

Computing Minimum Spanning Tree in Interval Valued Bipolar Neutrosophic Environment

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Abstract—Interval valued bipolar neutrosophic sets is a new generalization of fuzzy set, bipolar fuzzy set, neutrosophic set and bipolar neutrosophic set so that it can handle uncertain information more flexibly in the process of decision making. In this paper, an algorithm for finding minimum spanning tree (MST) of an undirected neutrosophic weighted connected graph (UNWCG) in which the edge weights is represented by a an interval valued bipolar neutrosophic number is presented. The proposed algorithm is based on matrix approach to design the MST of UNWCG. A numerical example is provided to show the effectiveness of the proposed algorithm. Lastly, a comparative study with other existing methods is proposed.

Index Terms—Score function, interval valued bipolar neutrosophic sets, Neutrosophic sets, Spanning tree problem.

I. INTRODUCTION

In 1998, Smarandache [1] explored the concept of neutrosophic set (NS) from the philosophical point of view, to represent uncertain, imprecise, incomplete, inconsistent, and indeterminate information that are exist in the real world. The concept of neutrosophic set is a generalization of the concept of the classic set, fuzzy set, intuitionistic fuzzy set (IFS). The neutrosophic sets are characterized by a truth-membership function (t), an indeterminate-membership function (i) and a false-membership function (f) independently, which are within the real standard or nonstandard unit interval $]0, 1^+[$. To apply the concept of neutrosophic sets (NS) in science and engineering applications, Smarandache [6] introduced for the first time, the single valued neutrosophic set (SVNS). Later on, Wang *et al.* [2] studied some properties related to single valued neutrosophic sets. The neutrosophic set model is an important tool for dealing with real scientific and engineering applications because it can handle not only incomplete information but also the inconsistent information and indeterminate information. Some more literature about the extension of neutrosophic sets and their applications in various fields can be found in the literature [17].

In classical graph theory, there are common algorithms for solving the minimum spanning tree including Prim and

kruskal algorithm. By applying the concept of single valued neutrosophic sets on graph theory, a new theory is developed and called single valued neutrosophic graph theory (SVNGT). The concept of SVNGT and their extensions finds its applications in diverse fields [6]-[16]. Very recently few researchers have used neutrosophic methods to find minimum spanning tree in neutrosophic environment. Ye [4] proposed a method to find minimum spanning tree of a graph where nodes (samples) are represented in the form of SVNS and distance between two nodes which represents the dissimilarity between the corresponding samples has been derived. Kandasamy [3] proposed a double-valued Neutrosophic Minimum Spanning Tree (DVN-MST) clustering algorithm, to cluster the data represented by double-valued neutrosophic information. Mandal and Basu [5] presented a solution approach of the optimum spanning tree problems considering the inconsistency, incompleteness and indeterminacy of the information. The authors consider a network problem with multiple criteria which are represented by weight of each edge in neutrosophic sets. The approach proposed by the authors is based on similarity measure. Recently Mullai [18] solved the minimum spanning tree problem on a graph in which a bipolar neutrosophic number is associated to each edge as its edge length, and illustrated it by a numerical example.

The principal objective of this paper is to propose a new version of Prim's algorithm based on matrix approach for finding the cost minimum spanning tree of an undirected graph in which an interval valued bipolar neutrosophic number [19] is associated to each edge as its edge length.

The rest of the paper is organized as follows. Section 2 briefly introduces the concepts of neutrosophic sets, single valued neutrosophic sets and the score function of interval valued bipolar neutrosophic number. Section 3 proposes a novel approach for finding the minimum spanning tree of interval valued bipolar neutrosophic undirected graph. In Section 4, an illustrative example is presented to illustrate the proposed method. In section 5, a comparative study with other existing methods is provided. Finally, Section 6 concludes the paper.

II. PRELIMINARIES

Some of the important background knowledge for the materials that are presented in this paper is presented in this section. These results can be found in [1], [2], [19].

Definition 2.1 [1] Let ξ be an universal set. The neutrosophic set A on the universal set ξ categorized in to three membership functions called the true $T_A(x)$,

Manuscript received October 1, 2017; revised October 30, 2017.

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indeterminate $I_A(x)$ and false $F_A(x)$ contained in real standard or non-standard subset of $]0, 1^+[$ respectively.

$$0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+ \quad (1)$$

Definition 2.2 [2] Let ξ be a universal set. The single valued neutrosophic sets (SVNs) A on the universal ξ is denoted as following

$$A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle : x \in \xi \} \quad (2)$$

The functions $T_A(x) \in [0, 1]$, $I_A(x) \in [0, 1]$ and $F_A(x) \in [0, 1]$ are named degree of truth, indeterminacy and falsity membership of x in A, satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \quad (3)$$

Definition 2.3 [4]. An interval valued bipolar neutrosophic set A in X is defined as an object of the form

$$A = \{ \langle x, [T_L^p, T_M^p], [I_L^p, I_M^p], [F_L^p, F_M^p], [T_L^n, T_M^n], [I_L^n, I_M^n], [F_L^n, F_M^n] \rangle : x \in X \}$$

where $T_L^p, T_M^p, I_L^p, I_M^p, F_L^p, F_M^p : X \rightarrow [0, 1]$ and $T_L^n, T_M^n, I_L^n, I_M^n, F_L^n, F_M^n : X \rightarrow [-1, 0]$. The positive interval membership degree where $T_L^p, T_M^p, I_L^p, I_M^p, F_L^p, F_M^p$ denotes the lower and upper truth membership, lower and upper indeterminate membership and lower and upper false membership of an element $\in X$ corresponding to a bipolar neutrosophic set A and the negative interval membership degree $T_L^n, T_M^n, I_L^n, I_M^n, F_L^n, F_M^n$: denotes the lower and upper truth membership, lower and upper indeterminate membership and lower and upper false membership of an element $\in X$ to some implicit counter-property corresponding to an interval valued bipolar neutrosophic set A.

Deli *et al.* [19], introduced a concept of score function. The score function is applied to compare the grades of IVBNS. This function shows that greater is the value, the greater is the interval valued bipolar neutrosophic sets and by using this concept paths can be ranked

Definition 2.4 [19]. Let

$$\tilde{A} = \langle [T_L^p, T_M^p], [I_L^p, I_M^p], [F_L^p, F_M^p], [T_L^n, T_M^n], [I_L^n, I_M^n], [F_L^n, F_M^n] \rangle$$

be an interval valued bipolar neutrosophic number, Then, the score function $s(\tilde{A})$, accuracy function $a(\tilde{A})$ and certainty function $c(\tilde{A})$ of an IVBNN are defined as follows:

$$(i) s(\tilde{A}) = \left(\frac{1}{12} \right) \times \left[\begin{array}{l} T_L^p + T_M^p + 1 - I_L^p + 1 - I_M^p + 1 - \\ F_L^p + 1 - F_M^p + 1 + T_L^n \\ + 1 + T_M^n - I_L^n - I_M^n - F_L^n - F_M^n \end{array} \right] \quad (4)$$

$$(ii) a(\tilde{A}) = T_L^p + T_M^p - F_L^p - F_M^p + T_L^n + T_M^n - F_L^n - F_M^n \quad (5)$$

$$(iii) c(\tilde{A}) = T_L^p + T_M^p - F_L^n - F_M^n \quad (6)$$

Comparison of interval valued bipolar neutrosophic numbers

Let $\tilde{A}_1 = \langle [T_{L1}^p, T_{M1}^p], [I_{L1}^p, I_{M1}^p], [F_{L1}^p, F_{M1}^p], [T_{L1}^n, T_{M1}^n], [I_{L1}^n, I_{M1}^n], [F_{L1}^n, F_{M1}^n] \rangle$ and $\tilde{A}_2 = \langle [T_{L2}^p, T_{M2}^p], [I_{L2}^p, I_{M2}^p], [F_{L2}^p, F_{M2}^p], [T_{L2}^n, T_{M2}^n], [I_{L2}^n, I_{M2}^n], [F_{L2}^n, F_{M2}^n] \rangle$ be two interval valued bipolar neutrosophic numbers then

If $s(\tilde{A}_1) > s(\tilde{A}_2)$, then \tilde{A}_1 is greater than \tilde{A}_2 , that is, \tilde{A}_1 is superior to \tilde{A}_2 , denoted by $\tilde{A}_1 \succ \tilde{A}_2$

If $s(\tilde{A}_1) = s(\tilde{A}_2)$, and $a(\tilde{A}_1) > a(\tilde{A}_2)$ then \tilde{A}_1 is greater than \tilde{A}_2 , that is, \tilde{A}_1 is superior to \tilde{A}_2 , denoted by $\tilde{A}_1 \succ \tilde{A}_2$

If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) > c(\tilde{A}_2)$ then \tilde{A}_1 is greater than \tilde{A}_2 , that is, \tilde{A}_1 is superior to \tilde{A}_2 , denoted by $\tilde{A}_1 \succ \tilde{A}_2$

If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) = c(\tilde{A}_2)$ then \tilde{A}_1 is equal to \tilde{A}_2 , that is, \tilde{A}_1 is indifferent to \tilde{A}_2 , denoted by $\tilde{A}_1 = \tilde{A}_2$

III. MINIMUM SPANNING TREE ALGORITHM OF IVBN-UNDIRECTED GRAPH

In this section, a neutrosophic version of Prim's algorithm is proposed to handle minimum spanning tree in a neutrosophic environment. In the following, we propose an interval valued bipolar neutrosophic minimum spanning tree algorithm (IVBNMST), whose steps are described below:

Algorithm:

Input: The weight matrix $M = [W_{ij}]_{n \times n}$ for the undirected weighted neutrosophic graph G.

Output: Minimum cost Spanning tree T of G.

Step 1: Input interval valued bipolar neutrosophic adjacency matrix A.

Step 2: Translate the IVBN-matrix into score matrix $[S_{ij}]_{n \times n}$ by using score.

Step 3: Iterate step 4 and step 5 until all (n-1) entries matrix of S are either marked or set to zero or other words all the nonzero elements are marked.

Step 4: Find the score matrix S either columns-wise or row-wise to find the unmarked minimum entries S_{ij} , which is the weight of the corresponding edge e_{ij} in S.

Step 5: If the corresponding edge e_{ij} of selected S_{ij} produce a cycle with the previous marked entries of the score matrix S then set $S_{ij} = 0$ else mark S_{ij} .

Step 6: Construct the graph T including only the marked entries from the score matrix S which shall be desired minimum cost spanning tree of G.

Step 7: Stop.

IV. NUMERICAL EXAMPLE

In this section, a numerical example of IVBNMST is used to demonstrate of the proposed algorithm. Consider the following graph $G=(V, E)$ shown in Figure 2, with five nodes and five edges. The different steps involved in the construction of the minimum cost spanning tree are described as follow:

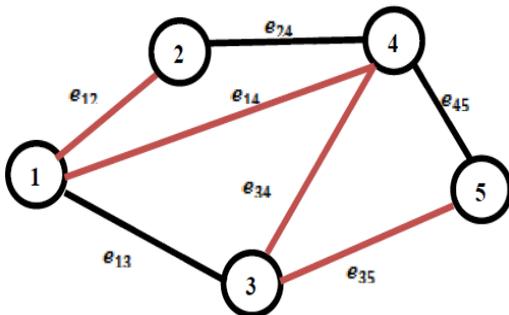


Fig. 2. Undirected IVBN- graphs.

e	Edge length
e_{12}	$\langle [0.3, 0.4], [0.1, 0.2], [0.2, 0.4], [-0.8, -0.7], [-0.5, -0.3], [-0.1, 0] \rangle$
e_{13}	$\langle [0.4, 0.5], [0.2, 0.6], [0.4, 0.6], [-0.2, -0.1], [-0.4, -0.2], [-0.5, -0.4] \rangle$
e_{14}	$\langle [0.6, 0.7], [0.7, 0.8], [0.8, 0.9], [-0.6, -0.5], [-0.4, -0.3], [-0.4, -0.2] \rangle$
e_{24}	$\langle [0.4, 0.5], [0.8, 0.9], [0.3, 0.4], [-0.2, -0.1], [-0.5, -0.1], [-0.7, -0.5] \rangle$
e_{34}	$\langle [0.2, 0.4], [0.3, 0.4], [0.7, 0.8], [-0.2, -0.1], [-0.4, -0.1], [-0.4, -0.3] \rangle$
e_{35}	$\langle [0.4, 0.5], [0.6, 0.7], [0.5, 0.6], [-0.4, -0.3], [-0.4, -0.2], [-0.3, -0.2] \rangle$
e_{45}	$\langle [0.5, 0.6], [0.4, 0.5], [0.3, 0.4], [-0.4, -0.2], [-0.5, -0.2], [-0.8, -0.6] \rangle$

The IVBN- adjacency matrix A is given below:

$$A = \begin{bmatrix} 0 & e_{12} & e_{13} & e_{14} & 0 \\ e_{12} & 0 & 0 & e_{24} & 0 \\ e_{13} & 0 & 0 & e_{34} & e_{35} \\ e_{14} & e_{24} & e_{34} & 0 & e_{45} \\ 0 & 0 & e_{35} & e_{45} & 0 \end{bmatrix}$$

Thus, using the score function, we get the score matrix

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 3. Score matrix.

According to the Fig. 3, we observe that the minimum entries 0.358 is selected and the corresponding edge (1, 4) is marked by the brown color. Repeat the procedure until the iteration will exist.

According to the Fig. 4 and Fig. 5, the next non zero minimum entries 0.408 is marked and corresponding edges (3, 5) are also colored

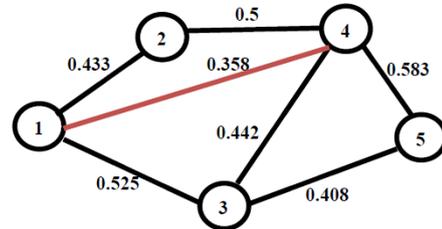


Fig. 4. The marked edge (1,4) of G in next iteration.

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 5. The marked next minimum entries 0.408 of S.

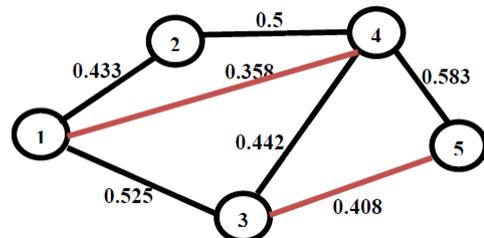


Fig. 6. The marked edge (3,5) of G in next iteration.

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 7. The marked next minimum entries 0.433 of S

According to the Fig. 7, the next minimum non zero element 0.433 is marked.

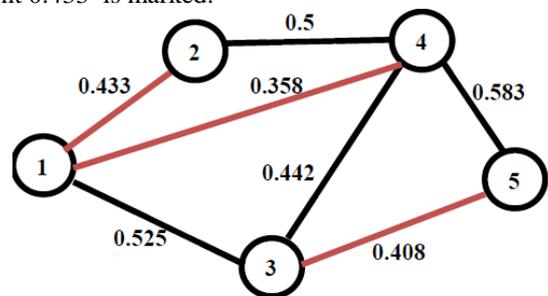


Fig. 8. The marked edge (1, 2) of G in next iteration.

According to the Fig. 9. The next minimum non zero element 0.442 is marked, and corresponding edges (3, 4) are also colored

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 9. The marked next minimum entries 0.442 of S.

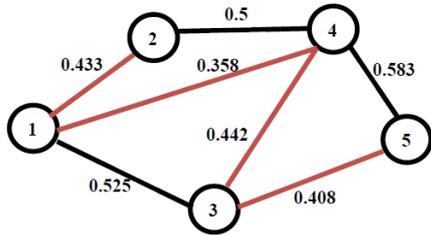


Fig. 10. The marked edge (3, 4) of G in next iteration.

According to the figure 11. The next minimum non zero element 0.5 is marked. But while drawing the edges it produces the cycle. So we delete and mark it as 0 instead of 0.5

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 11. The marked next minimum entries 0.5 of S.

The next non zero minimum entries 0.525 is marked it is shown in the Fig. 12. But while drawing the edges it produces the cycle. So, we delete and mark it as 0 instead of 0.525

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 & 0 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 & 0 \\ 0 & 0 & 0.408 & 0.583 & 0 & 0 \end{bmatrix}$$

Fig. 12. The marked next minimum entries 0.525 of S.

According to the Fig. 13. The next minimum non zero element 0.583 is marked. But while drawing the edges it produces the cycle so we delete and mark it as 0 instead of 0.583.

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 & 0 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 & 0 \\ 0 & 0 & 0.408 & 0.583 & 0 & 0 \end{bmatrix}$$

Fig. 13. The marked next minimum entries 0.583 of S.

After the above steps, the final path of minimum cost of spanning tree of G is portrayed in Fig. 14.

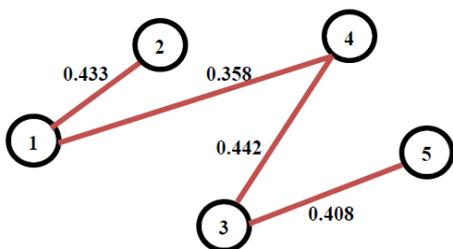


Fig. 14. Final path of minimum cost of spanning tree of G.

Using the above steps described in section 4, hence, the crisp minimum cost spanning tree is 1,641 and the final path of minimum cost of spanning tree is {2,1},{1, 4},{4, 3},{3, 5}.

V. Comparative STUDY

In order to illustrate the rationality and effectiveness of the proposed method, we apply the algorithm proposed by Mullai *et al.* [18] on our IVBN-graph presented in Section 4. Following the setps of Mullai's algorithm we obtained the results

Iteration 1:

Let $C_1 = \{1\}$ and $\bar{C}_1 = \{2, 3, 4, 5\}$

Iteration 2:

Let $C_2 = \{1, 4\}$ and $\bar{C}_2 = \{2, 3, 5\}$

Iteration 3:

Let $C_3 = \{1, 4, 2\}$ and $\bar{C}_3 = \{3, 5\}$

Iteration 4:

Let $C_4 = \{1, 4, 2, 3\}$ and $\bar{C}_4 = \{5\}$

Finally, IVBN minimal spanning tree is

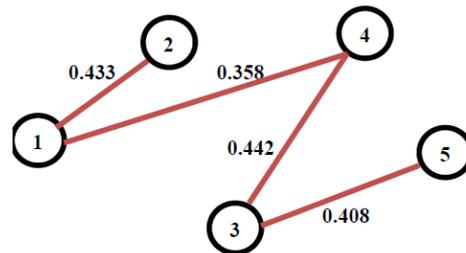


Fig. 15. IVBN minimal spanning tree obtained by Mullai's algorithm.

So, it can be seen that the IVBN minimal spanning tree {2, 1},{1, 4},{4, 3},{3, 5} obtained by Mullai's algorithm, After deneutrosophication of edges' weight using the score function, is the same as the path obtained by proposed algorithm.

The difference between the proposed algorithm and Mullai's algorithm is that our approach is based on Matrix approach, which can be easily implemented in Matlab, whereas the Mullai's algorithm is based on the comparison of edges the in each iteration of the algorithm, which leads to high computation.

VI. CONCLUSION

This paper deals with minimum spanning tree problem on a network where the edges weights are represented by an interval valued bipolar neutrosophic numbers. This work can be extended to the case of directed neutrosophic graphs and other types of neutrosophic graphs .

ACKNOWLEDGMENT

The authors are very grateful to the chief editor and reviewers for their comments and suggestions, which is helpful in improving the paper

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