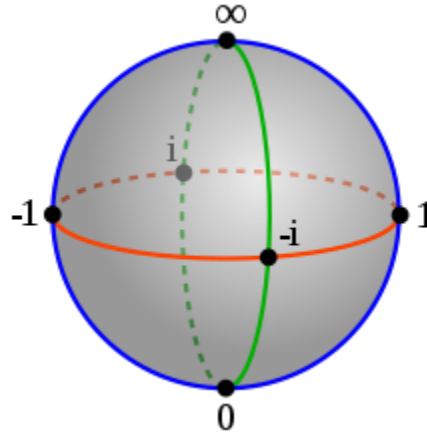


## Refutation of additive arithmetic operations in the Riemann sphere

© Copyright 2018 by Colin James III All rights reserved.

We assume Meth8/VL4 with the designated *proof* value of  $\tau$ autology and falsity value of  $\sigma$ contingency.

Taken from: [en.wikipedia.org/wiki/Riemann\\_sphere](http://en.wikipedia.org/wiki/Riemann_sphere)



Addition of complex numbers may be extended by defining, for  $z \in \mathbb{C}$ ,  
 $z + \infty = \infty$  for any complex number  $z$ , (1.1)

and multiplication may be defined by  $z \times \infty = \infty$   
 for all nonzero complex numbers  $z$ , with  $\infty \times \infty = \infty$ . (2.1)

Note that  $\infty - \infty$  and  $0 \times \infty$  are left undefined. (3.1)

Unlike the complex numbers, the extended complex numbers do not form a field, since  $\infty$  does not have a multiplicative inverse. Nonetheless, it is customary to define division on  $\mathbb{C} \cup \{\infty\}$  by  $z/0 = \infty$  and  $z/\infty = 0$  for all nonzero complex numbers  $z$ ,  
 with  $\infty/0 = \infty$  and  $0/\infty = 0$ . (4.1)

The quotients  $0/0$  and  $\infty/\infty$  are left undefined. (5.1)

LET  $p, q, r: \mathbb{C}, z, \infty$ ;  
 $\&$  And,  $\times$ ;  $+$  Or,  $+$ ,  $\cup$ ;  $-$  Not Or,  $-$ ;  $\setminus$  Not And,  $/$ ;  $<$  Not Imply,  $\in$ ;  
 $=$  Equivalent,  $=$ ;  $@$  Not Equivalent;  $\%$  possibility, for one or some;  $\#$  necessity, for all;  
 $(\%p>\#p)$  one, 1;  $((\%p>\#p)-(\%p>\#p))$  zero, 0;  $(r@r)$  undefined.

$$((q \times p) \times q) \times ((q+r)=r) ; \quad \text{TTCT TTTT TCTT TTTT} \quad (1.2)$$

$$((q \times p) \times (\%q@((\%p>\#p)-(\%p>\#p)))) \times (((q \times r)=r) \times ((r \times r)=r)) ;$$

$$\text{TTTT TTTT TTTT TTTT} \quad (2.2)$$

$$((r-r) \times (((\%p>\#p)-(\%p>\#p)) \times r)) = (r @ r) ; \quad \text{TTTT TTTT TTTT TTTT} \quad (3.2)$$

$$((p+r) \times (\%q@((\%p>\#p)-(\%p>\#p)))) \times$$

$$(((q \setminus ((\%p>\#p)-(\%p>\#p)))=r) \times ((q \setminus r)=((\%p>\#p)-(\%p>\#p))))$$

$$\& (((r \setminus ((\%p>\#p)-(\%p>\#p)))=r)$$

$$\& (((((\%p>\#p)-(\%p>\#p)) \setminus r)=((\%p>\#p)-(\%p>\#p)))) ;$$

$$\text{TTTC TTCC TTTC TTCC} \quad (4.2)$$

$$(((\%p>\#p)-(\%p>\#p)) \setminus ((\%p>\#p)-(\%p>\#p))) \times (r \setminus r) = (r @ r) ;$$

$$\text{CCCC TTTT CCCC TTTT} \quad (5.2)$$

Eqs. 2.2 and 3.2 as rendered are tautologous. This means the definition of multiplication for extended complex numbers and undefined values of  $\infty - \infty$  and  $0 \times \infty$  are theorems.

Eq. 1.2 is *not* tautologous. This means the definition of addition for extended complex numbers is not a theorem.

Eqs. 4.2 and 5.2 are *not* tautologous. This means the custom of forcing a field definition for extended complex numbers is mistaken as are the undefined values of the quotients  $0/0$  and  $\infty/\infty$ .