Refutation of Craig's interpolation theorem

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table (fragments) is row-major and horizontal.

LET r, s, t, w, x, y: B, S, T, ϕ lc_phi, ψ lc_psi, θ lc_theta;
& And; > Imply; % possibility, for one or some.

From: Feferman, S. (2008). Harmonious Logic: Craig's Interpolation Theorem and its Descendants
math.stanford.edu/~feferman/papers/Harmonious%20Logic.pdf

A common statement of Craig's theorem (initially referred to by him as a lemma) goes as follows:

> Suppose $\vdash \phi(R, S) \rightarrow \psi(S, T)$. Then there is a $\theta(S)$ such that $\vdash \phi(R, S) \rightarrow \theta(S)$ and $\vdash \theta(S) \rightarrow \psi(S, T)$. (1.1)

Here $\vdash$ is validity in classical first order logic with equality (FOL), ϕ, ψ, θ are sentences, and R, S, and T are sequences of relation symbols for which the sequence S is non-empty.

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((w&(r&s))>(x&(s&t))) > ((%(y&s)>(w&(r&s)))&(%(y&s)>(x&(s&t)))) ;
            NNNN NNNN NNNN NNNN, NNNN NNNN NNNN TTTT,
            NNNN NNNN FFFF FFFF, NNNN NNNN FFFF TTTT                (1.2)
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Eq. 1.2 is *not* tautologous, refuting Craig's interpolation theorem.