

Refutation of the Hilbert Grand Hotel paradox

© Copyright 2018 by Colin James III All rights reserved.

We assume the method and apparatus of Meth8/VL4 with \top as the designated *proof* value, \bot as contradiction, \mathbb{N} as truthity (non-contingency), and \mathbb{C} as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

From: en.wikipedia.org/wiki/Hilbert's_paradox_of_the_Grand_Hotel

LET p, q : rooms, guests;
 \sim Not; $\&$ And; $+$ Or; $-$ Not Or; $>$ Imply, greater than; $<$ Not Imply, less than;
 $@$ Not Equivalent; $\#$ necessity, for all; $\%$ possibility, for one or some;
 $(\%p\>\#p)$ 1, one.

"It is demonstrated that a fully occupied hotel with infinitely many rooms may still accommodate additional guests, even infinitely many of them, and this process may be repeated infinitely often." (1.1)

We take the expression "a fully occupied hotel with infinitely many rooms may still accommodate additional guests" as rooms are greater than guests.

We also take the expression "and this process may be repeated infinitely often" to mean the possibility that both the rooms outnumber the guests *and* the guests outnumber the rooms.

$$\begin{aligned}
 & ((\#(p>q)\&\sim((p-q)<(\%p\>\#p)))> \\
 & (((p-(\%p\>\#p))\&(q-(\%p\>\#p)))>((p+(\%p\>\#p))\&(q-(\%p\>\#p))))> \\
 & (((p-(\%p\>\#p))\&(q+(\%p\>\#p)))>((p+(\%p\>\#p))\&(q+(\%p\>\#p)))))) > \%((p>q)\&\sim(p>q)) ; \\
 & \qquad \qquad \qquad \text{CCCC CCCC CCCC CCCC}
 \end{aligned}
 \tag{1.2}$$

Eq. 1.2 as rendered is *not* contradictory but rather falsity. Hence this refutes the Hilbert Grand Hotel paradox.

Remark: We could not reduce this paradox to *one* variable because rooms and guests are distinctly counted.