

Sub and Superluminal Transformations of the Complex Vector Potential

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We consider in this chapter the transformations of an oscillatory form of A^μ in a complex 12D Minkowski space. The form of the transformation of A^μ or (\underline{A}, ϕ) depends on whether such a transformation is a superluminal Lorentz transformation (SLT) or a subluminal Lorentz transformation.

1. Introduction

In [1] we extended our examination of the properties of complex Minkowski spaces and the mixing of real and imaginary components of space and time under the influence of a superluminal boost in the x -direction. We determined that there is a unique mixing of position and time vectors in complex Minkowski space which is not present in real 4D Minkowski space. We take real M^4 4-space as a slice through the complex Minkowski space, $\hat{M}^4 + \mathbb{C}^4$.

In this chapter we examine in detail the transformations of the vector and scalar potential in complex Minkowski space under an x -direction superluminal boost. We find that we have a mixing of the temporal and spatial components in the laboratory frame but only mixing of temporal components in the moving frame. In the laboratory frame, Σ an oscillatory and damped solution, both expressed in terms of space and time, whereas in the moving frame, Σ' the damped term is expressed in terms of time components only. The oscillation in terms of the spatial coordinates vanishes in going from the Σ to Σ' frame. It is also interesting to note that the vector potential normalization term A_{ox} goes to ϕ' , the scalar potential term under the superluminal boost (SLT).

We also examine the relationship between the vector and scalar potential transformation under the SLT and compare this to the variation of E and B fields and their relationship to A_x and ϕ . The transformation from $v < c$ through c to $v > c$ produces a mixing or spacetime rupture which greatly modifies any existing vacuum fields. We examine the presence of tachyonic signals in [2,3], and here we demonstrate that the monopole structure may be associated with a tachyonic signal.

2. Complex Minkowski Spaces with Time Symmetry Considerations

In previous work, Rauscher and Ramon introduced the structure and properties of complex Minkowski spaces [1,2] and examined the mixing of real and imaginary components of space and time under the influence of superluminal boosts in the x direction and determined that the mixing is unique. We label the complex coordinates as

$$\underline{z}^\mu = x_{\text{Re}}^\mu + ix_{\text{Im}}^\mu \quad (1)$$

where \underline{z} is a complex quantity and x_{Re} and x_{Im} are real quantities, where “Re” and “Im” refer to the real and imaginary parts of the complex quantity \underline{z} . The index μ runs 0,1,2,3 where the index 0 represents the time component and 1,2,3 represent the spatial vector components. We denote these 4-component vectors as t, x, y, z .

In complex Minkowski space these vectors are complex quantities and are given as

$$\underline{t} = t_{\text{Re}} + it_{\text{Im}}, \quad \underline{x} = x_{\text{Re}} + ix_{\text{Im}} \quad (2)$$

$$\underline{y} = y_{\text{Re}} + iy_{\text{Im}}, \quad \underline{z} = z_{\text{Re}} + iz_{\text{Im}}$$

This set of vectors defines an 8D complex space [4]. A slice of this 8-space gives four real dimensions of M_4 forming a subspace in which the line elements are given by the real part of the complex quantities [5].

For a 12D space we consider time as a 3D complex quantity,

$$t = t_x \hat{x} + t_y \hat{y} + t_z \hat{z} \quad (3)$$

where we have the components

$$\begin{aligned} t_x &= t_{x\text{Re}} + it_{x\text{Im}} \\ t_y &= t_{y\text{Re}} + it_{y\text{Im}} \\ t_z &= t_{z\text{Re}} + it_{z\text{Im}} \end{aligned} \quad (4)$$

As before the subscripts Re and Im refer to the real and imaginary parts of the complex quantities.

Demers [6] introduced a symmetry principle between multidimensional time components which specify only one modulus as having physical meaning,

$$|t| = (t_x^2 + t_y^2 + t_z^2)^{\frac{1}{2}} \quad (5)$$

The modulus of the time vector is chosen to correspond to the usual physical time. A detailed discussion of this choice of modulus and its implications for Lorentz invariance is in [1]. In our HD model we imply that all complex temporal components are physically significant [7]. This probably makes correspondence to Cramer’s Transactional Interpretation of quantum theory where he implies that all off diagonal components of a transaction are physically real [8-10].

3. Complex Transformations of the Vector Potential

We start with the vector potential \underline{A} in the usual form

$$\underline{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \quad (6)$$

where we choose A_x to vary as

$$\underline{A}_x = \underline{A}_{0x} \exp i(\omega t \pm \underline{k}_x \cdot \underline{x}) \quad (7)$$

The sign in the exponent refers to the accelerated or retarded vector potentials, respectively, and k_x is the wave number in the x -direction. The potential A_{0x} is complex and is given by

$$A_{0x} = A_{0x\text{Re}} + iA_{0x\text{Im}} \quad (8)$$

The plus sign in the exponential in Eq. (7) is associated with the advanced or accelerated vector potential and the minus sign with the retarded vector potential [11].

Upon substitution of the real and imaginary parts of A_{0x} from Eq. (8) and for t and x from (2), we have

$$A_x = (A_{0x\text{Re}} + iA_{0x\text{Im}}) \exp \left[\omega(t_{x\text{Re}} + it_{x\text{Im}}) + ik_x(x_{\text{Re}} + x_{\text{Im}}) \right] \quad (9)$$

where the wave vectors are given as $k_x = \omega/c = k_y = k_z$ and $k_x = k = \omega/v_\phi$ where v_ϕ is the phase velocity. The frequency, ω and velocity, c are isotropic in all directions of the propagation of the potential. The phase velocity, v_ϕ is taken to be $v_\phi > c$ for a superluminal boost also called a superluminal Lorentz transformation, (SLT). We examine this case and determine for SLT's if $k_x = k_y = k_z$. That is, if the isotropy of the potential maintained. In the vacuum propagation of the vector potential, $k_x = k_y = k_z$, it is maintained but not in a material medium for the case where $k_x \neq k_y \neq k_z$, which is the case for uniaxial, biaxial or triaxial crystals, for example, in phase space for the vacuum for $v_\phi > c$.

In a vacuum, $k_x = k_y = k_z$, and in the moving frame traveling at $v > c$ for a superluminal boost (for example in the frame of a rocket for $v > c$ for deep space interstellar travel), the vector potential remains isotropic. This is not true for the observer in the laboratory frame, i.e. $v < c$, where the properties of the media are modified by the Lorentz transformation which affects the perceived properties of the object. For example, a photon appears as a photon in the "rocket" frame or superluminal frame, but it appears as a tachyon to an observer in the laboratory frame. Since the vector potential is defined by a phase velocity component and not by a group velocity parameter then we consider that the vector k is the same in dimensions of free space even under the SLT boost [2].

Then considering the form of the vector and scalar potentials under the action of a superluminal boost in the x direction and determine both a damping wave and an oscillatory wave for the advanced potential solutions only. This case will be expanded on further for the limit as the velocity approaches the velocity of light and where quantum mechanical considerations may demand that $k_x = k_y = k_z$, for example. In the limit of a transformation where nonlinear stresses exist one can define as a "rupture" in the extreme case of the subluminal, through c , to superluminal transformation.

Similar to equation (9) for the vector potential, we can define the complex scalar potential, ϕ as

$$\begin{aligned} \phi_x &= \phi_{0x} \exp \left[i(\omega t \pm \underline{k}_x \cdot \underline{x}) \right] \\ &= (\phi_{0x\text{Re}} + i\phi_{0x\text{Im}}) \exp i\omega(t_{x\text{Re}} + it_{x\text{Im}}) \\ &\quad \times \exp \left[\pm ik_x(x_{\text{Re}} + x_{\text{Im}}) \right] \end{aligned} \quad (10)$$

For equation (10) we again have assumed isotropic conditions of the vacuum in which the propagation constant is symmetric in all directions, i.e., $k_x = k_y = k_z = \omega/v_\phi$ where v_ϕ , is the phase velocity

propagation.

4. Superluminal Vector and Scalar Potential Transformation Laws

For simplicity we consider superluminal boost $v_x = \infty$ along the positive x direction [13,14]. See Chap. 2 on tachyonic signaling. See Fig. 1. The space and time vectors in the real 4D Minkowski space transform as follows [12]

$$x' = +t, \quad y' = -iy, \quad z' = iz, \quad t' = x \quad (11)$$

for real and imaginary parts separately, where x, y, z, t are real quantities in the laboratory Σ frame, and x', y', z', t' are the real quantities in the moving Σ' frame. Now in the 12D (M^2) complex Minkowski space, the above transformation laws for a superluminal boost ($v_x = +\infty$) in the positive x direction become

$$\begin{aligned} x'_{\text{Re}} + ix'_{\text{Im}} &= t_{x,\text{Re}} + it_{x,\text{Im}}, & y'_{\text{Re}} + iy'_{\text{Im}} &= y_{\text{Im}} - iy_{\text{Re}}, \\ z'_{\text{Re}} + iz'_{\text{Im}} &= z_{\text{Im}} - iz_{\text{Re}}; & t'_{x,\text{Re}} + it'_{x,\text{Im}} &= x_{\text{Re}} + ix_{\text{Im}}, \\ t'_{y,\text{Re}} + it'_{y,\text{Im}} &= t_{y,\text{Im}} - it_{y,\text{Re}}, & t'_{z,\text{Re}} + it'_{z,\text{Im}} &= t_{z,\text{Im}} - it_{z,\text{Re}} \end{aligned} \quad (12)$$

The transformation laws given by (12) preserve the magnitude of the line element but not the sign as

$$-x'^{\mu}x'^{\nu} = x^{\mu}x^{\nu} \quad (13)$$

where index μ and ν run over 0,1,2,3 representing 0 as time vector and 1,2,3 as spatial vectors. Therefore we have the signature (-+++).

Similar to the transformation laws for space and time vectors as given by (12) we can write the transformation laws for the vector and scalar potential. For a superluminal boost in positive x direction, the transformation laws for (\underline{A}, ϕ) are:

$$A'_x = \gamma \left(A_x - \frac{v_x^2}{c^2} \phi \right), \quad A'_y = A_y, \quad A'_z = A_z, \quad \phi' = \gamma(\phi - v_x A_x) \quad (14)$$

where ϕ is the scalar potential and γ is the usual Lorentz term

$$\gamma' \equiv \frac{1}{\left(\frac{v_x^2}{c^2} - 1 \right)^{\frac{1}{2}}}. \quad (15)$$

We consider A'_x , etc., transforming as a gauge. In Eq. (14), the vector potential A is considered to be a 4-vector real quantity, A_{μ} or $\tilde{A} = (A_x, A_y, A_z, \frac{i\phi}{c})$, which preserves the length of the line element but not the sign, i.e. we have for the gauge transform

$$A_\mu A_\mu = -A'_\mu A'_\mu \quad (16)$$

Eq. (14) then simplifies to the following relationships for the velocities approaching infinity, $v_x = \infty$.

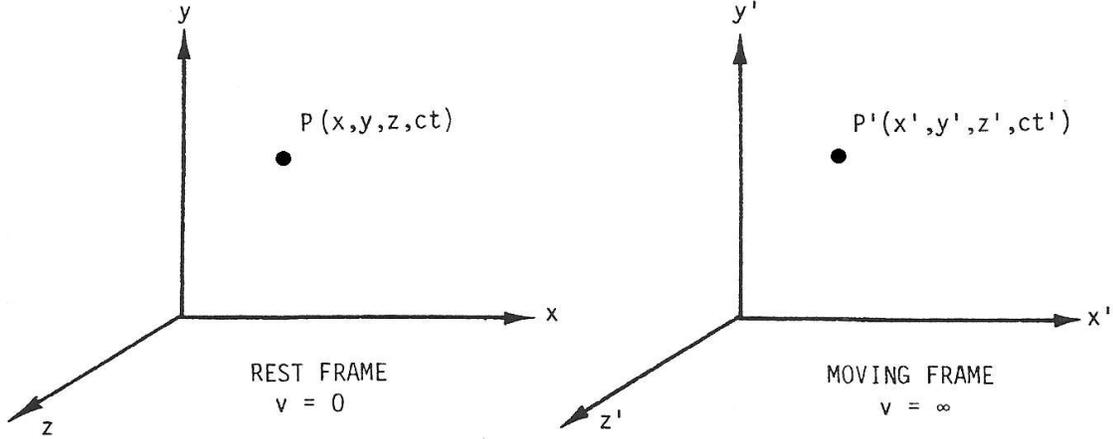


Figure 1. Schematic representation of a superluminal boost (Eq. 12) between an event P', in the moving frame and the corresponding event P in the rest frame, Σ . Relative velocity of the moving frame, Σ' is infinite, $v = \infty$. For an observer in the rest frame looking at event, P', he will see the x' coordinate transform to the time component, t in the rest frame and vice-versa.

The transformation laws for scalar and vector potentials under the superluminal boost in the positive x direction for $v_x = +\infty$. From the rest frame, Σ , to the moving frame, Σ' , for unaccelerated vector and scalar potentials, we have

$$A_x = -\phi', \quad A_y = A'_y, \quad A_z = A'_z, \quad \phi = -A'_x \quad (17)$$

From the moving frame, S', to the rest frame, S, for the unaccelerated vector and scalar potentials we obtain

$$A'_x = -\phi, \quad A'_y = A_y, \quad A'_z = A_z, \quad \phi' = -A_x \quad (18)$$

Eq. (18) is valid for real or complex vector and scalar potentials. Real and imaginary parts are easily separable in a complex quantity and they will transform according to Eq. (18) under the influence of a superluminal boost in the positive x direction. If these are the retarded (or accelerated or advanced) vector and scalar potentials, the transformation laws under the superluminal boosts will be different from the ones given by Eq. (18). These transformation laws are given by the combination of Eq. (18) and the transformation laws of the complex space and time vectors as given by Eq. (12).

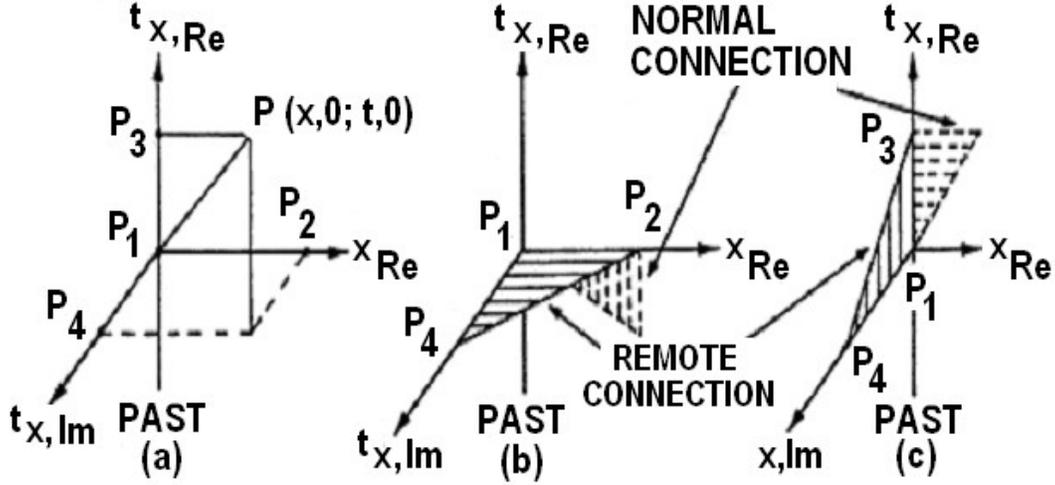


Figure 2. We represent the location of four points in the complex manifold. In Fig. 2a, point P_1 is the origin, and P is a generalized point which is spatially and temporally separated from P_1 . In Fig. 2b, the Points P_1 and P_2 are separated in space but synchronous in time. This could be a representation of real-time nonlocal spatial separation..In Fig. 2c, points P_1 and P_3 are separated temporally and spatially contiguous. This represents an anticipatory temporal connection.

These conditions are illustrated in Fig. 2. In 2a we represent a generalized point $P(x_{Re}, t_{Re}, t_{Im})$, displaced from the origin which is denoted as P_1 . This point can be projected on each dimension x_{Re} , t_{Re} and t_{Im} as points P_2 , P_3 , and P_4 respectively. In Fig. 2b, we denote the case where a real-time *spatial* separation exists between points, P_1 and P_2 on the x_{Re} axis, so that $\Delta x_{Re} \neq 0$, and there is no anticipation, so that $t_{Re} = 0$, and access to imaginary time t_{Im} , nonlocality can occur between the P_1 to P_4 interval, so that $\Delta t_{Im} \neq 0$. Then, our metric gives us $\Delta s^2 = 0$, where nonlocality is the contiguity between P_1 and P_2 by its access to the path to P_4 . By using this complex path, the physical spatial separation between P_1 and P_2 becomes equal to zero, allowing direct nonlocal connectedness of distant spatial locations, observed as a fundamental nonlocality of remote connectedness on the spacetime manifold.

Figure 2c represents the case where anticipation occurs between P_1 and an apparent future anticipatory accessed event, P_3 on the t_{Re} axis. In this case, no physical spatial separation between observer and event is represented in the figure. Often such separation on the x_{Re} exists. In the case where $x_{Re} = 0$, then access to anticipatory information, along t_{Re} can be achieved by access to the imaginary temporal component, t_{Im} . Hence, remote, nonlocal events in 4-space or the usual Minkowski space, appear contiguous in the complex eight space and nonlocal temporal events in the 4-space appear as anticipatory in the complex 8-space metric. Both nonlocality and anticipatory systems occur in experimental tests of Bell's Theorem and perhaps in all quantum measurement processes.

The propagation constant is considered to be isotropic in vacuum and defined as $d_x = \omega / v_\phi$, where v_ϕ is the phase velocity and ω is the radian frequency of the propagating signal. Usually in most cases the phase velocity of propagation in vacuum is a constant $v_\phi = c$, where c is the velocity of light in vacuum. For the purpose of this work, we will consider a tachyon traveling faster than light emitting an electromagnetic signal at frequency ω which propagates at the velocity of light. This assumption will

simplify the subject matter of this work. We examine the faster than light electromagnetic signals emitted by a traveling tachyon which might lead into a Doppler effect at velocities faster than light.

Considering only the advanced potential solution from (11), Equation (11) can now be rewritten as two separate terms, so that in the Σ frame,

$$A_x = (A_{0x,Re} + iA_{0x,Im}) \{ \exp i[\omega t_{x,Re} - kx_{Re}] \times \exp -[\omega t_{x,Im} - kx_{Im}] \} \quad (19)$$

where the first exponent represents the usual type of oscillatory terms and the second exponent represents a decaying component which is not present in the usual 4D spacetime model. Note also that we have used the isotropy of the vector, k in Eq. (19) as examined in the previous section.

Examine the complex exponential of Eq. (19) using the transformations of Eq. (11) as follows so that we have for the exponents

$$\exp i[\omega x'_{Re} - kt'_{x,Re}] \times \exp -[\omega x'_{Im} - kt'_{x,Im}] \quad (20)$$

We regroup terms in ω and k so that we have

$$\exp i[\omega(x'_{Re} + ix'_{Im}) - k(t'_{x,Re} - it'_{x,Im})] \quad (21)$$

Now using equations from for $\underline{x}' = x'_{Re} + ix'_{Im}$ we have

$$\exp i[\omega x' - k(t'_{x,Re} - it'_{x,Im})] \quad (22)$$

Note that the second part of the exponent for the k term does not reduce to t' since there is a minus before $it'_{x,Im}$. For the boost, $v_x \rightarrow \infty$ or $v > c$, we obtain for $\exp i[\omega t + kx]$ from Eq. (11) under this transformation going to

$$\exp i[\omega x'] \times \exp -k[t'_{x,Re} - it'_{x,Im}] \quad (23)$$

Let us look at the example of the transformation from A'_x (in the moving frame, Σ') to its form in the restframe, Σ a mixing vector and scalar potential. In the SLT from the restframe, Σ to the moving Σ' frames; we have a change of length of the time component vector in Eq. (23). The vector potential term, A_{0x} transforms as

$$A'_x = \gamma \left(A_x - \frac{v_x}{c^2} \phi \right) \quad (24)$$

which is the same as Eq. (15), so that for the superluminal boost $v_x \rightarrow \infty$, implies that

$$\gamma \equiv \frac{1}{\sqrt{\frac{v_x^2}{c^2} - 1}} = \frac{1}{\frac{v_x}{c} \sqrt{1 - \frac{c^2}{v_x^2}}} \cong \frac{c}{v_x} \quad (25)$$

where the $\sqrt{1 - c^2 / v_x^2}$ term approaches unity as $v_x \rightarrow \infty$. Then we rewrite the transformed vector potential as

$$A'_x = \frac{1}{\sqrt{\frac{v_x^2}{c^2} - 1}}, \quad A_x - \frac{\frac{v_x}{c}}{\sqrt{\frac{v_x^2}{c^2} - 1}} \phi \quad (26)$$

For $v_x \rightarrow \infty$ and from Eqs. (25) and (26),

$$A'_x = \frac{cA_x}{v_x} - \frac{v_x}{c^2} \frac{c}{v_x} \phi = 0 - \frac{1}{c\phi} \equiv -\phi \quad (27)$$

for units in which $c = 1$. Therefore $A'_x = -\phi$ for a superluminal boost, $v_x \rightarrow \infty$ and the transformation of the scalar potential, in analogy to Eq. (15), we have

$$\phi' = \gamma(\phi - v_x A_x) \quad (28)$$

and for $v_x \rightarrow \infty$, we have $\gamma \cong c / v_x$ so that in the limit of the SLT,

$$\phi' \lim_{v \rightarrow \infty} = \frac{c}{v_x} \phi - cA_x = -cA_x \quad (29)$$

for the units $c = 1$, then $\phi' = A_x$. Compare this equation to Eq. (27) and also for $A'_y = A_y$ and $A'_z = A_z$ we write

$$A_x = [A_{0x, \text{Re}} + iA_{0x, \text{Im}}] \exp i[\omega t + kx] = [-\phi'_{\text{Re}} - i\phi'_{\text{Im}}] \exp i\omega x' \times \exp \pm k_x [t_{x, \text{Re}} - it'_{x, \text{Im}}] \quad (30)$$

where $x' = x'_{\text{Re}} + ix'_{\text{Im}}$ and using the result of Eq. (27) and (29) for the non-exponent part and the exponential term which is given in Eq. (22), Eq. (30) gives us the vector and scalar form in the moving, Σ' frame.

If we consider only the accelerated potential, then we consider only the plus sign in Eq. (30). By use of the definition of complex quantities, Eq. (31) can be rewritten in a compact, simplified form:

$$A_x = -\phi'_x \exp(i\omega x') \cdot \exp(ik_x t'_x). \quad (31)$$

Using Eq. (31) we can describe the x component of the complex vector potential in moving frame, Σ' after a superluminal boost in the positive x direction. The same vector potential in the rest frame is defined. The transformation of the A_y and A_z components of the complex vector potential under a superluminal boost in the positive x direction can similarly be written as

$$\begin{aligned} A_y &= A'_{0y} \exp[-\omega(t'_{y,Re} + it'_{y,Im})] \cdot \exp[-ky(z'_{Re} + iy'_{Im})] \\ &= A'_{0z} \exp[-\omega(t'_{z,Re} + it'_{z,Im})] \cdot \exp[-ky(z'_{Re} + iz'_{Im})] \end{aligned} \quad (32)$$

The scalar potential is defined by a complex quantity, so that

$$\phi' = \phi'_{Re} + i\phi'_{Im} \quad (33)$$

which we use for the non-exponential term of Eq. (32) which then becomes

$$A_x = -\phi' \exp i\omega x' \cdot \exp k[t'_{x,Re} - it'_{x,Im}] \quad (34)$$

We compare the vector potential forms of A_x in Eq. (29) in the Σ or laboratory frame, and A_x of Eq. (34) in the Σ' frame or moving frame. (See Table 1)

TABLE 1 Comparison of The Exponential Part of the Vector Potential A_x In The Σ and Σ' Frames of Reference

	OSCILLATORY	DAMPED
Σ Frame	$A_{0x} \propto \exp i[\omega t_{x,Re} - kx_{Re}]$	$\exp -[\omega t_{x,Im} - kx_{Im}]$
Σ' Frame	$\phi' \propto \exp i[\omega x']$	$\exp k[t'_{x,Re} - it'_{x,Im}]$

In the oscillatory solution of the Σ' frame for ϕ' , we find no dependence on the wave number factor k and hence we have apparent media independence, recalling $x' = x_{Re} + ix_{Im}$, whereas in the Σ frame for A_{0x} , we have dependence on ω and k .

For the damped solution, we have ω and k dependence in the Σ frame for A_{0x} , which is a pure real exponential and hence not oscillatory. In the Σ' frame, ϕ' sometimes has a damped solution dependent on k which has a real and imaginary component. The exponential factor can be written a

$$t'_{x,Re} - it'_{x,Im} = x_{Re} - ix_{Im} \quad (35)$$

Time dilation and vector length are modified in the complex 12D space. We find that a superluminal, unidimensional x -dimensional boost in complex Minkowski space not only modifies space and time (as well as mass) by the γ factor, it also modifies $\underline{A} = (\underline{A}, \phi)$ and we find a mixing of \underline{A} and ϕ for $\underline{A} = A_j$

where j runs 1 to 3 for space-like quantities and ϕ transforms as a temporal quantity for subluminal transformations.

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