

# OFDM Performance Improvement Based on Active Constellation Extension and Active Interference Cancellation

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**Abstract**-Orthogonal Frequency Division Multiplexing (OFDM) is a transceiver technology able to achieve spectrally efficient and high data rate wireless transmissions. It is also able to transmit in a non-contiguous (NC) fashion by utilizing several separate spectral whitespaces. Cognitive radio systems based on non-contiguous orthogonal frequency division multiplexing (NC-OFDM) is a promising technique that can provide efficient bandwidth utilization for high data rate wireless communication systems under frequency selective fading environments. However, the high peak to average power ratio (PAPR) and large spectrum sidelobe power are the two major limitations at the transmitter side of the NC-OFDM based Cognitive Radio (CR) system, which may significantly reduce the power efficiency and performance of the system. In this paper, we present a joint method for the PAPR reduction and sidelobe suppression based on Active Constellation Extension(ACE) and Active Interference Cancellation(AIC).

The key idea of the proposed method is to dynamically extend part of the constellation points on the secondary user (SU) subcarriers based on ACE and add several SC symbols on the primary user (PU) subcarriers based on AIC to generate the appropriate cancellation signal for joint PAPR reduction and sidelobe suppression. Since the convex optimization is computationally complex in nature, a suboptimal method is also proposed whose simulation result shows that there is a significant improvement in terms of PAPR and sidelobe reduction.

**Keywords**-NC-OFDM, ACE, AIC, PAPR, Sidelobe

## 1. Introduction

Bandwidth requirement for wireless communication increases exponentially as the number of existing and new wireless services increases. The basic necessity of all wireless service is spectrum availability, which is a sparse resource now-a-days. Cognitive radio is an efficient technology for the development of future wireless communication systems since the radio frequency spectrum is a scarce natural resource that cannot be expanded. By sensing the operating spectrum, cognitive radio systems can use those portions of the allocated spectrum band that are not presently used by the licensed users. Noncontiguous orthogonal frequency division multiplexing (NC-OFDM) is a well known multicarrier modulation technique for cognitive radio systems. This provides cognitive radio systems to avail the benefits of OFDM modulation such as high spectral efficiency, efficient transmission in frequency selective fading environment and simplified channel equalization. At the same time, it is plagued by the inherent issue of the OFDM implementation i.e., high peak to average power ratio (PAPR). High PAPR causes non-linear distortion to the transmitted signal

when passed through the high power amplifier (HPA) at the front end of the transmitter. The large sidelobe power of the cognitive user (secondary user) subcarriers in NC-OFDM spectrum induces interference between the licensed user (primary user) and the cognitive user. Hence, it is highly desirable to reduce the sidelobe power and PAPR significantly [1]. Recently, various methods have been proposed to reduce the PAPR for the NC-OFDM-based CR system in the literature, such as clipping [2], partial transmit sequence [3], [4], active constellation extension [5], and tone reservation [6], [7]. These PAPR reduction techniques can efficiently reduce the PAPR of transmitted signals; however, they do not take sidelobe suppression into account. Moreover, to suppress the sidelobe of the NC-OFDM-based CR system, many schemes have been proposed in the literature, such as active interference cancellation (AIC) [8], constellation adjustment (CA) [8], pulse shaping (PS) [10], spectrum precoding (SP) [11], and sidelobe suppression with orthogonal projection (SSOP) [12]. Thus, the CA method must send the selected weights as side information to the receiver for data recovery, resulting in the decrease in the data rate. Both the PS and SP methods can efficiently suppress the sidelobe of the NC-OFDM-based CR system by shaping the waveform of NC-OFDM signals. However, they suffer from high computational complexity.

The SSOP method utilizes an orthogonal projection matrix for sidelobe suppression and adopts several reserved subcarriers for recovering the distorted signal in the receiver. Thus, the SSOP method suffers from the decrease in the data rate. Furthermore, although these sidelobe suppression methods can suppress the sidelobe power in the NC-OFDM-based CR system, all of them do not consider PAPR reduction. Since the PAPR reduction methods may affect the spectrum sidelobe of the NC-OFDM-based CR system, we must jointly consider PAPR reduction and sidelobe suppression. To the authors' best knowledge, few researches investigate the joint PAPR reduction and sidelobe suppression in the NC-OFDM-based CR system. In [13], the selected mapping (SLM) technique is employed to jointly reduce the PAPR and suppress the sidelobe in the NC-OFDM-based CR system, and its key idea is to generate several alternative signals and select the signal with low PAPR and sidelobe as the transmitted signal. However, the SLM method cannot achieve significant PAPR reduction and sidelobe suppression performances. Moreover, the SLM method must reserve several subcarriers to transmit the side information, resulting in the decrease in the data rate.

This paper discusses a technique in which both the PAPR and sidelobe power are reduced jointly. There are not many studies in the area of joint PAPR and side lobe reduction as per the technical literature. The method is referred as signal cancellation (SC) which accomplishes the task of joint PAPR and sidelobe reduction by dynamically extending a part of the outer constellation points on SU subcarriers by the addition of constellation adjustment (Ca) symbols, whereas SC symbols (Cs) are added on the PU subcarriers, to generate the cancellation signal. For the proposed method, after dynamically extending the external constellation points of the SU subcarriers, add an auxiliary signal to the SU subcarriers and add SC symbols to the PU subcarriers so as to jointly reduce the effect of both PAPR and sidelobe power. The proposed method formulates the problem as a quadratically constrained quadratic program (QCQP), and the optimal signal can be obtained by convex optimization. Due to the computational complexity for finding out the optimal signal, a sub-optimal method is also presented in this paper which shows a significant reduction in PAPR and sidelobe power.

The remainder of this paper is organized as follows: The section II deals with the system model and introduces the definition of PAPR and spectrum sidelobe. In section III, the proposed method for joint PAPR reduction and sidelobe suppression is discussed in detail. Simulation results are discussed in section IV. Finally, section V presents the conclusion.

## 2. SYSTEM DESCRIPTION

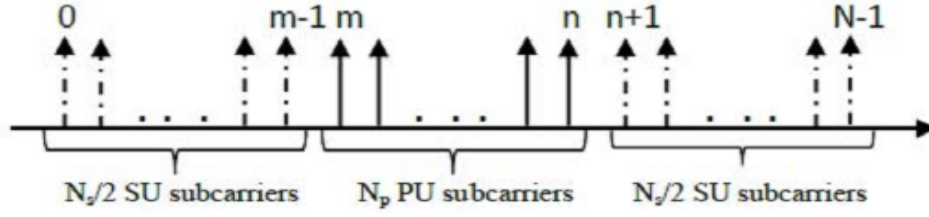


Fig.1: NC-OFDM system coexisting with PUs and SUs.

Consider an NC-OFDM system with  $B$  bandwidth in which  $N$  orthogonal subcarriers are occupied as illustrated in Fig.1, such that  $N_p$  subcarriers (from  $m$  to  $n$ ) at the center of the frequency band is occupied by the PUs.

Then, the remaining subcarriers belongs to SUs, i.e.,  $N_s = N - N_p$  (from  $0$  to  $m-1$  and  $n+1$  to  $N-1$ ).

In order to create spectrum notch in the conventional NC-OFDM system, PU subcarriers are turned off by the SU to limit the interference to the PUs.

Even if the PU subcarriers are turned off, still there will be some amount of interference.

Therefore, the high PAPR and large sidelobe power are the two major bottle necks of the CR based NC-OFDM. For an  $N$  subcarrier system, the NC-OFDM symbol,  $X$  is given by  $X = [X(0), X(1), \dots, X(N-1)]$ , in which the  $N_p$  subcarriers are turned off.

Then, the time domain NC-OFDM signal can be generated by employing  $N$ -point inverse discrete Fourier transform (IDFT) to  $X$  can be represented as:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi k n \Delta f}{B}}, \quad n=0,1,\dots,N-1 \quad (1)$$

where  $\Delta f$  denotes the frequency interval between the adjacent subcarriers and  $B = N\Delta f$  forms the total bandwidth of the system. The peak value of NC-OFDM signal will be very high when compared to the average value of the whole system.

This ratio of peak to average value is defined as PAPR.

$$PAPR = \frac{\max_{0 \leq n \leq N-1} |x(n)|^2}{E[|x(n)|^2]} \quad (2)$$

where the  $x(n)$  represents the transmitting signal and  $E[\cdot]$  is the average power.

To measure the sidelobe power in the frequency band of PU, we consider the spectrum at each sample points  $\{f_0, f_1, \dots, f_{M-1}\}$ .

Then, the sidelobe at sample point  $f_m$  can be given as:

$$S_a(m) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} X(k) e^{j 2\pi n (k\Delta f - f_m)}, \quad m = 0, 1, \dots, M-1 \quad (3)$$

The (3) can be expressed in matrix form as:

$$S_d = P_d X \quad (4)$$

$P_d$  is a  $M \times N$  matrix indicating the power of the input data  $X$ .

Thus, the average sidelobe power can be given by equation

$$SL_{avg} = \sum_{m=1}^{M-1} |S_d(m)|^2 \quad (5)$$

### 3. Proposed Method

The input data  $X'$  is QAM (Quadrature Amplitude Modulation) modulated in the system we have considered and is converted into parallel stream of data. By turning off the PU subcarriers generates the NC-OFDM system as illustrated in the Fig.2.



Fig.2: Block diagram representation of proposed method.

To extend the outer points of the constellation points on to the SU subcarriers, constellation adjustment symbol  $C_a = [C_a(0), C_a(1), \dots, C_a(N-1)]$  is added to  $X$ . Also, on to the PU subcarriers, signal cancellation symbol is added as  $C_s = [C_s(0), C_s(1), \dots, C_s(N-1)]$ .

It combines the idea of active constellation extension and tone reservation.

Thus, the overall cancellation symbol can be represented as

$$C = C_a + C_s \quad (6)$$

The transmitted symbol can be expressed as follows:

$$\hat{X} = X + C_a + C_s = X + C \quad (7)$$

When taking the N-IDFT of  $\hat{X}$ , we are able to obtain the time domain transmitting signal  $\hat{x}$ . Thus, the proposed method can be modeled as shown in Fig. 2.

#### 3.1. Convex Optimization Problem Formulation

The convex optimization problem for the proposed system can be written as follows:

$$\min_c \|FX + FC\|_\infty \quad (8a)$$

$$\text{Subject to } \|P_d X + P_d C\|_\infty \leq \beta \|P_d X\|_2 \quad (8b)$$

$$\|X + C\|_2^2 \leq (1 + \mu) \|X\|_2^2 \quad (8c)$$

where the  $F$  represents the Fast Fourier transform (FFT). The equation (8a) is a convex function and equations (8b) and (8c) forms a convex set [17]. Thus, the joint reduction problem is a convex QCQP and it is very difficult to obtain the optimal solution [12]. The equation (8a) points out the minimization of PAPR of the proposed method. The sidelobe constraint is described in (8b), i.e. the sidelobe power of the

proposed method must be less than or equal to  $\beta$  times the sidelobe power of the conventional NC-OFDM system, where  $\beta$  represents the parameter to control the maximum spectrum sidelobe power. The equation (8c) shows that the power of the proposed method must be less than or equal to  $(1+\mu)$  times the power of the conventional NC-OFDM system, where  $\mu$  is the parameter to adjust the threshold power.

### 3.2. Sub-Optimal method

Sub-optimal method is an iterative algorithm in which it first solves the problem of PAPR and then moves on to the second problem. In the first stage of sub-optimal method, the constellation adjustment symbol  $C_a$  is added on to the SU subcarriers to eliminate high PAPR of the system. The next goal is to suppress the sidelobe power. For this  $C_s$  is added onto the PU subcarriers. Thus, sub-optimal method solves the two problems separately as a repeating process till it reduces both PAPR and sidelobe. Since  $C_a$  and  $C_s$  are added on to the SU and PU subcarriers, the average power of the system is increased.

Therefore, the sub-optimal method is able find a key way to jointly reduce the PAPR and sidelobe.

#### 3.2.1. PAPR Reduction

For the sub-optimal method, first initialize the value of  $C_a$  and  $C_s$  to a constant value.

During each iteration, the sub-optimal technique tackles the primary issue initially and afterward proceeds to the second issue. For the first problem, the  $C_s$  on PU subcarriers is seen as a constant, which is obtained by in the last iteration of sidelobe suppression. Then, the  $C_a$  on SU subcarriers is employed to generate the peak-canceling signal for PAPR reduction in the NC-OFDM-based CR framework.

Thus, the first problem can be formulated as follows:

$$\min_{C_a} \|FX + FC_a + FC_s\|_{\infty} \quad (9)$$

$C_a$  can be obtained by the following algorithm given below:

1. Take IDFT operation to the data block  $X + C_s$  to obtain,  
 $\check{x} = [\check{x}(0), \check{x}(1), \dots, \check{x}(N-1)]$
2. Clip signal whose amplitude is larger than A (threshold value).  

$$\bar{x}(n) = \begin{cases} \check{x}(n)e^{j\theta(n)} & ; |\check{x}(n)| < A \\ Ae^{j\theta(n)} & ; |\check{x}(n)| \geq A \end{cases}$$
3. Calculate the clipping noise,

$$C_{clip}(n) = \bar{x} - \check{x}(n)$$

4. Take N-DFT of  $C_{clip}$  to obtain

$$C_{clip} = [C_{clip}(0), C_{clip}(1), \dots, C_{clip}(N-1)]$$

5. For the ACE part, keep only those components of  $C_{clip}$ , which belong to extension directions of the SU subcarrier constellations in  $C_a$ . Store all components of  $C_{clip}$ , which belong to PU

subcarriers, into  $C_{asus}$  such that the elements are

$$C_{asus,k} = \begin{cases} C_{clip,k}, & k \in I_p \\ 0, & \text{otherwise} \end{cases}$$

6. obtain  $C_a$  by applying IFFT.
7. Update the data signal,  $X + \mu C_a$

### 3.2.2. Sidelobe Suppression

During each iteration, after solving the first problem and obtaining appropriate value for  $C_a$ , then the system tries to solve the second problem. For the second problem, the  $C_a$  obtained is seen as a constant, and the SC symbol  $C_s$  on PU subcarriers is utilized as a variable to suppress the sidelobe in NC-OFDM-based CR systems. Thus, the second problem can be expressed as:

$$\min_{C_s} \|P_d X + P_d C\|_{\infty} \quad (10)$$

$C_s$  can be obtained as:

1. Let,

$$C_s = T \quad (11)$$

where T is a matrix that transfers  $\hat{C}_s$  to  $C_s$ .

2. Then

$$\hat{C}_s = -(B^H B)^{-1} B^H P_d (X + C_a) \quad (12)$$

where  $B = P_d T$ , and  $B^H$  denotes the complex conjugate transpose matrix of B.

3. Substituting (12) in (11), we get

$$C_s = Q(X + C_a) \quad (13)$$

where  $Q = T(B^H B)^{-1} B^H P_d$

Therefore, the total added symbol C can be expressed as,  $C = C_a + C_s$

### 3.2.3. Algorithm

1. Set the maximum iteration number K and threshold clipping level A.
2. Set  $i=0$ , and for  $i = 0$ th iteration, set  $C_a^0 = 0, C_s^0 = 0$ . Now, randomly generate the data block  $X = [X(0), X(1), \dots, X(N-1)]$ .
3. Take  $C_s^i$  PU subcarriers as a constant and obtain  $C_a^{i+1}$  by solving (9) as described in sub-section 3.2.1.
4. Now, take  $C_a^{i+1}$  SU subcarriers as a constant and obtain the SC symbol  $C_s^{i+1}$  by (10).

5. Update the total symbol  $C^{i+1}$  by (19) and calculate the total power  $\|X + C\|_2^2$ . If the power constraint  $\|X + C^{i+1}\|_2^2 \leq (1 + \mu)$  is not satisfied. Then output  $\hat{x} = F(X + C^{i+1})$  as the transmitted signal and terminate the algorithm.
6. Set  $i = i + 1$ , if  $i < K$ , go to Step 3; Otherwise, take N-point IDFT operation to  $X + C^{i+1}$  to obtain the transmitted signal  $\hat{x}$  and terminate the algorithm.

## 4. Simulation result

This section evaluates the simulation results obtained from the formulation of optimization problem and sub-optimal method for the joint PAPR reduction and sidelobe suppression

of NC-OFDM. The simulation results were obtained by considering a data that is been modulated by QAM. The system considers a total number of subcarriers as  $N = 64$ , in which  $m = 29$  and  $n = 38$  will be the subcarriers of PUs and the remaining subcarriers will be acting as SUs as illustrated in Fig.1. The subcarrier interval is  $\Delta f = 22.5\text{kHz}$ . The PUs occupies the spectrum from  $30\Delta f$  to  $39\Delta f$ , in which 37 sampling frequency are calculated for the total sidelobe power are placed with the equivalent space of  $\Delta f/4$ . For comparison purpose, a conventional NC-OFDM system was simulated by turning off the PUs in order to avoid interference with the PUs and also, the SC method.

Fig. 3 shows the PAPR reduction of the proposed SC method with  $\mu = 0.5$  when  $\beta$  is different for QAM. As shown in Fig. 3, the ACE method can slightly reduce the PAPR of the transmitted NC-OFDM signals, and its PAPR is 9.1 dB when  $\text{CCDF} = 10^{-3}$ . The proposed SC method with different  $\beta$  values also achieves significant PAPR reduction. For example, when  $\text{CCDF} = 10^{-3}$ , for  $\beta = 0.2, 0.3, 0.4,$  and  $0.5$ , the PAPR of the proposed SC method is 10.2, 9.5, 8.3, and 7.3 dB, respectively. Moreover, the PAPR reduction performance of the SC method improves when the value of  $\beta$  increases. Because there is a tradeoff between PAPR reduction and sidelobe suppression, and the sidelobe suppression constraint becomes more relaxed with the increase of  $\beta$ , this leads to the improvement of the PAPR reduction performance and the degradation of the sidelobe suppression performance.

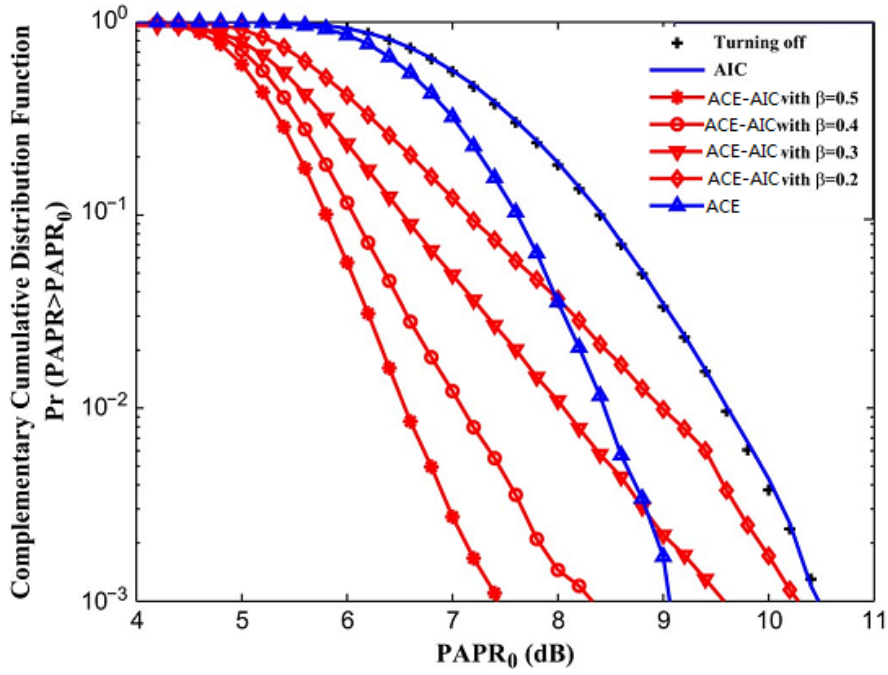


Fig. 3. PAPR reduction with the SC method when  $\mu = 0.5$  and  $\beta$  is different for QAM.

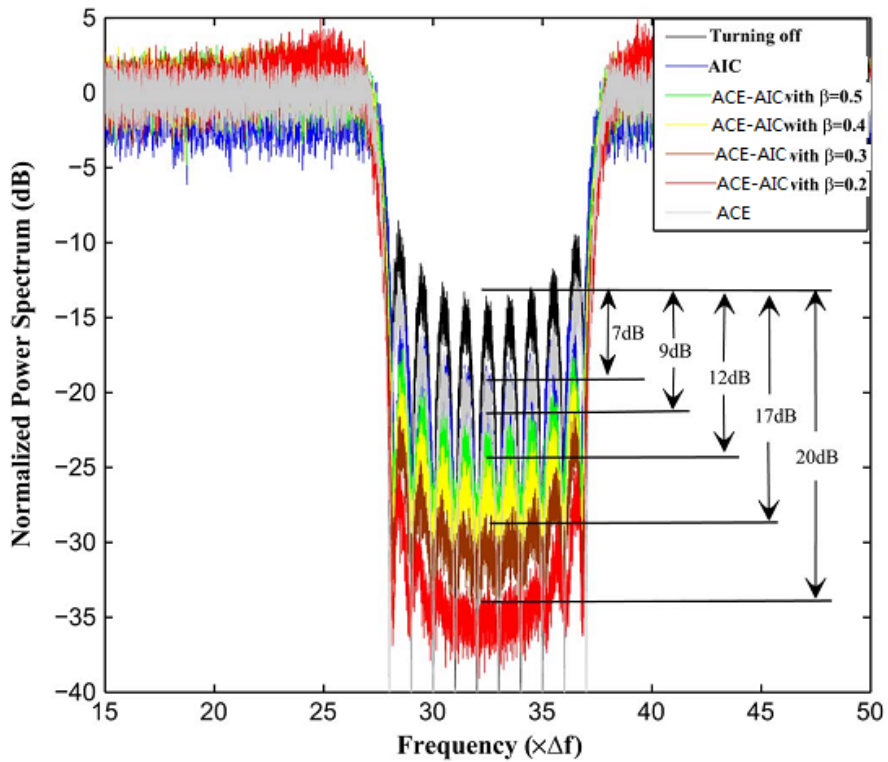


Fig. 4. Power spectrum density of the NC-OFDM signals with the SC method when  $\mu = 0.5$  and  $\beta$  is different for QAM.

Fig. 4 shows the normalized power spectrum of the NC-OFDM signals with the proposed SC method when  $\mu = 0.5$  and  $\beta$  is different for QAM. As shown in Fig. 4, the turning-off method can achieve a spectrum notch of only 13 dB in the target spectrum band from  $29\Delta f$  to  $38\Delta f$ , which means a considerable interference to the PUs. The AIC method can obtain a spectrum notch that is 7 dB deeper than that of the turning-off method. In addition, the ACE method can achieve the same sidelobe suppression performance with the AIC method. Moreover, it is obvious that the sidelobe suppression performance of the proposed SC method is



much better than those of the turning-off, AIC, and ACE methods. For example, compared with the turning-off method, when  $\beta=0.2, 0.3, 0.4,$  and  $0.5,$  the SC method can improve the sidelobe suppression by 20, 17, 12, and 9 dB, respectively. Thus, as shown in Figs. 3 and 4, the SC method can achieve much better PAPR reduction and sidelobe suppression performances than the turning-off, AIC, and ACE methods. Furthermore, the sidelobe suppression performance of the SC method improves when the value of  $\beta$  decreases, since there is a tradeoff between PAPR reduction and sidelobe suppression. Therefore, the SC method can provide both sufficient PAPR reduction and sidelobe suppression performances by adjusting parameters  $\mu$  and  $\beta$ .

## 5. Conclusion

In this paper, we have proposed an SC method for joint PAPR reduction and sidelobe suppression in NC-OFDM-based CR systems. The proposed SC method dynamically extends part of constellation points on the SU tones and adds several SC symbols on the PU tones to jointly reduce the PAPR and suppress the sidelobe of NC-OFDM signals. Moreover, we also proposed a sub-SC method to efficiently reduce the PAPR and suppress the sidelobe with low computational complexity. Simulation results show that both the SC and sub-SC methods can provide significant PAPR reduction and sidelobe suppression performances.

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