The Cherry on *Tau*

Frederik Vantomme
Calle de Omega 32A, 4°C, 28032 Madrid, Spain
frederik.vantomme@gmail.com

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Abstract

I propose to rewrite the volume equation for the non-euclidian spherical Universe in terms of *tau* instead of π. Written this new way, a truly elegant equation and deeper structure becomes visible. Further, I postulate that the Universe *is* the Fundamental Theorem of Calculus, i.e. that the 3 dimensional Universe we live in is the derivative-surface of its 4 dimensional hypersphere volume.

> *"What really worries me is that the first thing we broadcast to the cosmos to demonstrate our "intelligence" is 3.14 .... I am a bit concerned about what the lifeforms who receive it will do after they stop laughing at creatures who must rarely question orthodoxy."*
>
> — Bob Palais, π *Is Wrong!*

$2\pi^2 r^3$

The above is the volume equation for the non-Euclidian spherical Universe, first proposed by Riemann, Einstein and Friedmann. Here is a link to Albert's original manuscript (Courtesy of the Einstein Archives Online ©)[1] [2] and the printed transcript from his book *Relativty: The Special and the General Theory, Chapter XXXI. The Possibility of a "Finite" and Yet "Unbounded" Universe[3]*:

"Perhaps the reader will wonder why we have placed our "beings" on a sphere rather than on another closed surface. But this choice has its justification in the fact that, of all closed surfaces, the sphere is unique in possessing the property that all points on it are equivalent. I admit that the ratio of the circumference C of a circle to its radius r depends on r, but for a given value of r it is the same for all points of the "world-sphere"; in other words, the "world-sphere" is a "surface of constant curvature".

To this two-dimensional sphere-universe there is a three-dimensional analogy, namely, the three-dimensional spherical space which was discovered by Riemann. Its points are likewise all equivalent. It possesses a finite volume, which is determined by its "radius" $2\pi^2 r^3$. Is it possible to imagine a spherical space? To imagine a space means nothing else than that we imagine an epitome of our "space" experience, i.e. of experience that we can have in the movement of "rigid" bodies. In this sense we can imagine a spherical space."

The volume equation for the 3d-Euclidian Universe is $\mathrm{v} = \frac{4}{3}\pi r^3$. Einstein's General Relativity, however, established non-Euclidian geometry as the actual geometry of space.

The volume equation for the non-euclidian spherical Universe, i.e. the 3d hypersurface volume of a 4 dimensional sphere (a so-called hypersphere or 3-sphere S^3), is $\mathrm{v} = 2\pi^2 r^3$. Mathematically, the value of the maximum volume of the entire Friedmann Universe is $2\pi^2 r^3(t)$ [4] [5].

In my previous article *On the geometry of Space*[6], I used the same equation $2\pi^2 r^3$ for the 3d hypersurface and postulated that "*[...] (2) Space might be the collapsed-compactified (fibration) of its higher 4d$_{L+R}$ S^3 hyper-sphere to its 3d transverse slice, this surface adopting the topology of a closed and flat left+right handed trefoil knot. [...]*", the blueprint of which is still visible in the Cosmic Microwave Background (CMB) radiation images.

However, $2\pi^2 r^3$ as equation for the volume of the Universe is not satisfactory. It makes not much sense.

$$(\tau r).(\tfrac{1}{2}\tau r^2)$$

But look, just by splitting up and re-arranging, a truly elegant equation rises to the surface:

$2\pi^2 r^3$

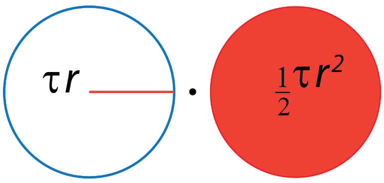
$2\pi.\pi.r^2.r$

$(2\pi r).(\pi r^2)$

Make yourself familiar with *Tau* (τ) and behold it frequently in spirit. Here is some literature:

- Joseph Lindenberg: *Tau Before It Was Cool* and many other links;
- Bob Palais: *π Is Wrong!*
- Michael Hartl: *The Tau Manifesto*, also with many other links.

Setting $\tau = 2\pi$ and thus $\pi = \frac{1}{2}\tau$ we find:



Imagine Einstein had used *tau* instead of π. You immediately see that the first part of the equation is the Circumference $C = \tau r$ of the circle and the second part is the Area formula $A = \frac{1}{2}\tau r^2$. The latter, as Lindenberg and Hartl recall, is a *quadratic form.*

Quadratic forms - which can be found in many equations in physics - arise whenever you integrate a function $f(x) = \mathtt{k}.x$.

On his website *Tau Before It Was Cool* Joseph Lindenberg sums up the following ‘Examples of the Same Pattern in Physics Formulas’:

$f(x) = (\mathbf{k}.x)$		Quadratic form: $\int (k.x)\, dx$	
1d Circumference C	$\tau.r$	2d Area A	$\frac{1}{2}\tau.r^2$
Momentum p	$\mathrm{m}.v$	Kinetic Energy (KE)	$\frac{1}{2}\mathrm{m}.v^2$
Angular Momentum L	$\mathrm{I}.\omega$	Rotational Energy	$\frac{1}{2}\mathrm{I}.\omega^2$
Applied Torque τ	$\mathrm{k}.\theta$	Potential Energy	$\frac{1}{2}\mathrm{k}.\theta^2$
Applied Force F in Hooke’s Law	$\mathrm{k}.x$	Elastic Potential Energy	$\frac{1}{2}\mathrm{k}.x^2$
Electric Flux Density D	$\varepsilon.E$	Electric Field Energy Density	$\frac{1}{2}\varepsilon.E^2$
Magnetic Flux Density B	$\mu.H$	Magnetic Field Energy Density	$\frac{1}{2}\mu.H^2$
Charge q	$\mathrm{C}.V$	Energy stored in the electric field of the Capacitor	$\frac{1}{2}\mathrm{C}.V^2$
Flux Φ	$\mathrm{L}.I$	Energy stored in the magnetic field of the Inductor	$\frac{1}{2}\mathrm{L}.I^2$

As you will notice, by remembering only the basic volume equation for the non-Euclidian spherical Universe, $(\tau r).(\frac{1}{2}\tau r^2)$, you get many equations for free.

But there is more to that! This very basic pattern $(kx).\int(kx)dx$ or $(\tau r).(\frac{1}{2}\tau r^2) = \frac{1}{2}\tau^2 r^3$ requires that C and A *are mixed together*, they are intertwined into a conserved quantity, there is a *dot* between them. And the mix between them results in a Conservation Law.

From Michael Weiss and John Baez: *Is Energy Conserved in General Relativity?*

> “In ordinary high-school analytic geometry, a (two-dimensional) vector v has components, say (v_1, v_2). While v is a geometrical object, existing free from the confines of any coordinate system, the same is not true for the components v_1 and v_2 – they depend on the choice of the axes and their scales. If you change the coordinate system – say you rotate the axes – then the components change according to a standard formula. We say that v has an *invariant meaning*, while the components are *coordinate dependent*.
>
> Now in relativity, neither energy nor momentum by themselves have invariant meaning, just like time and space.
>
> *But if we weld the energy and momentum together, we get a geometric object called a 4-vector that is invariant.* That is, if E is the energy and $p = (p_1, p_2, p_3)$ is the momentum, then (E, p_1, p_2, p_3) is the *energymomentum 4-vector*. The energymomentum 4-vector basks in celebrity, being the second most famous 4-vector; the top spot is held by the *time-space 4-vector*, (t, x, y, z). That’s why we say that the energy is the *time component* of the energymomentum 4-vector: it occupies the same slot that time does in the time-space 4-vector.
>
> If we want to conserve something, it better have an invariant meaning. The energymomentum 4-vector p fills the bill.”

Likewise, we can extend the table as follows (Annex 1 contains the table in full):

$(\mathbf{k}.\boldsymbol{x}).\int(\boldsymbol{k}.\boldsymbol{x})\,\boldsymbol{dx}$		$\int[(\mathbf{k}.\boldsymbol{x}).\int(\mathbf{k}.\boldsymbol{x})\,\boldsymbol{dx}]\,\boldsymbol{dx}$		
3d **hypersurface** of S^3 (a 4d sphere)	$\frac{1}{2}\tau^2.r^3$	4d **hypervolume** of S^3 (a 4d sphere)	$\frac{1}{8}\tau^2.r^4$	**Associated Conservation Laws**
		The relativistic **Energy-Momentum tensor** (or **Stress-Energy tensor**) $\boldsymbol{T^{\mu\nu}}$. The energy-momentum tensor is conserved when $\nabla_\mu T^{\mu\nu} = 0$.		The 4-divergence of the Stress–Energy tensor $T^{\mu\nu}$ gives **Conservation of Energy** and **Conservation of Linear Momentum.**
In Newtonian mechanics in 3d, the Angular Momentum is defined as $\mathbf{L} = I\boldsymbol{\omega}$, which can be reduced to $\mathbf{L} = \mathbf{r}\,.\,m\mathbf{v} = \mathbf{r}\,.\,\mathbf{p}$.		In relativistic mechanics in 4d, this is generalised to the Angular-Momentum tensor $\boldsymbol{M} = X \wedge P$ or, in tensor components, $\boldsymbol{M^{\mu\nu}} = X^\mu P^\nu - X^\nu P^\mu$ where X^μ is the four-position and P^ν the energy-momentum four-vector.		**Conservation of Angular Momentum.**
		In special relativity, the **torque** acting on a point-like particle is defined as the **derivative of the** Angular Momentum tensor given above with respect to proper time: $\Gamma = \frac{d\mathrm{M}}{d\tau} = \mathbf{X} \wedge \mathbf{F}$ or, in tensor components, $\boldsymbol{\Gamma^{\mu\nu}} = X^\mu F^\nu - X^\nu F^\mu$ where F is the 4-Force $F = \frac{dP^\mu}{d\tau}$.		
		The **Elastic Modulus tensor** (or **Elastic Stiffness tensor**) of fourth rank c_{ijkl} relates the *stress tensor* σ_{ij} and the *strain tensor* u_{ki} in the linear Hooke's law $\sigma_{ij} = c_{ijkl}\,u_{ki}$.		**Conservation of Energy.**
		▪ The **Electromagnetic Displacement tensor** $\boldsymbol{D^{\mu\nu}}$ cross-combines the $\boldsymbol{D}$ and $\boldsymbol{H}$ fields; ▪ The **Electromagnetic Field tensor** $\boldsymbol{F^{\mu\nu}}$ cross-combines the $\boldsymbol{B}$ and $\boldsymbol{E}$ fields; ▪ The **Magnetization-Polarization tensor** $\boldsymbol{M^{\mu\nu}}$ combines the $\boldsymbol{P}$ and $\boldsymbol{M}$ fields; The three field tensors are related by: $D^{\mu\nu} = \frac{1}{\mu_0}F^{\mu\nu} - M^{\mu\nu}$; and $\partial_\mu D^{\mu\nu} = J^\mu$.		**Conservation of Electromagnetic Energy and Momentum.**
		▪ The charge $\boldsymbol{q}$ becomes charge density $\boldsymbol{\rho}$; the current $\boldsymbol{I}$ becomes current density $\boldsymbol{j}$. The **current four-vector** $\boldsymbol{J^\mu} = (c\rho, j^1, j^2, j^3)$ cross-combines charge density and current density. ▪ The flux $\boldsymbol{\Phi}$ of the Magnetic Flux Density $\boldsymbol{B}$ becomes the magnetic vector potential $\boldsymbol{A}$ where $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. The **potential four-vector** $\boldsymbol{A^\mu} = (\frac{V}{c}, A^1, A^2, A^3)$ cross-combines electric scalar potential and magnetic vector potential.		The 4-divergence of the 4-current J^μ gives the **Conservation of Charge.** The 4-divergence of the 4-potential A^μ gives the **Conservation of EM 4-potential.**

The Universe *is* the Fundamental Theorem of Calculus

The following is from S. Gong, 1989 [7] *The problem of the maximum volumes and particle horizon in the Friedmann Universe Model, Astrophysics and Space Science, Springer, 1989, 158, 1-7*:

...

From the Robertson-Walker metric (Weinberg, 1972)

$$ds^2 = c^2dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\phi^2\right] \tag{1}$$

the volume element is

$$dv = \sqrt{g}\ \ dr\, d\theta\, d\phi = R^3(t)(1-kr^2)^{-1/2}\ \ r^2dr\sin\theta\, d\theta\, d\phi \tag{2}$$

where $R(t)$ is the cosmological scale factor, k is a parameter related to the space-time curvature with $k = 0, -1$ for an open universe and $k = +1$ for the closed universe.

After integrating (2), the volume becomes

$$v = 4\pi R^3(t)\int_0^r r^2(1-kr^2)^{-1/2}\ \ dr \tag{3}$$

$$v = 2\pi R^3(t)(\chi - sin\chi cos\chi) \tag{4}$$

where $\chi = sin^{-1}r$; thus (4) can be written as

$$v = 2\pi R^3(t)\ \ (\sin^{-1}r \pm r\sqrt{1-r^2}) \tag{5}$$

- sign for $sin^{-1}r \leqslant \frac{1}{2}\pi$; + sign for $sin^{-1}r > \frac{1}{2}\pi$

In (4) or (5), the maximum value within the bracket is π, thus the maximum volume V_{mv} is readily taken as $2\pi^2R^3(t)$ in current books (Weinberg 1972; Heidmann, 1980 [8]; and Zel'dovich and Novikov, 1983 [9]). The maximum volume is usually called the full volume of the closed universe.

...

However, an easier way to come to the same solution exists in taking the derivative of the volume of the 4d hypersphere $\frac{1}{8}\tau^2r^4$ (or $\frac{1}{4}\pi^2r^4$) with respect to r which gives you the surface area $\frac{1}{2}\tau^2r^3$ (or $2\pi^2r^3$).

As a matter of fact, for each and every sphere up to dimension n, the derivative of the volume of the sphere with respect to r equals its surface area.

What this means is that the 3d Universe we live in, is the derivative-surface of its 4d hypersphere volume and that implies that each and every point in space is, in the end, a compressed-compactified *infinitismal point* that has a *virtual* (with a real and *imaginary part*) pre-image. This pre-image is an auxiliary tool, embracing each infinitismal point, and contains more mathematical degrees of freedom than the number of physical degrees of freedom in our 3d hypersurface. This virtual but necessary pre-image gives an overcomplete description of physical reality that, at the same time, is conditioned to a real outcome (only virtual images that give real outcomes seem to be allowed).

The equations of General Relativity are defined in terms of - *truly virtual, higher dimensional* - tensors. In Quantum Mechanics, observables are described in terms of - *truly virtual, higher dimensional* - Hermitian operators which result in real-valued outcomes on the 3d hypersurface. General Relativity and Quantum Mechanics have 'real-based fantasy' in comon.

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Annex 1

$f(x) = (\mathbf{k}.x)$		Quadratic form: $\int (k.x)\, dx$		$(\mathbf{k}.x).\int (k.x)\, dx$		$\int [(\mathbf{k}.x).\int (\mathbf{k}.x)\, dx]\, dx$		
1d Circumference C	$\tau.r$	**2d Area A**	$\frac{1}{2}\tau.r^2$	**3d hypersurface** of S^3 (a 4d sphere)	$\frac{1}{2}\tau^2.r^3$	**4d hypervolume** of S^3 (a 4d sphere)	$\frac{1}{8}\tau^2.r^4$	**Associated Conservation Laws**
Momentum p	$m.v$	Kinetic Energy (KE)	$\frac{1}{2}m.v^2$			The relativistic **Energy-Momentum tensor** (or **Stress-Energy tensor**) $T^{\mu\nu}$. The energy-momentum tensor is conserved when $\nabla_\mu T^{\mu\nu} = 0$.		The 4-divergence of the Stress–Energy tensor $T^{\mu\nu}$ gives **Conservation of Energy** and **Conservation of Linear Momentum.**
Angular Momentum L	$I.\omega$	Rotational Energy	$\frac{1}{2}I.\omega^2$	In Newtonian mechanics in 3d, the Angular Momentum is defined as $\mathbf{L} = I\boldsymbol{\omega}$, which can be reduced to $\mathbf{L} = \mathbf{r}.m\mathbf{v} = \mathbf{r}.\mathbf{p}$.		In relativistic mechanics in 4d, this is generalised to the Angular-Momentum tensor $M = X \wedge P$ or, in tensor components, $M^{\mu\nu} = X^\mu P^\nu - X^\nu P^\mu$ where X^μ is the four-position and P^ν the energy-momentum four-vector.		**Conservation of Angular Momentum.**
Applied Torque τ	$k.\theta$	Potential Energy	$\frac{1}{2}k.\theta^2$			In special relativity, the **torque** acting on a point-like particle is defined as the **derivative of the** Angular Momentum tensor given above with respect to proper time: $\Gamma = \frac{d\mathbf{M}}{d\tau} = \mathbf{X} \wedge \mathbf{F}$ or, in tensor components, $\Gamma^{\mu\nu} = X^\mu F^\nu - X^\nu F^\mu$ where F is the 4-Force $F = \frac{dP^\mu}{d\tau}$.		
Applied Force F in Hooke's Law	$k.x$	Elastic Potential Energy	$\frac{1}{2}k.x^2$			The **Elastic Modulus tensor** (or **Elastic Stiffness tensor**) of fourth rank c_{ijkl} relates the *stress tensor* σ_{ij} and the *strain tensor* u_{ki} in the linear Hooke's law $\sigma_{ij} = c_{ijkl}\, u_{ki}$.		**Conservation of Energy.**
Electric Flux Density D	$\varepsilon.E$	Electric Field Energy Density	$\frac{1}{2}\varepsilon.E^2$			▪ The **Electromagnetic Displacement tensor $D^{\mu\nu}$** cross-combines the D and H fields; ▪ The **Electromagnetic Field tensor $F^{\mu\nu}$** cross-combines the B and E fields; ▪ The **Magnetization-Polarization tensor $M^{\mu\nu}$** combines the P and M fields; The three field tensors are related by: $D^{\mu\nu} = \frac{1}{\mu_0}F^{\mu\nu} - M^{\mu\nu}$; and $\partial_\mu D^{\mu\nu} = J^\mu$.		**Conservation of Electromagnetic Energy and Momentum.**
Magnetic Flux Density B	$\mu.H$	Magnetic Field Energy Density	$\frac{1}{2}\mu.H^2$					
Charge q	$C.V$	Energy stored in the electric field of the Capacitor	$\frac{1}{2}C.V^2$			▪ The **charge q** becomes charge density ρ; the **current I** becomes current density j. The **current four-vector** $J^\mu = (c\rho, j^1, j^2, j^3)$ cross-combines charge density and current density. ▪ The flux Φ of the Magnetic Flux Density B becomes the magnetic vector potential A where $B = \nabla \times A$. The **potential four-vector** $A^\mu = (\frac{V}{c}, A^1, A^2, A^3)$ cross-combines electric scalar potential and magnetic vector potential.		The 4-divergence of the 4-current J^μ gives the **Conservation of Charge.** The 4-divergence of the 4-potential A^μ gives the **Conservation of EM 4-potential.**
Flux Φ	$L.I$	Energy stored in the magnetic field of the Inductor	$\frac{1}{2}L.I^2$					

$I =$	The Moment of Inertia I is a tensor of the second rank whose terms are a property of the body and relate L to ω by $L_i = \sum_j I_{ij}\omega_j$.
$\omega =$	Angular velocity.
k =	Spring constant.
$\varepsilon =$	Permittivity of the medium (or material), or electric constant, a physical value characterizing the response of a medium (e.g. a solid) to the electric component E of an external electromagnetic field, where the permittivity ε is a scalar. If the medium is anisotropic, the permittivity is a second rank tensor.
$\varepsilon_0 =$	Permittivity of the free space or the vacuum.
$\varepsilon_r =$	The relative permittivity of the medium or 'dielectric constant'.
$\mu =$	Permeability of the medium (or material) which, in general, is a tensor quantity, a measure of how easily a magnetic field can pass through a medium.
$\mu_0 =$	Magnetic permeability of the free space or the vacuum.
$\mu_r =$	The relative magnetic permeability of the medium.
$\boldsymbol{E} =$	Electric Field Intensity/Strength; or strength of the Electric field.
$\boldsymbol{H} =$	Magnetic Field Intensity/Strength; or strength of the Magnetic field.
$\boldsymbol{D} =$	Electric Flux Density; also called Electric Displacement Field. $\boldsymbol{D} = \varepsilon\boldsymbol{E} = \varepsilon_0\varepsilon_r\boldsymbol{E}$.
$\boldsymbol{B} =$	Magnetic Flux Density (magnetic field vector) (magnetic induction). $\boldsymbol{B} = \mu\boldsymbol{H} = \mu_0\mu_r\boldsymbol{H}$.
$q =$	Electric charge.
$\rho =$	Electric charge density. The amount of electric charge per unit length, surface area, or volume.
$I =$	Electric current.
$j =$	Electric current density. The amount of electric current per unit area of cross section.
$C =$	Capacitance. A quantitative measure of the ability of an object or material to retain electric charge q. Charged particles of the same sign attempt to go out of a charged body due to electrostatic repulsion. In a system of conductors, a correspondence between their electric charges q and electric potentials V is determined by the linear relation $q_i = \sum_j C_{ij}V_j$ (C_{ij} is called the *capacitance matrix* of the i^{th} and j^{th} conductors).
$V =$	Electric potential difference, a scalar quantity.
$\Phi =$	Magnetic flux. The flux of the Magnetic Flux Density (magnetic field vector) $\boldsymbol{B}$ through a given surface area S.
$L =$	Self-Inductance, a physical quantity (in *henries*). In electrical circuits, any changing current I produces a magnetic field around the current-carrying wire (e.g. a loop or coil of wire). In other words, when the current in the wire-circuit itself is changing, this generates a magnetic flux Φ acting on the circuit. The magnetic field created by the changing current in the circuit itself induces a voltage (a back EMF) in the same circuit that counters or tends to reduce the rate of change in the current, and thus also opposes changes in the magnetic flux. The ratio of the magnetic flux Φ to the current I is called the 'Self-Inductance' $L = \frac{\Phi}{I}$, which is usually simply referred to as the inductance of the circuit.

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