

Gravitational Interaction between Photons and Unification of All the Fundamental Interactions

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Recently, it has been reported an experiment where a very weak laser beam passes through a dense cloud of *ultracold rubidium atoms*. Under these circumstances, it was observed that the photons bound together in pairs or triplets, suggesting an unexpected *attractive* interaction between them. Here, it is shown that mentioned interaction can be related to the *gravitational interaction*, and that this possibility permits the formulation of a solid approach for a grand *unification* of the fundamental forces of the Universe.

Key words: Interaction Gravitational, Casimir Force, Interaction between Photons.

1. Introduction

In a paper recently published in *Science* [1], researchers have reported that when they have put a very weak laser beam through a dense cloud of *ultracold rubidium atoms* (as a *quantum nonlinear medium*), the photons bound together in pairs or triplets, suggesting an unexpected *attractive* interaction between them.

Here, it is shown that mentioned interaction is related to the *gravitational interaction*.

2. Theory

I have show in the *Mathematical Foundations of the Relativistic Theory of Quantum Gravity* [2] that, by combination of Gravitation and the *Uncertainty principle* it is possible to derive the expression for the *Casimir force*. The starting point is the expression of correlation between gravitational mass m_g and *rest* inertial mass, m_{i0} , obtained in the mentioned paper, i.e.,

$$\chi = \frac{m_g}{m_{i0}} = \left\{ 1 - 2 \left[\sqrt{1 + \left(\frac{p}{m_{i0}c} \right)^2} - 1 \right] \right\} \quad (1)$$

where p is the variation in the particle's *kinetic momentum*; c is the light speed.

Thus, an uncertainty Δm_{i0} in m_{i0} produces an uncertainty Δp in p and therefore an uncertainty Δm_g in m_g , which according to Eq.(1), is given by

$$\Delta m_g = \Delta m_{i0} - 2 \left[\sqrt{1 + \left(\frac{\Delta p}{\Delta m_{i0}c} \right)^2} - 1 \right] \Delta m_{i0} \quad (2)$$

From the uncertainty principle for position and momentum, we know that the product of the uncertainties of the simultaneously measurable values of the corresponding position and momentum components are at least of the magnitude order of \hbar , i.e.,

$$\Delta p \Delta r \sim \hbar$$

Substitution of $\Delta p \sim \hbar / \Delta r$ into (2) yields

$$\Delta m_g = \Delta m_i - 2 \left[\sqrt{1 + \left(\frac{\hbar / \Delta m_i c}{\Delta r} \right)^2} - 1 \right] \Delta m_i \quad (3)$$

Therefore if

$$\Delta r \ll \frac{\hbar}{\Delta m_i c} \quad (4)$$

Then the expression (3) reduces to:

$$\Delta m_g \cong - \frac{2\hbar}{\Delta r c} \quad (5)$$

Note that, Δm_g does not depend on m_g .

Consequently, an uncertainty ΔF in the gravitational force $F = -G m_g m'_g / r^2$, will be given by

$$\begin{aligned} \Delta F &= -G \frac{\Delta m_g \Delta m'_g}{(\Delta r)^2} = \\ &= - \left[\frac{2}{\pi (\Delta r)^2} \right] \frac{hc}{(\Delta r)^2} \left(\frac{G\hbar}{c^3} \right) \end{aligned} \quad (6)$$

The amount $(G\hbar/c^3)^{1/2} = 1.61 \times 10^{-35} m$ is called the *Planck length*, l_{planck} , (the length scale on which quantum fluctuations of the metric of the space time are expected to be of order unity).

Thus, we can write the expression of ΔF as follows

$$\begin{aligned}\Delta F &= -\left(\frac{2}{\pi}\right)\frac{hc}{(\Delta r)^4}l_{planck}^2 = \\ &= -\left(\frac{\pi}{480}\right)\frac{hc}{(\Delta r)^4}\left[\left(\frac{960}{\pi^2}\right)l_{planck}^2\right] = \\ &= -\left(\frac{\pi A_0}{480}\right)\frac{hc}{(\Delta r)^4}\end{aligned}\quad (7)$$

or

$$F_0 = -\left(\frac{\pi A_0}{480}\right)\frac{hc}{r^4}\quad (8)$$

which is the expression of the *Casimir force* for $A = A_0 = (960/\pi^2)l_{planck}^2$.

Now, multiplying Eq. (8) (the expression of F_0) by n^2 we obtain

$$F = n^2 F_0 = -\left(\frac{\pi n^2 A_0}{480}\right)\frac{hc}{r^4} = -\left(\frac{\pi A}{480}\right)\frac{hc}{r^4}\quad (9)$$

This is the general expression of the *Casimir force*.

We can then conclude that *the Casimir effect* is just a *gravitational* effect related to the *uncertainty principle*. In this context, the nature of the *Casimir force* is clearly *gravitational* as shown in the derivation of Eq. (9), which expresses, in turn, the intensity of the *gravitational force in the case of very small scale* (r very small)*.

Now consider the discovery reported recently in *Science* [1]. When the researchers have put a very weak laser beam through a dense cloud of *ultracold rubidium atoms*†, the photons bound together in pairs or triplets, suggesting an unexpected *attractive* interaction between them. Now, we will show that the nature of this interaction is *gravitational*.

According mentioned in the paper, the length of the cloud of ultracold rubidium atoms

* The Casimir force is only significative when the value of r is very small (*microcosm scale*).

† The velocities of the photons through the cloud of *ultracold rubidium atoms* are strongly reduced. This is the reason for the laser to pass through the mentioned cloud. Lene Hau et al., [3] showed that light speed through a cloud of *ultracold rubidium atoms* reduces to values much smaller than $100m.s^{-1}$.

were of approximately $130\mu m$ (along the propagation direction), while the transverse extent of the probe beam waist had about $4.5\mu m$. Therefore, the distances r between the photons of the cloud were very small. As we have already seen, at very small scale, the *gravitational interaction* cannot be treated via usual Newton's equation of gravitation. In this case, Eq. (9) must be used. Thus, assuming $A \approx \lambda^2 = (c/f)^2 \cong 10^{-13}m^2$, and substituting this value into Eq. (9), we obtain:

$$F \approx 10^{-40}/r^4\quad (10)$$

Using the above equation, and considering the dimensions of the mentioned cloud ($130\mu m \times 4.5\mu m$), we can calculate the intensity of the *gravitational force* between two photons of the cloud, when the distance r between them were, for example, of the order of $1\mu m$, i.e.,

$$F \approx 10^{-16}N\quad (11)$$

The intensity of this *gravitational force* is highly significative. Compare for example, with the *Coulombian attractive force* between an *electron* and a *proton*, separated by *the same distance* ($r \approx 1\mu m$), which is given by

$$F_c = \frac{e^2}{4\pi\epsilon_0 r^2} \cong \frac{10^{-28}}{r^2} \approx 10^{-16}N\quad (12)$$

The *Coulombian repulsive force* between two *protons* in an atomic *nucleus*, considering that, $r_{proton} = 1.4 \times 10^{-15}m$, and that the distance between them is $r = 4 \times 10^{-15}m$ [4], is given by

$$F_c = \frac{e^2}{4\pi\epsilon_0 r^2} \cong 14N\quad (13)$$

This enormous repulsive force *must be overcome* by the intense *attractive nuclear force* (*strong nuclear force*).

Now consider Eq. (9), where we put $A = \pi r_{proton}^2 \cong 6 \times 10^{-30}m^2$ and $r = 4 \times 10^{-15}m$, then the result is

$$F = -\left(\frac{\pi A}{480}\right)\frac{hc}{r^4} \cong 30N \quad (14)$$

Comparing Eq. (14) with Eq. (13), we can conclude that the *attractive gravitational force* (30N) is sufficient to overcome the *repulsive Coulombian force* expressed by Eq. (13).

These results lead us to formulate the following question: What is the true nature of the “strong nuclear force”? Is it *gravitational* as shown above?

This possibility is reinforced by the derivation the *Coupling Constants for the Fundamental Forces* that we will make hereafter, starting from Eq. (9).

It is known that the *weak force*, F_w , which is related to the *strong force*, F_s , by means of the following expression:

$$\frac{F_w}{F_s} = \frac{\alpha_w}{\alpha_s} \quad (15)$$

where α_w is the *weak force coupling constant*, and α_s is the *strong force coupling constant* [‡].

Assuming that $F_s = F$, where F is given by Eq. (9), then Eq.(15) can be rewritten as follows

$$F_w = \left(\frac{\alpha_w}{\alpha_s}\right)\left(\frac{\pi A}{480}\right)\frac{hc}{r^4} \quad (16)$$

At $r \cong 3 \times 10^{-18} m$ (0.1% of the diameter of a proton), the weak interaction has a strength of a similar magnitude to electromagnetic force, $F_E = e^2/4\pi\epsilon_0 r^2$ [5]. Thus, making $F_w = F_E$, and substituting the above mentioned value of r , we obtain

[‡] Similarly, the weak force is related to the electromagnetic force, F_E , by means of the expression: $F_w/F_E = \alpha_w/\alpha_E$; and the strong force is related to the electromagnetic force, by means of the expression: $F_s/F_E = \alpha_s/\alpha_E$; and the gravitational force, F_G , is related to the electromagnetic force, by means of the expression: $F_G/F_E = \alpha_G/\alpha_E$.

$$\frac{\alpha_w}{\alpha_s} = \frac{480r^2 e^2}{4\pi^2 \epsilon_0 A hc} = \frac{480r^2 e^2}{4\pi^3 \epsilon_0 r_p^2 hc} \approx 3 \times 10^{-7} \quad (17)$$

This is the same value mentioned in the literature for α_w/α_s [6].

Now, considering that $F_w/F_E = \alpha_w/\alpha_E$, where α_E is the *electromagnetic force coupling constant*, then we can write that

$$F_w = \left(\frac{\alpha_w}{\alpha_E}\right)\frac{e^2}{4\pi\epsilon_0 r^2} \quad (18)$$

At the maximum range of the weak interaction, r_{\max} , we have the minimum value of the weak force, F_w^{\min} , which can be expressed by Eq. (16) or Eq. (18) as follows

$$F_w^{\min} = \left(\frac{\alpha_w}{\alpha_s}\right)\left(\frac{\pi A}{480}\right)\frac{hc}{r_{\max}^4} \quad (19)$$

$$F_w^{\min} = \left(\frac{\alpha_w}{\alpha_E}\right)\frac{e^2}{4\pi\epsilon_0 r_{\max}^2} \quad (20)$$

By comparing these equations, we obtain

$$\left(\frac{\alpha_w}{\alpha_s}\right)\left(\frac{\pi A}{480}\right)\frac{hc}{r_{\max}^4} = \left(\frac{\alpha_w}{\alpha_E}\right)\frac{e^2}{4\pi\epsilon_0} \quad (21)$$

or

$$\begin{aligned} \frac{\alpha_s}{\alpha_E} &= \frac{4\pi^2 A \epsilon_0 hc}{480 e^2 r_{\max}^2} = \frac{4\pi^3 r_p^2 \epsilon_0 hc}{480 e^2 r_{\max}^2} = \\ &= \left(\frac{4\pi\epsilon_0 \hbar c}{e^2}\right)\left(\frac{2\pi^3 r_p^2}{480 r_{\max}^2}\right) \quad (22) \end{aligned}$$

Experimental data, describing the strong force between nucleons is consistent with a strong force coupling constant of about 1 [6]. Thus, making $\alpha_s = 1$ (*strong force coupling constant*) in Eq. (22), we obtain

$$\alpha_E = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\left(\frac{480 r_{\max}^2}{2\pi^3 r_p^2}\right) \quad (23)$$

The maximum range of the weak interaction, r_{\max} , is of the order of $10^{-16} m$ [7]. Equation above shows that, for $r_{\max} \cong 5 \times 10^{-16} m$ the term

$$\left(\frac{480r_{\max}^2}{2\pi^3 r_p^2} \right) \cong 1 \quad (24)$$

Consequently, Eq. (23) reduces to

$$\alpha_E = \frac{e^2}{4\pi\epsilon_0\hbar c} \cong \frac{1}{137} \quad (25)$$

this is the expression of the *electromagnetic force coupling constant*.

Multiplying α_w/α_s (given by Eq. (17)) by α_s/α_E (given by Eq. (22)), we get

$$\frac{\alpha_w}{\alpha_E} = \left(\frac{480r^2 e^2}{4\pi^3 \epsilon_0 r_p^2 \hbar c} \right) \left(\frac{4\pi\epsilon_0 \hbar c}{e^2} \right) \left(\frac{2\pi^3 r_p^2}{480r_{\max}^2} \right)$$

$$\begin{aligned} \alpha_s &= 1 \\ \alpha_E &= 1/137 \\ \alpha_w &\approx 3 \times 10^{-7} \\ \alpha_G &\cong 5.9 \times 10^{-39} \end{aligned}$$

whence we obtain

$$\begin{aligned} \alpha_w &= \left(\frac{480r^2 e^2}{4\pi^3 \epsilon_0 r_p^2 \hbar c} \right) \left(\frac{4\pi\epsilon_0 \hbar c}{e^2} \right) \left(\frac{2\pi^3 r_p^2}{480r_{\max}^2} \right) \alpha_E = \\ &= \left(\frac{480r^2 e^2}{4\pi^3 \epsilon_0 r_p^2 \hbar c} \right) \left(\frac{2\pi^3 r_p^2}{480r_{\max}^2} \right) \cong \\ &\cong \left(\frac{r^2 e^2}{2\epsilon_0 r_{\max}^2 \hbar c} \right) \cong 3 \times 10^{-7} \end{aligned} \quad (26)$$

Now, we will obtain the *gravitational force coupling constant*, α_G , starting of the fact that the *strong force*, F_G , is related to the *electromagnetic force*, F_E , by means of the following expression:

$$\frac{F_G}{F_E} = \frac{\alpha_G}{\alpha_E} \quad (27)$$

Then, we can write that

$$\alpha_G = \alpha_E \left(\frac{F_G}{F_E} \right) = \alpha_E \left(\frac{Gm_p^2}{e^2} \right) \cong 5.9 \times 10^{-39} \quad (28)$$

The relative strength of interactions varies with distance [8]. Here, starting from the fact that *the strong nuclear force* and the

weak nuclear force are *gravitational forces* expressed by Eq. (9), we have showed that, at the range of about 10^{-15} m ($r_{\max} \cong 5 \times 10^{-16}$ m), the *strong force* ($\alpha_s = 1$) is approximately 137 times as strong as electromagnetic force ($\alpha_E = 1/137$), about a million times as strong as the weak force ($\alpha_w \cong 3 \times 10^{-7}$), and about 10^{38} times as strong as gravitation ($\alpha_G \cong 5.9 \times 10^{-39}$). All these values are in strong accordance with the values widely mentioned in the literature [9, 10], given below

Finally, we complete the *unification* of the Fundamental Forces of the Universe, by deriving from Eq. (9) the equations of the *Coulombian Force* and of the *Newtonian Force*.

Consider two electric charges q_1 and q_2 separated by a distance r . If we define the *area* A in Eq. (9) by means of the following expression

$$\begin{aligned} A &= \sqrt{A_1 A_2} = \sqrt{k_e \left(\frac{q_1}{e} \right)^2 r^2 \times k_e \left(\frac{q_2}{e} \right)^2 r^2} = \\ &= k_e \left(\frac{q_1 q_2}{e^2} \right) r^2 \end{aligned} \quad (29)$$

where k_e is a constant to be determined, and $e = 1.6 \times 10^{-19}$ C, then Eq. (9) can rewritten as follows

$$F = \frac{\pi \hbar c k_e q_1 q_2}{480 e^2 r^2} = \frac{1}{4\pi\epsilon_0} \left(\frac{4\pi^2 \hbar c k_e \epsilon_0}{480 e^2} \right) \frac{q_1 q_2}{r^2} \quad (30)$$

Note that, the term in brackets is equal to 1 for $k_e = 120e^2/\pi^2 \hbar c \epsilon_0 \cong 0.1769$. In this case, Eq. (30) reduces to

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad (31)$$

which is the expression of the *Coulombian Force*.

In a similar way, we can derive the expression of the *Newtonian Force* for two particles with masses m_1 and m_2 respectively, separated by a distance r . First we define the area A in Eq. (9) by means of the following expression

$$\begin{aligned} A = \sqrt{A_1 A_2} &= \sqrt{k_g \left(\frac{m_1}{m_0}\right)^2 r^2 \times k_g \left(\frac{m_2}{m_0}\right)^2 r^2} = \\ &= k_g \left(\frac{m_1 m_2}{m_0^2}\right) r^2 \end{aligned} \quad (32)$$

where k_g is a constant to be determined, and m_0 , is a *minimum* value of mass that will be calculated hereafter. Then substitution of Eq. (32) into Eq. (9) yields

$$F = \frac{\pi \hbar c k_g m_1 m_2}{480 m_0^2 r^2} = G \left(\frac{2\pi^2 \hbar c k_g}{480 G m_0^2} \right) \frac{m_1 m_2}{r^2} \quad (33)$$

The term in brackets is equal to 1 for

$$\frac{k_g}{m_0^2} = \left(\frac{60}{\pi^2} \right) \left(\frac{4G}{\hbar c} \right) \quad (34)$$

Equation (34) can be rewritten as follows

$$\frac{k_g}{m_0^2} = \frac{\left(\frac{60}{\pi^2} \right)}{\left(\sqrt{\frac{\hbar c}{4G}} \right)^2} \quad (35)$$

where $\sqrt{\hbar c/4G} = 1.08 \times 10^{-8} \text{ kg}$.

Equation (35) shows that, the term $(60/\pi^2)$ is a *pure number* such as k_g , and the term $\sqrt{\hbar c/4G}$ is expressed in *kg* such as m_0 , then we can conclude that

$$k_g = \frac{60}{\pi^2} \quad (36)$$

and

$$m_0 = \sqrt{\frac{\hbar c}{4G}} \quad (37)$$

This expression it was first derivated by Hawking (1971) [11], and it is known as *Hawking mass limit*. Starting from the principle that the gravitational collapse is a process essentially classic, Hawking have concluded that black-holes could not exist with radius less than the *Planck length* $(\sqrt{G\hbar/c^3})$ (limit for which *quantum* fluctuations in the metric of the spacetime are considered of the order of 1). In this way, the *minimum* radius of Schwarzschild, $r_s = 2Gm_0/c^2$, would have this value and, to this radius would correspond to a *minimum* value of mass m_0 , given by

$$m_0 = \frac{r_s c^2}{2G} = \frac{c^2 \sqrt{G\hbar/c^3}}{2G} = \sqrt{\frac{\hbar c}{4G}} \quad (38)$$

This would be, obviously, the *smaller* mass value for any *macroscopic* gravitational systems (black-holes, etc).

Now, just substitute Eq. (36) and Eq. (37) into Eq.(33), in order to obtain the expression of the *Newtonian Force*.

$$F = G \frac{m_1 m_2}{r^2} \quad (39)$$

The derivation of the Equations (31) and (39) via Eq. (9), shows clearly the *unification* of the Fundamental Forces of the Universe, i.e. shows that the *nature* of all the fundamental interactions is *Gravitational*.

Starting from Eq. (15), which expresses the correlation between F_w e F_s , i.e.,

$$\frac{F_w}{F_s} = \frac{\alpha_w}{\alpha_s}$$

we can write that

$$\frac{F_w}{F_s} = \frac{\alpha_w \left[\left(\frac{\pi A}{480} \right) \frac{\hbar c}{r^4} \right]}{\alpha_s \left[\left(\frac{\pi A}{480} \right) \frac{\hbar c}{r^4} \right]} \quad (40)$$

whence we obtain

$$F_w = \alpha_w \left(\frac{\pi A}{480} \right) \frac{\hbar c}{r^4} \quad (41)$$

$$F_s = \alpha_s \left(\frac{\pi A}{480} \right) \frac{\hbar c}{r^4} \quad (42)$$

In addition, since

$$\frac{F_W}{F_E} = \frac{\alpha_W}{\alpha_E} \quad \text{and} \quad \frac{F_W}{F_G} = \frac{\alpha_W}{\alpha_G}$$

we get

$$F_E = \left(\frac{\alpha_E}{\alpha_W} \right) F_W = \alpha_E \left(\frac{\pi A}{480} \right) \frac{hc}{r^4} \quad (43)$$

$$F_G = \left(\frac{\alpha_G}{\alpha_W} \right) F_W = \alpha_G \left(\frac{\pi A}{480} \right) \frac{hc}{r^4} \quad (44)$$

Before 10^{-43} seconds after the Big Bang the forces F_S , F_W , F_E and F_G become equal in strength, unifying themselves into a single force, F_U , i.e.,

$$\begin{aligned} F_U &= \alpha_S \left(\frac{\pi A}{480} \right) \frac{hc}{r^4} = \alpha_W \left(\frac{\pi A}{480} \right) \frac{hc}{r^4} = \\ &= \alpha_E \left(\frac{\pi A}{480} \right) \frac{hc}{r^4} = \alpha_G \left(\frac{\pi A}{480} \right) \frac{hc}{r^4} \end{aligned} \quad (45)$$

where $\alpha_S = \alpha_W = \alpha_E = \alpha_G$.

Since $\alpha_S = 1$, we then conclude that

$$F_U = \left(\frac{\pi A}{480} \right) \frac{hc}{r^4} \quad (46)$$

Under these conditions, the energy U_U of a system of two particles with masses m_1 and m_2 , separated by a distance r , is given by

$$U_U = F_U r = \left(\frac{\pi A}{480} \right) \frac{hc}{r^3} \quad (47)$$

Since, $A = k_g (m_1 m_2 / m_0^2) / r^2$ where $k_g = 60 / \pi^2$, (See Eqs. (32) and (36)). Then, Eq. (47) can be rewritten as follows

$$U_U = \left(\frac{m_1 m_2}{m_0^2} \right) \frac{hc}{8\pi r} \quad (48)$$

Assuming that, in the mentioned conditions (at the beginning of the Universe), $m_1 = m_2 = m_0$, and that the total mass of the Universe, $M_{Univ} \cong 10^{54} \text{ kg}$ [§], at this epoch, it was formed by a number $N = M_{Univ} / m_0 \cong 10^{62}$ of particles with mass m_0 and radius r_0 , compressed into the volume of the Initial Universe, $V_{Univ} = \frac{4}{3} \pi r_{Univ}^3$, where $r_{Univ} \cong 10^{-14} \text{ m}$, then we can write that

[§] $M_{Univ} = c^3 / 2GH_0 \cong 10^{54} \text{ kg}$ [12]. H_0 is the Hubble constant.

$$N \left(\frac{4}{3} \pi r_0^3 \right) \approx \frac{4}{3} \pi r_{Univ}^3 \quad (49)$$

where we get

$$r_0 \approx \frac{r_{Univ}}{\sqrt[3]{N}} \cong 10^{-35} \text{ m} \quad (50)$$

Therefore, we can assume that the distances r among the particles it was of the order of the Planck length, $l_{Planck} = \sqrt{G\hbar/c^3} \cong 1.6 \times 10^{-35} \text{ m}$ ^{**}.

Substitution of $m_1 = m_2 = m_0$ and $r \cong l_{Planck}$ into Eq. (48) gives

$$U_U \cong \frac{1}{4} \sqrt{\frac{\hbar c^5}{G}} \cong \frac{1}{4} E_p \cong 3 \times 10^{18} \text{ GeV} \quad (51)$$

where $E_p = \sqrt{\hbar c^5 / G} \cong 1.22 \times 10^{19} \text{ GeV}$ is the called *Planck Energy*

Note that the energy U_U is above, but very close to, the energy level of the **Total Unification Energy** Λ_{Total} ^{††} (the energy level above which the *electromagnetic force*, *weak force*, *strong force* and the *gravitational force* become equal in strength and unified in a single force), i.e., $U_U \gtrsim \Lambda_{Total}$. Thus, as $U_U \cong 1/4 E_p$, we write that

$$\Lambda_{Total} \lesssim U_U \cong 3 \times 10^{18} \text{ GeV}$$

Therefore, the *Total Unification Energy*, Λ_{Total} is about 300 times greater than the called *Grand Unification Energy*, $\Lambda_{GUT} \cong 1 \times 10^{16} \text{ GeV}$ ^{‡‡}.

^{**} The unification of the *electromagnetic force*, *weak force*, and *strong force* with the *gravitational force* is generally assumed to be close to the *Planck Scale*.

^{††} Above Λ_{Total} , as showed in Eq. (45) and (47). Very close to Λ_{Total} because the unification of the 3 fundamental forces with the *gravitational force* requires that Λ_{Total} has a value close to the *Planck Energy*.

^{‡‡} The *grand unification energy* Λ_{GUT} , is the energy level above which, it is believed, the *electromagnetic force*, *weak force*, and *strong force* become equal in strength and unified in a single force.

References

- [1] Liang, Qi-Yu, et al., (2018) *Observation of three-photon bound states in a quantum nonlinear medium*, Science, Vol. 359, **6377**, pp. 783-786.
- [2] De Aquino, F. (2010) *Mathematical Foundations of the Relativistic Theory of Quantum Gravity*, Pacific Journal of Science and Technology, **11** (1), pp. 173-232.
- [3] Hau, L.V, et al., (1999) *Light Speed Reduction to 17 Meters per Second in an Ultracold Atomic Gas*, Nature 397, 594-598.
- [4] Halliday, D. and Resnick, R. (1968) *Physics*, J. Willey & Sons, Portuguese Version, Ed. USP, p.718.
- [5] HyperPhysics. Georgia State University.
<http://hyperphysics.phy-astr.gsu.edu/hbase/Forces/funfor.html>
- [6] *Coupling Constants for the Fundamental Forces*. (2011) HyperPhysics. Georgia State University.
<http://hyperphysics.phy-astr.gsu.edu/hbase/Forces/couple.html>
- [7] S. B.,Treiman, (1959)*The Weak Interactions*, Scientific American.
- [8] Strassler, M. (2013). *The Strengths of the Known Forces*. Available at:
<https://profmattstrassler.com/articles-and-posts/particle-physics-basics/the-known-forces-of-nature/the-strength-of-the-known-forces/>
- [9] Rohlf, J.W.(1994) *Modern Physics from a to Z0*,Wiley.
- [10] *Coupling Constants for the Fundamental Forces*. (2011) HyperPhysics. Georgia State University.
<http://hyperphysics.phy-astr.gsu.edu/hbase/Forces/couple.html>
- [11] Hawking, S.W (1971) *Gravitationally Collapsed Objects of Very low Mass*. *MNRAS.*, **152**, 75-78.
- [12] Kragh, H. (1999). *Cosmology and Controversy: The Historical Development of Two Theories of the Universe*: Princeton University Press.