

## Refutation of Fitch's paradox of knowability

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We assume the method and apparatus of Meth8/VL4 with  $\tau$ autology as the designated *proof* value,  $F$  as contradiction,  $N$  as truthity (non-contingency), and  $C$  as falsity (contingency). The 16-valued truth table fragment ) is row-major and horizontal.

LET  $p$   $x$ ;  $\sim$  Not;  $\&$  And;  $=$  Equivalent;  $>$  Imply;  
 $\%$  possibility, for one or some,  $L$ ;  $\#$  necessity, for all,  $K$  *discernible* (instead of known);  
 $(p=p)$   $\tau$ autology.

Note: The parser for Meth8 explicitly decomposes the negation of concatenated modal operators on literals as follows:  $\sim\% \#(p>q)$  is  $\sim(\% \#(p>q)=(p=p))$ .

From: [en.wikipedia.org/wiki/Fitch's\\_paradox\\_of\\_knowability](http://en.wikipedia.org/wiki/Fitch's_paradox_of_knowability), of which please see because we do not reproduce here.

Fitch rules:

$\#p>p$ ;	TTTT TTTT TTTT TTTT	(A.2)
$\#(p\&q)>(\#p\&\#q)$ ;	TTTT TTTT TTTT TTTT	(B.2)
$p>\#\%p$ ;	TTTT TTTT TTTT TTTT	(C.2)
$\sim p>\sim\%p$ ;	NTNT NTNT NTNT NTNT	(D.1.2)
Since Eq. D.1.1 is not tautologous, it should correctly read: $"K\sim p>\sim Lp"$		(D.2.1)
$\#\sim p>\sim\%p$ ;	TTTT TTTT TTTT TTTT	(D.2.2)

Fitch steps:

$\#(p\&\sim p)=(p=p)$ ;	FFFF FFFF FFFF FFFF	(1.2)
$\#(p\&\sim p)>(\#p\&\#\sim p)$ ;	TTTT TTTT TTTT TTTT	(2.2)
$(\#p\&\#\sim p)>\#p$ ;	TTTT TTTT TTTT TTTT	(3.2)
$(\#p\&\#\sim p)>\#\sim p$ ;	TTTT TTTT TTTT TTTT	(4.2)
$(\#p\&\#\sim p)>\sim\#p$ ;	TTTT TTTT TTTT TTTT	(5.2)
$(\#p\&\sim p)>\sim(\#(p\&\sim p)=(p=p))$ ;	TTTT TTTT TTTT TTTT	(6.2)
$\sim(\#(p\&\sim p)=(p=p)) > \sim(\% \#(p\&\sim p)=(p=p))$ ;	TTTT TTTT TTTT TTTT	(7.2)
$(p\&\sim p)=(p=p)$ ;	FCFC FCFC FCFC FCFC	(8.2)
$(p\&\sim p) > \% \#(p\&\sim p)$ ;	TNTN TNTN TNTN TNTN	(9.2)
$(\sim(\% \#(p\&\sim p)=(p=p))\&(\% \#(p\&\sim p)))>\sim(p\&\sim p)$ ;	TTTT TTTT TTTT TTTT	(10.2)
$\sim(p\&\sim p)>(p>\#p)$ ;	TTTT TTTT TTTT TTTT	(11.2)

As rendered, Eqs. D.1.2, 1.2, 8.2, and 9.2 are *not* tautologous. However Eqs. 7.2 and 11.2 are tautologous. This means the alleged paradox is *not* contradictory, *not* a paradox, and hence a theorem.

It states that "every truth is discernible", and we add, "by the instant modal logic model checker".

Some writers invoke Gödel incompleteness then jettison the knowability rule (C.1) to generalize and solve the paradox, rewritten as:

$$\%x((x \& \neg Kx) \& LKx) \& LK((x \& \neg Kx) \& LKx) \tag{C'.1.1}$$

$$\%p \& (((p \& \sim \#p) \& \% \#p) \& \% \#((p \& \sim \#p) \& \% \#p)) ; \tag{C'.1.2}$$

undistributed quantifier ;  
FFFF FFFF FFFF FFFF

$$(\%p \& ((p \& \sim \#p) \& \% \#p)) \& (\%p \& \% \#((p \& \sim \#p) \& \% \#p)) ; \tag{C'.1.3}$$

distributed quantifier ;  
FFFF FFFF FFFF FFFF

Eqs. C'.1.2 and C'.1.3 are *not* tautologous but contradictory. Therefore, that artifice solves nothing.