SIMPLE DIRECT DERIVATION OF FERMAT'S LAST THEOREM

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Abstract. For $n \in \mathbb{N}$: We devise an algebraic identity $r^n + s^n = t^n$ holding for $r, s, t > 0 \in \mathbb{R}$, to relate to $x^n + y^n = z^n$ holding for co-prime $x, y, z \geq 1$. The detailed identity has an unrestricted variable, which allows us to directly infer that $\{r, s, t\} = \{x, y, z\}$. We show for $n > 2$ that there exists no co-prime (r, s, t). Hence, for $n > 2$, there exists no co-prime nor integral (x, y, z).

1. Introduction

Fermat's Last theorem (FLT) states, for integral $n \geq 3$, that no positive integral x, y, z satisfy $x^n + y^n = z^n$. A simple proof of FLT does not exist for every $n \geq 3$.

2. The Direct, Deductive Argument, Not By Way Of Contradiction

We start a chain of reasoning with an algebraic identity that is *detailed enough, sufficient* for implying FLT, unlike identities of similar form, viz., our identity:

$$(1) \qquad \left(\left(2^{p+1}q^n\right)^{\frac{1}{n}}\right)^n + \left(\left(m - 2^p q^n\right)^{\frac{1}{n}}\right)^n = \left(\left(m + 2^p q^n\right)^{\frac{1}{n}}\right)^n.$$

For all integral $n \geq 1$: *Let* this identity *hold specifically* for all positive rational values of q, all positive odd p, and all positive real values of m such that $m > 2^p q^n$.
Exclusively rational q would be sufficient for our argument, per Prop. 2.3, below.

Let real $(2^{p+1}q^n)^{\frac{1}{n}}$; $(m-2^p q^n)^{\frac{1}{n}}$; $(m+2^p q^{n_1})^{\frac{1}{n}}$, respectively, *be denoted as* r, s, t.
Each existing $r, s, t \in \mathbb{N} \subset \mathbb{R}$ is a unique n-th root with $r^n, s^n, t^n \geq 1$.
For $n = 2$ with even p, equation (1) does not hold for $r, s, t \in \mathbb{N} \subset \mathbb{R}$, per Sect. 3.

For any given value of n, note that r, s, t are functions of variables m, q, p.
However, for convenience of notation, we suppress this functional dependency.

For any given $n \in \mathbb{N}$: *Let* A be $\{r, s, t \in \mathbb{R} | r, s, t > 0\}$ satisfying equation (1).

It is easy to establish, for $n = 1, 2$ that a $r, s, t \in \mathbb{N} \subset A$ satisfies equation (1).

Example 2.1. *For* $n = 1$, *values* $m = \frac{3}{4}$, $p = 1$, *and* $q = \frac{11}{2}$ *result in* $3 + 4 = 7$.

Example 2.2. *For* $n = 2$, *values* $m = \frac{3}{2}$, $p = 1$ *and* $q = \frac{41}{2}$ *result in* $3^2 + 4^2 = 5^2$.

For any given $n \in \mathbb{N}$: *Let* B be co-prime $\{x, y, z \in \mathbb{N}\}$ satisfying $x^n + y^n = z^n$.

We relate $\{x, y, z \in B\}$ to $\{r, s, t \in A\}$, and subsequently to $\{r, s, t \in \mathbb{N} \subset A\}$.

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We infer values of n for $\{x, y, z \in B, \in \varnothing\}$, a *hypothetical example* being $n = 3$.

We use three techniques to keep the argument as general as possible for n :
1) We stress the truth of the following : For $n = 1, 2$ *at minimum*, equation (1) and $x^n + y^n = z^n$, respectively, hold for $\{r, s, t \in \mathbb{N} \subset A\}$, and for $\{x, y, z \in B\}$.
This is our strategy to keep open the possibility of existing such sets for $n \geq 3$.
2) For $n \geq 3$, we suppress the pre-established "fact" that $\{x, y, z \in \mathbb{N}, \in \varnothing\}$.
3) For $n \geq 3$, we reveal solely in Sect. 3, below, that $\{r, s, t \in \mathbb{N} \subset A \in \varnothing\}$.

For any given $n \in \mathbb{N}$: *Let* C be $\{\frac{rs}{t} | r, s, t \in A\}$.
For any given $n \in \mathbb{N}$: *Let* D be $\{\frac{xy}{z} | x, y, z \in B\}$.
For any given $n \in \mathbb{N}$: *Let* E be co-prime $\{r, s, t \subset A | r, s, t \geq 1\}$ satisfying (1).
For any given $n \in \mathbb{N}$: *Let* F be $\{\frac{rs}{t} \subset C | r, s, t \in E\}$.

Proposition 2.3. *For any given value of n for which there exists* $\{(x, y, z) \in B\}$, *it is true that* $\{(r, s, t) \in E\} = \{(x, y, z) \in B\}$.

Proof. For any given n for which set B is not null : Due *solely* to unrestricted m, real $\frac{rs}{t} \in C$ or $\frac{(2^{p+1}q^n)^{\frac{1}{n}}(m-2^p q^n)^{\frac{1}{n}}}{(m+2^p q^n)^{\frac{1}{n}}}$ takes every value of rational $\frac{xy}{z} \in D$.
Simultaneously, due solely to m, real $(m - 2^p q^n)^{\frac{1}{n}}$ take every value of integral y.
Since, simultaneously, real values of s take every integral value of y, hence (with co-prime x, y, integral xy, and integral rs) real values of r necessarily take every integral value of x. Consequently, it is true that $r = x$, $s = y$, and $t = z$.
These equations just mean that $(r, s, t) \in A$ take each value of $(x, y, z) \in B$. □

Rational q is sufficient for the truth of proposition 2.3, for the following reason:
Irrational values of q are irrelevant because values taken by m, p, q, with p, q independent of determining proposition 2,3, are *sufficient* for prop. 2.3 to be true.
Values of n for $r^n + s^n = t^n$ *and for* $x^n + y^n = z^n$ *are equal*, as follows :
This is implied by $\{(r, s, t) \in E\} = \{(x, y, z) \in B\}$, as per proposition 2.3.
Allowing q to vary thusly results in $\{r, s, t \in A\}$ *being restricted to* $\{r, s, t \in E\}$.
We choose to restrict odd p to $p = 1$ since, per Sect. 3, below, with $p = 1$, equation (1) yields, for the largest $\{n | n \in \mathbb{N}, n \geq 3\}$, a $\{r, s, t \in \mathbb{N}, \in \varnothing\}$.
Hence, for (1), the value of $(r, s, t) \in E$ is $((4q^n)^{\frac{1}{n}}; (m - 2q^n)^{\frac{1}{n}}; (m + 2q^n)^{\frac{1}{n}}$.

3. Results and Conclusion

We show for $r^n + s^n = t^n$ that no values of $n \geq 3$ exist such that (1) holds for values of positive co-prime triple r, s, t or $((4q^n)^{\frac{1}{n}}, (m - 2q^n)^{\frac{1}{n}}, (m + 2q^n)^{\frac{1}{n}})$:
By inspection, $(4q^n)^{\frac{1}{n}} = 2^{\frac{2}{n}}q$. Therefore, with rational q and $n \geq 3$, the set of all rational values of $2^{\frac{2}{n}}q$, so, the subset of all integral values of $2^{\frac{2}{n}}q$, is null.

Remark 3.1. *For* $p = 1, ..., 19...$ *we get, respectively, integral values* $r = 2^{\frac{2}{n}}m, ...2^{\frac{20}{n}}m...$, *which have progressively smaller sets of n for which r is not integral.*
For example, with $p = 19$, *the values of n for which r is not integral show as* $n = 3, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19$, *plus* $\{n | n \in \mathbb{N}, n > 20\}$.
Intermittent integral values of r provide no useful information about values of n.

Hence, for $n \geq 3$ there exists no positive co-prime nor integral (r, s, t).
Thus, per Prop. 2.3, for $n \geq 3$ there exists no positive integral (x, y, z).QED