

The antisymmetric wave function for a state with Q identical particles is:

$$\Psi(\mathbf{r}_1, s_1, \dots, \mathbf{r}_Q, s_Q)$$

I suppose that the wave function is given by a convergent antisymmetric power series (I write to simplify a two particles system in two dimensions):

$$\Psi(\mathbf{r}_1, s_1, \mathbf{r}_2, s_2) = \sum_{p+q+r=2N+1} P(\mathbf{r}_1, s_1, \mathbf{r}_2, s_2)_{ijklmnopqr} \begin{vmatrix} x_1^i & x_2^i \\ y_1^j & y_2^j \end{vmatrix}^p \begin{vmatrix} x_1^k & x_2^k \\ x_1^l & x_2^l \end{vmatrix}^q \begin{vmatrix} y_1^m & y_2^m \\ y_1^n & y_2^n \end{vmatrix}^r$$

$P_{ijklmnopqr}$ is an elementary symmetric polynomial multiplied for a constant.

The antisymmetric elementary polynomial for two identical particle, in a three-dimensional space, is:

$$P(x_1^a-x_2^a)^p(y_1^b-y_2^b)^q(z_1^c-z_2^c)^r \begin{vmatrix} x_1^d & x_2^d \\ y_1^e & y_2^e \end{vmatrix}^s \begin{vmatrix} x_1^f & x_2^f \\ z_1^g & z_2^g \end{vmatrix}^t \begin{vmatrix} y_1^h & y_2^h \\ z_1^i & z_2^i \end{vmatrix}^u \begin{vmatrix} x_1^j & x_2^j \\ x_1^k & x_2^k \end{vmatrix}^v \begin{vmatrix} y_1^l & y_2^l \\ y_1^m & y_2^m \end{vmatrix}^w \begin{vmatrix} z_1^n & z_2^n \\ z_1^o & z_2^o \end{vmatrix}^{\alpha}$$

the space can be covered by a three-dimensional grid, and I can choose some representative points to obtain an estimate of the ground state:

$$\frac{\langle \Psi(\alpha) | H | \Psi(\alpha) \rangle}{\langle \Psi(\alpha) | \Psi(\alpha) \rangle} \leq \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} \forall \Phi$$

so that I can approximate the potential on a infinitesimal grid cube described by $\max\{|x - x_i|, |y - y_i|, |z - z_i|\} \leq \epsilon$:

$$V(x, y, z) = V(x_i, y_i, z_i) + (x - x_i) \frac{V(x_i + \epsilon, y_i, z_i) - V(x_i, y_i, z_i)}{\epsilon} + (y - y_i) \frac{V(x_i, y_i + \epsilon, z_i) - V(x_i, y_i, z_i)}{\epsilon} + (z - z_i) \frac{V(x_i, y_i, z_i + \epsilon) - V(x_i, y_i, z_i)}{\epsilon}$$

and it is possible to integrate the complex function in these infinitesimal cube, and for a large number of cubes, an estimate of the energy of the ground state is obtained, because of there is a neighbourhood of the cube where the potential and the wave function have little variations: the sum of the integrals over the orthogonal cubes is an approximation of the expectation value of the Hamiltonian: the integrations are only simple power series integrations, for an unique wave function over infinitesimal cubes.

The free parameters of the polynomial approximation can be minimized using optimization algorithm, using a unique polynomial series for the wave function in all the space.

I think that this solution is a good approximation on a finite region of space, but to infinity it is not zero, so that if it is necessary a true wave function in all the space one can use an exponential reduction like a symmetric function.