Probability and entanglement

G. Quznetsov
mailto: gunnqu@gmail.com

january 2, 2018

Abstract

Here the concept of "TRUE" is defined according to Alfred Tarski, and the concept "OCCURING EVENT" is derived from this definition.

From here, we obtain operations on the events and properties of these operations and derive the main properties of the CLASSICAL PROBABILITY. PHYSICAL EVENTS are defined as the results of applying these operations to DOT EVENTS.

Next, the 3 + 1 vector of the PROBABILITY CURRENT and the EVENT STATE VECTOR are determined.

The presence in our universe of Planck's constant gives reason to presume that our world is in a CONFINED SPACE. In such spaces, functions are presented by Fourier series. These presentations allow formulating the ENTANGLEMENT phenomenon.

Contents

1 Truth

Science presents its ideas and results with language texts. Therefore, we will begin by considering narrative sentences:

By Alfred Tarski [1]

A sentence $\ll \Theta \gg$ is *true* if and only if Θ.

For example, sentence ≪It is raining≫ is true if and only if it is raining

A sentence $\ll \Theta \gg$ is *false* if and only if there is not that Θ.

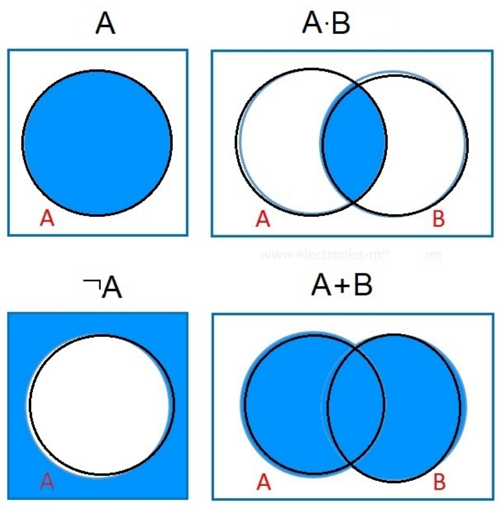

Figure 1: The Venn diagrams for A, B events

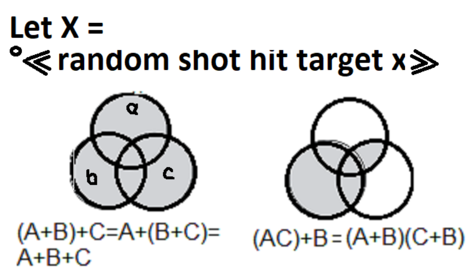

Figure 2: The Venn diagrams the associativity and the distribution

For example, $\ll 2+3=4 \gg$.

Still an example: Obviously, the following sentence isn't true and isn't false [2]:

$\ll$This sentense is false. $\gg$

Those sentences which can be either true, or false, are called as *meaningful* sentences. The previous example sentence is meaningless sentence.

Further, we consider only meaningful sentences which are either true, or false.

Sentences A and B are *equal* (design.: $A = B$) if A is true, if and only if B is true.

2 Events

A set $\mathtt{B}$ of sentences is called *an event, expressed by sentence* C, if the following conditions are fulfilled:

1. $C \in \mathtt{B}$;
2. if $A \in \mathtt{B}$ and $D \in \mathtt{B}$ then $A = D$;
3. if $D \in \mathtt{B}$ and $A = D$ then $A \in \mathtt{B}$.

In this case let us denote: $\mathtt{B} := {}^{\circ}C$.

An event $\mathtt{B}$ *occures* if a true sentence A exists such that $A \in \mathtt{B}$.

Events $\mathtt{A}$ and $\mathtt{B}$ *equal* (denote: $\mathtt{A} = \mathtt{B}$) if $\mathtt{A}$ occures if and only if $\mathtt{B}$ occures.

Event $\mathtt{C}$ is called *product* of event $\mathtt{A}$ and event $\mathtt{B}$ (denote: $\mathtt{C} = (\mathtt{A} \cdot \mathtt{B})$) if $\mathtt{C}$ occures if and only if $\mathtt{A}$ occures and $\mathtt{B}$ occures.

Events $\mathtt{C}$ is called *complement* of event $\mathtt{A}$ (denote: $\mathtt{C} = (\neg \mathtt{A})$) if $\mathtt{C}$ occures if and only if $\mathtt{A}$ does not occure.

$(\mathtt{A} + \mathtt{B}) := (\neg((\neg \mathtt{A}) \cdot (\neg \mathtt{B})))$. Event$(\mathtt{A} + \mathtt{B})$ is called *sum* of event $\mathtt{A}$ and event $\mathtt{B}$.

Therefore, a sum of event occures if and only if there at least one of the addends occures.

Events obey following properties (you can test these formulas by Venn diagrams – see Figure 1, Figure 2. Venn diagrams):

1. associarivity: $(\mathtt{A} \cdot \mathtt{B}) \cdot \mathtt{C} = \mathtt{A} \cdot (\mathtt{B} \cdot \mathtt{C}) = \mathtt{A} \cdot \mathtt{B} \cdot \mathtt{C}$,
   $(\mathtt{A} + \mathtt{B}) + \mathtt{C} = \mathtt{A} + (\mathtt{B} + \mathtt{C}) = \mathtt{A} + \mathtt{B} + \mathtt{C}$;
2. distributiviy: $(\mathtt{A} \cdot \mathtt{B}) + \mathtt{C} = (\mathtt{A} + \mathtt{C})\,(\mathtt{B} + \mathtt{C})$,
   $(\mathtt{A} + \mathtt{B}) \cdot \mathtt{C} = (\mathtt{A} \cdot \mathtt{C}) + (\mathtt{B} \cdot \mathtt{C})$;
3. if $\mathtt{C}$ occures then for every $\mathtt{A}$: $(\mathtt{A} \cdot \mathtt{C}) = \mathtt{A}$;
4. $(\neg\,(\neg \mathtt{A})) = \mathtt{A}$
5. cummutativity: $(\mathtt{A} \cdot \mathtt{B}) = (\mathtt{B} \cdot \mathtt{A})$, $(\mathtt{A} + \mathtt{B}) = (\mathtt{B} + \mathtt{A})$.

3 Classical Probability

Let $\mathsf{P}(X)$ be *a probability function*[3, pp.49–57] defined on the set of events.

Hence,

1. this function has values on the real numbers segment $[0; 1]$;
2. for all events $\mathtt{A}$ and $\mathtt{B}$: $\mathsf{P}(\mathtt{A} \cdot \mathtt{B}) + \mathsf{P}(\mathtt{A} \cdot (\neg \mathtt{B})) = \mathsf{P}(\mathtt{A})$;

3. for ever event $\mathtt{A}$: if $\mathsf{P}(\mathtt{A}) = 1$ then $\mathtt{A}$ occures.
Let there exists an event $\mathtt{C}_0$ such that $\mathsf{P}(\mathtt{C}_0) = 1$.

By this definition: $\mathsf{P}(\mathtt{C}_0 \cdot \mathtt{B}) + \mathsf{P}(\mathtt{C}_0 \cdot (\neg\mathtt{B})) = \mathsf{P}(\mathtt{C}_0) = 1$.
Because $\mathtt{C}_0 \cdot \mathtt{B} = \mathtt{B}$ and $\mathtt{C}_0 \cdot (\neg\mathtt{B}) = (\neg\mathtt{B})$ then

$$\mathsf{P}(\mathtt{B}) + \mathsf{P}(\neg\mathtt{B})) = 1 \tag{1}$$

Let us calculate:
$\mathsf{P}(\mathtt{A}+\mathtt{B}) = \mathsf{P}(\neg((\neg\mathtt{A})\cdot(\neg\mathtt{B}))) = 1 - \mathsf{P}((\neg\mathtt{A})\cdot(\neg\mathtt{B})) =$
$= 1 - (\mathsf{P}(\neg\mathtt{A}) - \mathsf{P}((\neg\mathtt{A})\cdot\mathtt{B})) = 1 - \mathsf{P}(\neg\mathtt{A}) + \mathsf{P}((\neg\mathtt{A})\cdot\mathtt{B}) =$
$= 1 - 1 + \mathsf{P}(\mathtt{A}) + \mathsf{P}(\mathtt{B}) - \mathsf{P}(\mathtt{A}\cdot\mathtt{B})$
Hence,

$$\mathsf{P}(\mathtt{A}+\mathtt{B}) = \mathsf{P}(\mathtt{A}) + \mathsf{P}(\mathtt{B}) - \mathsf{P}(\mathtt{A}\cdot\mathtt{B}) \tag{2}$$

this is *the formula for adding probabilities.*
The events $\mathtt{A}$ and $\mathtt{B}$ are *incompatible* if and if only $\mathsf{P}(\mathtt{A}\cdot\mathtt{B}) = 0$.
The formula for adding probabilities for incompatible events:

$$\mathsf{P}(\mathtt{A}+\mathtt{B}) = \mathsf{P}(\mathtt{A}) + \mathsf{P}(\mathtt{B}) \tag{3}$$

The events $\mathtt{A}$ and $\mathtt{B}$ are *independent* if $\mathsf{P}(\mathtt{A}\cdot\mathtt{B}) = \mathsf{P}(\mathtt{A})\cdot\mathsf{P}(\mathtt{B})$.

4 Physics Events

Events of the type ° ≪At the point $(t.\mathbf{x})$ $\mathtt{A}$ ≫ (for example, ° ≪At the point $(t.\mathbf{x})$ the temperature dropped to 0C≫) are called *dot events*. *Physical events* are dot events and events derived from physical events by the "·", "+", and "¬" operations.

Let[1] $\langle X_{\mathtt{A},0}, X_{\mathtt{A},1}, X_{\mathtt{A},2}, X_{\mathtt{A},3}\rangle$ be random coordinates of event $\mathtt{A}$.

Let $F_{\mathtt{A}}$ be a *Cumulative Distribution Function* i.e.:

$$F_{\mathtt{A}}(x_0, x_1, x_2, x_3) = \mathsf{P}((X_{\mathtt{A},0} < x_0)\cdot(X_{\mathtt{A},1} < x_1)\cdot(X_{\mathtt{A},2} < x_2)\cdot(X_{\mathtt{A},3} < x_3)).$$

If

$$\begin{aligned} j_0 &:= \frac{\partial^3 F}{\partial x_1 \partial x_2 \partial x_3}, \\ j_1 &:= -\frac{\partial^3 F}{\partial x_0 \partial x_2 \partial x_3}, \\ j_2 &:= -\frac{\partial^3 F}{\partial x_0 \partial x_1 \partial x_3}, \\ j_3 &:= \frac{\partial^3 F}{\partial x_0 \partial x_1 \partial x_2} \end{aligned}$$

then $\langle j_0, j_1, j_2, j_3\rangle$ is *a probability current vector of event.*

If $\rho := j_0/\mathrm{c}$ then ρ is *a a probability density function.*

[1] $x_0 = \mathrm{c}t$

If $\mathbf{u}_{\mathtt{A}} := \mathbf{j}_{\mathtt{A}}/\rho_{\mathtt{A}}$ then vector $\mathbf{u}_{\mathtt{A}}$ is *a velocity of a probability* of $\mathtt{A}$ propagation.

(for example for u_2:

$$u_2 = \frac{j_2}{\rho} = \frac{\left(-\frac{\partial^3 F}{\partial x_0 \partial x_1 \partial x_3}\right) c}{\left(\frac{\partial^3 F}{\partial x_1 \partial x_2 \partial x_3}\right)} = \left(-\frac{\Delta_{013} F}{\Delta_{123} F} \frac{\Delta x_2}{\Delta x_0}\right) c$$

)

Probability, for which $u_1^2 + u_2^2 + u_3^2 \leq \mathrm{c}$ are called *traceable probability.*

Denote:

$$1_2 := \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right], 0_2 := \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array}\right], \beta^{[0]} := -\left[\begin{array}{cc} 1_2 & 0_2 \\ 0_2 & 1_2 \end{array}\right] = -1_4,$$

the Pauli matrices

$$\sigma_1 = \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right], \sigma_2 = \left[\begin{array}{cc} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{array}\right], \sigma_3 = \left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right].$$

A set $\widetilde{C}$ of complex $n \times n$ matrices is called *a Clifford set of rank n* [4] if the following conditions are fulfilled:

if $\alpha_k \in \widetilde{C}$ and $\alpha_r \in \widetilde{C}$ then $\alpha_k \alpha_r + \alpha_r \alpha_k = 2\delta_{k,r}$;

if $\alpha_k \alpha_r + \alpha_r \alpha_k = 2\delta_{k,r}$ for all elements α_r of set $\widetilde{C}$ then $\alpha_k \in \widetilde{C}$.

If $n = 4$ then a Clifford set either contains 3 matrices (*a Clifford triplet*) or contains 5 matrices (*a Clifford pentad*).

For example, *light pentad* β:

$$\beta^{[k]} := \left[\begin{array}{cc} \sigma_k & 0_2 \\ 0_2 & -\sigma_k \end{array}\right], \gamma^{[0]} := \left[\begin{array}{cc} 0_2 & 1_2 \\ 1_2 & 0_2 \end{array}\right], \beta^{[4]} := \mathrm{i} \cdot \left[\begin{array}{cc} 0_2 & 1_2 \\ -1_2 & 0_2 \end{array}\right]$$

The following set of four real equations with eight real unknowns: b^2 with $b > 0$, α, β, χ, θ, γ, υ, λ:

$$\left\{\begin{array}{c} b^2 = \rho_{\mathtt{A}},, \\ b^2\left(\cos^2(\alpha)\sin(2\beta)\cos(\theta-\gamma) - \sin^2(\alpha)\sin(2\chi)\cos(\upsilon-\lambda)\right) = -\frac{j_{\mathtt{A},1}}{\mathrm{c}}, \\ b^2\left(\cos^2(\alpha)\sin(2\beta)\sin(\theta-\gamma) - \sin^2(\alpha)\sin(2\chi)\sin(\upsilon-\lambda)\right) = -\frac{j_{\mathtt{A},2}}{\mathrm{c}}, \\ b^2\left(\cos^2(\alpha)\cos(2\beta) - \sin^2(\alpha)\cos(2\chi)\right) = -\frac{j_{\mathtt{A}.3}}{\mathrm{c}}. \end{array}\right| \quad (4)$$

has solutions for any traceable $\rho_{\mathtt{A}}$ and $j_{\mathtt{A},k}$[3, pp.62–63].

If

$$\begin{aligned} \varphi_{\mathtt{A}.1} &:= b\exp(\mathrm{i}\gamma)\cos(\beta)\cos(\alpha), \\ \varphi_{\mathtt{A},2} &:= b\exp(\mathrm{i}\theta)\sin(\beta)\cos(\alpha), \\ \varphi_{\mathtt{A},3} &:= b\exp(\mathrm{i}\lambda)\cos(\chi)\sin(\alpha), \\ \varphi_{\mathtt{A},4} &:= b\exp(\mathrm{i}\upsilon)\sin(\chi)\sin(\alpha) \end{aligned} \quad (5)$$

then you can calculate that

$$\rho_{\mathtt{A}} = \sum_{s=1}^{4} \varphi_{\mathtt{A},s}^{*} \varphi_{\mathtt{A},s}, \tag{6}$$

$$\frac{j_{\mathtt{A},\alpha}}{\mathrm{c}} = -\sum_{k=1}^{4} \sum_{s=1}^{4} \varphi_{\mathtt{A},s}^{*} \beta_{s,k}^{[\alpha]} \varphi_{\mathtt{A},k}$$

Function $\varphi_{\mathtt{A}}$ is *a state vector* of event $\mathtt{A}$.

5 Entanglement

The presence in our universe of Planck's constant gives reason to presume that our world is in a confined space: $|\mathbf{x}| \leq \pi\mathrm{c}/\mathrm{h}$. Functions

$$\phi_{\mathbf{n}}(\mathbf{x}) := \left(\frac{h}{2\pi c}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(\mathbf{n}\mathbf{x})\right).$$

form an orthonormal basis of this space with scalar product of the following shape:

$$(\widetilde{\varphi}(t), \widetilde{\chi}(t)) := \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{\mathrm{h}}} dx_1 \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{\mathrm{h}}} dx_2 \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{\mathrm{h}}} dx_3 \cdot \widetilde{\varphi}(t, \mathbf{x})^{\dagger} \widetilde{\chi}(t, \mathbf{x}).$$

For the state vector:

$$(\varphi_{\mathtt{A}}(t), \varphi_{\mathtt{A}}(t)) = \mathsf{P}(\mathtt{A}(t)).$$

Let

$$\varphi_{\mathtt{A}}(t, \mathbf{x}) = \sum_{\mathbf{n}} a_{\mathtt{A},\mathbf{n}}(t) \phi_{\mathbf{n}}(\mathbf{x})$$

be a Fourier series of $\varphi_{\mathtt{A}}(t, \mathbf{x})$.
That is:

$$\mathsf{P}(\mathtt{A}(t)) = \sum_{\mathbf{n}} a_{\mathtt{A},\mathbf{n}}^{\dagger}(t) a_{\mathtt{A},\mathbf{n}}(t) = \sum_{\mathbf{n}} \mathsf{P}(\mathtt{A}_{\mathbf{n}}(t)).$$

Hence,

$$\mathtt{A}(t) = \sum_{\mathbf{n}} \mathtt{A}_{\mathbf{n}}(t).$$

there $\mathtt{A}_n(t)$ are incompatible, independent events:Therefore, if $\mathsf{P}(\mathtt{A}(t)) = 1$ then $\mathtt{A}(t)$ occures. Hence, one among $\mathtt{A}_{\mathbf{n}}(t)$ occures.

Operator $\check{r}$, defined on the state function φ set and has values on this set, is *an realization operator* if $(\check{r}\varphi, \check{r}\varphi) = 1$.

This operator can act in the measurement process or as a result of some other external disturbance.

Let $\varphi_{\mathtt{A},\mathtt{B}}((t, \mathbf{x})(\tau, \mathbf{y}))$ be a state vector of event $\mathtt{A}(t, \mathbf{x})$ and event $\mathtt{B}(\tau, \mathbf{y})$.

In this case the basis of space present the following functions:

$$\phi_{\mathbf{n},\mathbf{s}}(\mathbf{x},\mathbf{y}) := \left(\frac{h}{2\pi c}\right)^3 \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(\mathbf{nx}+\mathbf{sy})\right).$$

The scalar product has the following shape:

$$\begin{aligned}&(\widetilde{\varphi}(t,\tau),\widetilde{\chi}(t,\tau)) :=\\ &\int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dx_1 \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dx_2 \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dx_3 \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy_1 \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy_2 \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy_3 \cdot\\ &\widetilde{\varphi}(t,\tau,\mathbf{x},\mathbf{y})^\dagger \widetilde{\chi}(t,\tau,\mathbf{x},\mathbf{y}).\end{aligned}$$

The Fourier series:

$$\varphi_{\mathtt{A},\mathtt{B}}((t,\mathbf{x})(\tau,\mathbf{y})) = \sum_{\mathbf{n}}\sum_{\mathbf{s}} a_{\mathtt{A},\mathtt{B},\mathbf{n},\mathbf{s}}(t,\tau)\,\phi_{\mathbf{n},\mathbf{s}}(\mathbf{x},\mathbf{y}).$$

Hence,

$$\begin{aligned}(\varphi_{\mathtt{A},\mathtt{B}}(t,\tau),\varphi_{\mathtt{A},\mathtt{B}}(t,\tau)) &= \sum_{\mathbf{n}}\sum_{\mathbf{s}} a^\dagger_{\mathtt{A},\mathtt{B},\mathbf{n},\mathbf{s}}(t,\tau)\,a_{\mathtt{A},\mathtt{B}.\mathbf{n},\mathbf{s}}(t,\tau),\\ (\varphi_{\mathtt{A},\mathtt{B}}(t,\tau),\varphi_{\mathtt{A},\mathtt{B}}(t,\tau)) &= \mathsf{P}(\mathtt{A}(t)\cdot\mathtt{B}(\tau)),\\ \mathsf{P}(\mathtt{A}(t)\cdot\mathtt{B}(\tau)) &= \sum_{\mathbf{n}}\sum_{\mathbf{s}} \mathsf{P}\left(\mathtt{A}_{\mathbf{n}}(t)\cdot\mathtt{B}_{\mathbf{s}}(\tau)_{,}\right).\end{aligned}$$

For example: Let

$$\begin{aligned}&\varphi_{\mathtt{A},\mathtt{B}}((t,\mathbf{x})(\tau,\mathbf{y})) =\\ &a_{\mathtt{A},\mathtt{B},\mathbf{1},\mathbf{4}}(t,\tau)\,\phi_{\mathbf{1},\mathbf{4}}(\mathbf{x},\mathbf{y}) + a_{\mathtt{A},\mathtt{B},\mathbf{2},\mathbf{3}}(t,\tau)\,\phi_{\mathbf{2},\mathbf{3}}(\mathbf{x},\mathbf{y}) +\\ &a_{\mathtt{A},\mathtt{B},3,\mathbf{2}}(t,\tau)\,\phi_{3,\mathbf{2}}(\mathbf{x},\mathbf{y}) + a_{\mathtt{A},\mathtt{B},4,\mathbf{1}}(t,\tau)\,\phi_{4,\mathbf{1}}(\mathbf{x},\mathbf{y}).\end{aligned}$$

Such event called *entangled* if $a_{\mathtt{A},\mathtt{B},\mathbf{n},\mathbf{s}}$ are not factorized.
In that case:

$$\begin{aligned}&(\varphi_{\mathtt{A},\mathtt{B}}(t,\tau),\varphi_{\mathtt{A},\mathtt{B}}(t,\tau)) =\\ &a^\dagger_{\mathtt{A},\mathtt{B},\mathbf{1},\mathbf{4}}(t,\tau)\,a_{\mathtt{A},\mathtt{B}.\mathbf{1},\mathbf{4}}(t,\tau) + a^\dagger_{\mathtt{A},\mathtt{B},\mathbf{2},\mathbf{3}}(t,\tau)\,a_{\mathtt{A},\mathtt{B}.\mathbf{2},\mathbf{3}}(t,\tau) +\\ &a^\dagger_{\mathtt{A},\mathtt{B},3,\mathbf{2}}(t,\tau)\,a_{\mathtt{A},\mathtt{B}.3,\mathbf{2}}(t,\tau) + a^\dagger_{\mathtt{A},\mathtt{B},4,\mathbf{1}}(t,\tau)\,a_{\mathtt{A},\mathtt{B}.4,\mathbf{1}}(t,\tau).\end{aligned}$$

$$\begin{aligned}&\mathsf{P}(\mathtt{A}(t)\cdot\mathtt{B}(\tau)) =\\ &\mathsf{P}\left(\mathtt{A}_{\mathbf{1}}(t)\cdot\mathtt{B}_{\mathbf{4}}(\tau)_{,}\right) + \mathsf{P}\left(\mathtt{A}_{\mathbf{2}}(t)\cdot\mathtt{B}_{\mathbf{3}}(\tau)_{,}\right) +\\ &\mathsf{P}\left(\mathtt{A}_{\mathbf{3}}(t)\cdot\mathtt{B}_{\mathbf{2}}(\tau)_{,}\right) + \mathsf{P}\left(\mathtt{A}_{\mathbf{4}}(t)\cdot\mathtt{B}_{\mathbf{1}}(\tau)_{,}\right).\end{aligned}$$

Let to $\varphi_{\mathtt{A},\mathtt{B}}((t,\mathbf{x})(\tau,\mathbf{y}))$ acts realization operator: $(\check{r}\varphi,\check{r}\varphi) = 1$. Then $\mathtt{A}(t)\cdot\mathtt{B}(\tau)$ occures. Therefore, in accordance with the sum definition, one among events $\mathtt{A}_{\mathbf{n}}(t)\cdot\mathtt{B}_{\mathbf{s}}(\tau)$ occures.

6 Conclusion

Hence, events of entangled pair can occure in any time moments.

References

[1] Tarski, A. The Semantic Conception of Truth and the Foundations of Semantics, *Philosophy and Phenomenological Research*, **4**, 1944.

[2] Eubulides of Miletus, Liar paradox, fl. 4th century BCE

[3] Gunn Quznetsov *Final Book on Fundamental Theoretical Physics*, American Research Press, 2011

[4] For instance, Madelung, E., *Die Mathematischen Hilfsmittel des Physikers.* Springer Verlag, (1957) p.29