.1.2: The first and second status connectivity index of K_n^{+k} are given by,

$$S_1(K_n^{+k}) = n(n-1)(2nk+n-k-1) + nk\,(5nk+3n-2k-4)$$

$$S_2(K_n^{+k}) = (nk)C_2(3nk+2n-k-3)^2 + nC_2(3nk+2n-k-3)(2nk+n-k-1)$$

Proof : The graph K_n^{+k} is of diameter 3 and there are two types of vertices in it. A set of '*nk*' pendent vertices and *n* vertices of degree 'n+1'.Let u_i i =1,2…nk denote the pendent vertices and v_i i =1,2….n denote the vertices of degree 'n+1.

Then the status of pendent vertex is

$\sigma(u_i) = 1+2(k-1)+2(n-1)+3k(n-1) = 3nk+2n-k-3$

and the status of the internal vertex v_i , is $\sigma(v_i) = 1.(n-1)+k+2k(n-1) = 2nk+n-k-1$.

Now in K_n^{+k} there are $n(n-1)/2$ adjacent pairs internal vertices and *nk* pairs of vertices forming edges formed by one internal and one external vertex. Hence by definition the status connectivity index of K_n^{+k} is

$$S_1(K_n^{+k}) = \frac{n(n-1)2(2nk+n-k-1)}{2} + nk\,(3nk+2n-k-3+2nk+n-k-1)$$

$$= n(n-1)(2nk+n-k-1) + nk\,(5nk+3n-2k-4)$$

Also in K_n^{+k} $(nk)C_2$ pairs of nonadjacent pendent vertices and $nk(n-1)$ pairs of nonadjacent pairs of vertices formed by one pendant and one internal vertex. So that status connectivity coindex of K_n^{+k} is,

$$S_2(K_n^{+k}) = (nk)C_2(3nk+2n-k-3)^2 + nC_2(3nk+2n-k-3)(2nk+n-k-1)$$

Theorem 2.1.3: If G is 'r' regular graph of diameter 2 then the status index of G^{+k} is

$$HS(G^{+k}) = \frac{nr}{2}\frac{1}{(2n+2kr+k-r-2)} + nk\,\frac{2}{(5n+5kr+3k-2r-6)}$$

Proof: First we observe that if G has diameter 2 then G^{+k} has diameter 4.Hence from the structure we have the status of each internal vertex v_i as ,

$$\sigma(v_i) = 1.(k+r) + 2\,k\,r + 2.(n-1-r) = 2n+2k\,r+k-r-2$$

Also the status of each pendant vertex u_i as,

$\sigma(u_i) = 1 + 2.r + 2(k\text{-}1) + 3(n\text{-}1\text{-}r) = 3n + 3rk + 2k - r - 4$.

There are $\frac{nr}{2}$ internal edges giving harmonic status contribution,

$$\frac{nr}{2}\frac{2}{2(2n+2kr+k-r-2)} = \frac{nr}{2}\frac{1}{(2n+2kr+k-r-2)}$$

Similarly the pendent 'nk' vertices adjacent to 'n' internal vertices contribute,

$$nk\,\frac{2}{(5n+5kr+3k-2r-6)}$$

Hence the harmonic status index of G^{+k}

$$HS(G^{+k}) = \frac{nr}{2}\frac{1}{(2n+2kr+k-r-2)} + nk\,\frac{2}{(5n+5kr+3k-2r-6)}$$

Theorem 2.1.4: If G is 'r' regular graph of diameter 2 then the harmonic status coindex of G^{+k} is,

$$\overline{HS}(G^{+k}) = (nk)C_2\,\frac{1}{(3n+3kr+2k-r-4)} + nk(n-1)\,\frac{2}{(5n+5rk+3k-2r-6)} + \left(n\,C_2 - \frac{nr}{2}\right)\frac{1}{(2n+2kr+k-r-2)}$$

Proof: We note that there are $n\,(k+1)C_2 - \left(\frac{nr}{2} + nk\right)$ non adjacent pairs of vertices in G^{+k}

There are $nk\,C_2$ pendent nonadjacent pendent vertices, $nk\,(n\text{-}1)$ pairs of nonadjacent vertices combining one pendant and one internal vertex and finally $n\,C_2 - \frac{nr}{2}$ nonadjacent internal vertices. Taking contribution from each of them we have status connectivity coindex of G^{+k} as,

$$\overline{HS}(G^{+k}) = (nk)C_2\,\frac{2}{(6n+6kr+4k-4r-8)} + nk(n-1)\,\frac{2}{5n+5rk+3k-2r-6} + \left(n\,C_2 - \frac{nr}{2}\right)\frac{2}{2(2n+2kr+k-r-2)}$$

$$= (nk)C_2\,\frac{1}{(3n+3kr+2k-2r-4)} + nk(n-1)\,\frac{2}{5n+5rk+3k-2r-6} + \left(n\,C_2 - \frac{nr}{2}\right)\frac{1}{(2n+2kr+k-r-2)}$$

We get the following in a routine manner.

Proposition 2.1.5: The first and second status connectivity index of G^{+k} are given by,

$$S_1(G^{+k}) = nr(2n + 2kr + k - r - 2) + nk(5n + 5kr + 3k - 3r - 6)$$

$$S_2(G^{+k}) = \frac{nr}{2}(2n + 2kr + k - r - 2)^2 + nk(3n + 3rk + 2k - r - 4)(2n + 2kr + k - r - 2).$$

Proposition 2.1.6: The first and second status connectivity co index of G^{+k} are given by,

$$\overline{S_1}(G^{+k}) = \frac{nk(nk-1)}{2} 2(3n + 3kr + 2k - r - 4) + nk(n-1)(5n + 5kr + 3k - 3r - 6) + (nC_2 - \frac{nr}{2})(4n + 4kr + 2k - 2r - 4) = nk(nk-1)(3n + 3kr + 2k - r - 4) + nk(n-1)(5n + 5kr + 3k - 3r - 6) + (nC_2 - \frac{nr}{2})(4n + 4kr + 2k - 2r - 4).$$

$$\overline{S_2}(G^{+k}) = \frac{nk(nk-1)}{2}(3n + 3kr + 2k - r - 4)^2 + nk(n-1)(3n + 3kr + 2k - 2r - 4)(2n + 2kr + k - r - 2) + (nC_2 - \frac{nr}{2})(2n + 2kr + k - r - 2)^2$$

2.2 Status connectivity indices of zero level thorn graphs

Now we discuss the harmonic status index and coindex of first level thorn graphs.We need to calculate status of each vertex and number of pairs of adjacent vertices and pairs of non adjacent vertices in $G^{\wedge 1 (+k)}$.With respect to degree there are three types of vertices in $G^{\wedge 1 (+k)}$.nk pendent vertices, n vertices of degree 'k +1' we call them as internal and lastly 'n' vertices having degree sequence added by 1.We call them external, in particular if G is 'r' regular their degrees will become 'r + 1'.

Theorem 2.2.1: If G is 'r' regular graph of diameter 2 then the status connectivity index of $G^{\wedge 1 (+k)}$ is, $HS(G^{\wedge 1 (+k)})$

$$= \frac{nr}{2} \frac{1}{(5n+4nk-rk-2r-2k-4)} + n \frac{2}{(5n+4nk-rk-2r-2k-4+7n+5nk-2r-4k-rk-6)} + n^2 k \frac{2}{(5n+4nk-rk-2r-2k-4+9n+6nk-rk-4k-2r-8)}$$

Proof: First we observe that if G has diameter 2 then $G^{\wedge 1 (+k)}$ has diameter 6.Hence from the structure we have the status of each internal vertex v_i as ,

$\sigma(v_i) = 1.(r + 1) + 2.(r + k)$ $+2.(n\text{-}1\text{-}r)$ $++3\,r\,k + 3.(n\text{-}1\text{-}r) + 4k\,(n\text{-}1\text{-}r) = 5n + 4nk - rk - 2r - 2k - 4$

Also the status of each external vertex u_i as,

$\sigma(u_i) = 1.(k + 1) + 2.r + 3\,r + 3.(n\text{-}1\text{-}r) + 4(n\text{-}1\text{-}r) + 4rk + 5k\,(n\text{-}1\text{-}r) = 7n + 5nk - 2r - 4k - rk - 6$

Finally the pendent vertices being the only vertices on the diametrical path have the status

$\sigma(w_i) = 1+2.1 + 2\,(k - 1) + 3.r + 4(n\text{-}1\text{-}r) + 4r + 5kr + 5(n\text{-}1\text{-}r) + 6k\,(n\text{-}1\text{-}r) = 9\,n + 6\,nk - rk - 4k - 2r - 8.$

In $G^{\wedge 1\,(+k)}$ there are $\frac{nr}{2}$ pairs of internal adjacent vertices, n pair of adjacent vertices formed of one internal and one external vertex and finally n^2k pairs of adjacent vertices formed of one internal and one pendant vertex.

Hence the harmonic status index of $G^{\wedge 1\,(+k)}$ is given by,

$$HS(G^{\wedge 1\,(+k)}) = \frac{nr}{2}\frac{1}{(5n+4nk-rk-2r-2k-4)} + n\,\frac{2}{(5n+4nk-rk-2r-2k-4+7n+5nk-2r-4k-rk-6)} + n^2k\,\frac{2}{(5n+4nk-rk-2r-2k-4+9n+6nk-rk-4k-2r-8)}$$

$$= \frac{nr}{2}\frac{1}{(5n+4nk-rk-2r-2k-4)} + n\frac{2}{(12n+9nk-2rk-4r-4k-10)} + n^2k\frac{2}{(14n+10nk-2rk-4r-6k-12)}$$

$$= \frac{nr}{2}\frac{1}{(5n+4nk-rk-2r-2k-4)} + n\frac{2}{(12n+9nk-2rk-4r-4k-10)} + n^2k\frac{1}{(7n+5nk-rk-2r-3k-6)}$$

Theorem 2.2.2: If G is 'r' regular graph of diameter 2 then the harmonic status coindex of $G^{\wedge 1\,(+k)}$ is given by, $\overline{HS}(G^{\wedge 1\,(+k)}) =$

$$(nk)C_2\frac{1}{(9n+6nk-rk-4k-2r-8)} + \left(nC_2 - \frac{nr}{2}\right)\frac{1}{(5n+4nk-rk-2r-2k-4)}\ \ nC_2\frac{1}{(7n+5nk-rk-2r-3k-6)} + n^2k\frac{1}{(7n+5nk-rk-2r-3k-6)} + n(n-1)\frac{2}{(12n+9nk-2rk-4r-6k-10)}$$

Proof: In $G^{\wedge 1\,(+k)}$ there are $(nk)C_2$ pairs of nonadjacent pendent vertices, $nC_2 - \frac{nr}{2}$ pairs of nonadjacent vertices formed by internal vertices, nC_2 pairs of nonadjacent vertices formed by external vertices, n^2k nonadjacent pair of vertices formed by one pendant and one internal vertex and finally $n(n\text{-}1)$ pairs of nonadjacent vertices formed by one internal and one external vertex.

Hence the harmonic status connectivity coindex is given by,

$$\overline{HS}\,(G^{\wedge 1\,(+k)}) = (nk)C_2\frac{2}{2(9n+6nk-rk-4k-2r-8)} + \left(nC_2 - \frac{nr}{2}\right)\frac{2}{2(5n+4nk-rk-2r-2k-4)} + nC_2\frac{2}{2(7n+5nk-rk-2r-3k-6)} + n^2k\frac{2}{(5n+4nk-rk-2r-2k-4+9n+6nk-rk-4k-2r-8)} + n(n-1)\frac{2}{(5n+4nk-rk-2r-2k-4+7n+5nk-2r-4k-rk-6)}$$

$$= (nk)C_2\frac{1}{(9n+6nk-rk-4k-2r-8)} + \left(nC_2 - \frac{nr}{2}\right)\frac{1}{(5n+4nk-rk-2r-2k-4)} + nC_2\frac{1}{(7n+5nk-rk-2r-3k-6)} + n^2k\frac{1}{(7n+5nk-rk-2r-3k-6)} + n(n-1)\frac{2}{(12n+9nk-2rk-4r-6k-10)}.$$

Following follows on the same lines.

Proposition 2.2.3: The first and second status connectivity index and coindex of a 'r' regular graph of order 'n' are given by,

$$S_1(G^{1(+k)}) = \frac{nr}{2}(5n + 4nk - rk - 2r - 2k - 4) + n(12n + 9nk - 2rk - 4r - 4k - 10) + 2n^2k(7n + 5nk - rk - 2r - 3k - 6)$$

$$S_2(G^{1(+k)}) = \frac{nr}{2}(5n + 4nk - rk - 2r - 2k - 4)^2 + n\,(5n + 4nk - rk - 2r - 2k - 4)(7n + 5nk - 2r - 4k - rk - 6)$$

$$\overline{S_1(G^{1(+k)}} = (nk)C_2\; 2(9n + 6nk - rk - 4k - 2r - 8) + \left(nC_2 - \frac{nr}{2}\right) 2(5n + 4nk - rk - 2r - 2) + 2\,(nC_2)(7n + 5nk - rk - 2r - 3k - 6) + n^2k(7n + 5nk - rk - 2r - 3k - 6) + 2n(n-1)(12n + 9nk - 2rk - 4r - 6k - 10)$$

$$\overline{S_2(G^{1(+k)}} = (nk)C_2\,(9n + 6nk - rk - 4k - 2r - 8)^2 + \left(nC_2 - \frac{nr}{2}\right)(5n + 4nk - rk - 2r - 2)^2 + (nC_2)(7n + 5nk - rk - 2r - 3k - 6)^2 + n^2k(7n + 5nk - rk - 2r - 3k - 6)^2 + n(n-1)(12n + 9nk - 2rk - 4r - 6k - 10)^2$$

References:

1. Harary F.,"Status and Contrastatus Sociometry Vol 22 1(1959), 23-43.

2. H.S.Ramane & A.S.Yalnaik, "Bounds for the status connectivity index of Line graphs" ,International Journal of Computational and Applied Mathematics,Vol 12(3) 2017.

3. H.S.Ramane & A.S.Yalnaik, "Status connectivity indices of graphs and its applications to the boiling point of benzenoid hydrocarbons".Journal of Applied Mathematics and Computing 2016.

4. H.S.Ramane,B.Basavangoud and A.S.Yalnaik. "Harmonic Status Index of Graphs, Bulletin of Mathematical Sciences and applications 17 (2016), pp 24-32.

5. S.Fajtlowicz.,On conjectures of Graffitti- II ,Congr.Numer 60 (1987).187-197.

6.Y.Hu., and X.Zhou,. "On the Harmonic Index of unicyclic and Bicyclic Graphs" Wseas Tran.Math. 12 (2013),716-726.

7. J.Liu., "On the Harmonic Index of triangle free graphs"Appl.Math.,4(2013) 1204-1206.