A CLASSIFICATION OF GEOMETRIC INTERACTIONS

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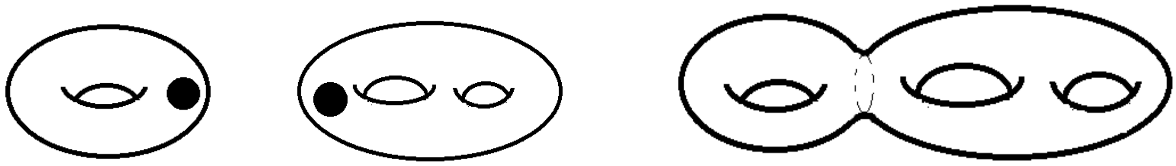
Abstract: In this work we discuss the possibility to classify geometric interactions with respect to the dimensions of the submanifolds which are decomposed and emitted from a differentiable manifold. The manifold, which is assumed to be an elementary particle, can be assumed to have the mathematical structure of a CW complex which is composed of *n*-cells. The decomposed *n*-cells will be identified with force carriers. In particular, for the case of differentiable manifolds of dimension three, there are four different types of geometric interactions associated with 0-cells, 1-cells, 2-cells and 3-cells. We discuss in more details the case of geometric interactions that are associated with the decomposition of 3-cells from a differentiable manifold and show that the physical interactions that are associated with the evolution of the geometric processes can be formulated in terms of general relativity.

In our previous works, we consider elementary particles as differentiable manifolds and physical interactions are identified with geometric processes [1,2,3]. We also assume that the evolution of the geometric processes that involve with the intrinsic geometric structure of a manifold can be described by the Ricci flow and Einstein field equations of the gravitational field. These assumptions can be verified by that fact that both the Ricci flow and Einstein field equations of general relativity can be derived from a purely geometrical formulation that uses the Bianchi identities as starting field equations. Together with the fact that both Dirac equation in quantum mechanics and Maxwell field equations of electromagnetism can be formulated purely from a general system of linear first order partial differential equations using only mathematical hypotheses in the forms of commutation relations, physics can be regarded as a physical mathematics when physical entities are identified with mathematical objects [4,5,6,7]. One of the interesting results that are obtained from this kind of purely mathematical formulation is the explanation of the charge of an elementary particle as a collective effect due to the force carrying particles that are emitted from a quantum system as 2-spheres and *n*-tori [2]. This duality between a physical interaction and a geometric process is only a particular case of a more complete classification of geometric interactions due to the decomposition of submanifolds from a three-dimensional differentiable manifold. In general, we may consider elementary particles as dimensional differentiable manifolds of dimension n which can emit submanifolds of dimension $m \leq n$ by decomposition. However, in order to formulate a physical theory we would need to devise a mathematical framework that allows us to account for the amount of subspaces that are emitted or absorbed by an elementary particle, which are assumed to be a differentiable manifold. This assumption leads to the visualisation of elementary particles as CW complexes which are constructed from *m*-

dimensional closed cells topologically glued together through the operation of connected sum as a modification of intrinsic geometric structures on manifolds. The effect of the operation results in a joint of two given manifolds. This construction plays a key role in the classification of closed surfaces. For example, two manifolds are glued to form a new manifold by deleting a ball inside each of the given manifolds and gluing them together along the resulting boundaries



More generally, manifolds can be glued together along identical submanifolds. Let M_1 and M_2 be two smooth oriented manifolds of equal dimension and V a smooth closed oriented manifold embedded as a submanifold into both M_1 and M_2. The connected sum of M_1 and M_2 along V is then the space $(M_1, V)\#(M_2, V)$ [8]. For the case of three-dimensional manifolds, the decomposition will produce three types of prime manifolds, which are the spherical types, $S^2 \times S^1$ and $K(\pi, 1)$. Only the prime manifold $K(\pi, 1)$ can be decomposed along embedded tori [9]. In order to describe the evolution of a geometric process as a physical interaction we assume that an assembly of cells of a specified dimension will give rise to a certain form of physical interactions and the intermediate particles, which are the force carriers of physical fields decomposed during a geometric evolution, may possess the geometric structures of the n-spheres and the n-tori. This speculation leads to a more profound speculation that physical properties assigned to an elementary particle, such as charge, are in fact manifestations due to the force carriers rather than physical quantities that are contained inside the elementary particle. If this is the case then the analysis of physical interactions will be reduced to the analysis of the geometric processes that are related to the geometric structures of the force carriers. Therefore, for observable physical phenomena, the study of physical dynamics reduces to the study of the geometric evolution of differentiable manifolds. In particular, if an elementary particle is considered to be a three-dimensional manifold then there are four different types of physical interactions that are resulted from the decomposition of 0-cells, 1-cells, 2-cells and 3-cells, and we will discuss this situation further in the following.

Forces associated with 0-cells: For a definite perception of a physical existence, we assume that space is occupied by mass points which interact with each other through the decomposition of 0-cells. However, since 0-cells have dimension zero therefore there is only contact forces between the mass points, which can be assumed to be constant $F = k_0$. When the mass points join together through the contact forces they form elementary particles. The 0-cells with contact forces can be arranged to form a particular topological structure [10].

Force associated with 1-cells: Depending on the topological structure of the cells it is possible to devise different forms of force associated with the cells. For the case of 1-cells, it is anticipated that they will manifest as either a linear force $F{\sim}r$ or a force of inverse law $F{\sim}1/r$ or a combination of the two

$$\mathbf{F}=k_1\mathbf{r} \tag{1}$$

$$\mathbf{F}=\frac{k_2\mathbf{r}}{r^2} \tag{2}$$

$$\mathbf{F}=\left(k_1+\frac{k_2}{r^2}\right)\mathbf{r} \tag{3}$$

Force associated with 2-cells: The decomposed 2-cells from an elementary particle can manifest either as a square force $F{\sim}r^2$ or a force of inverse square law $F{\sim}1/r^2$ or a combination of the two

$$\mathbf{F}=k_3r\mathbf{r} \tag{4}$$

$$\mathbf{F}=\frac{k_4\mathbf{r}}{r^3} \tag{5}$$

$$\mathbf{F}=\left(k_3r+\frac{k_4}{r^3}\right)\mathbf{r} \tag{6}$$

Force associated with 3-cells: For the decomposition of 3-cells from a manifold, even though it should be considered as a manifestation of either a cube force $F{\sim}r^3$ or a force of inverse cube law $F{\sim}1/r^3$ or a combination of the two, however, as will be discussed later, this form of geometric evolution can be applied to explain the cosmological evolution in general relativity. The cube force and the inverse cube law are given as

$$\mathbf{F}=k_5r^2\mathbf{r} \tag{7}$$

$$\mathbf{F}=\frac{k_6\mathbf{r}}{r^4} \tag{8}$$

$$\mathbf{F}=\left(k_5r^2+\frac{k_6}{r^4}\right)\mathbf{r} \tag{9}$$

From the above considerations, we can assume a general force which is a combination of those forces resulted from the decomposition of *n*-cells of all dimensions. For the case of dimension three, the general force takes the form

$$F=\sum_{n=-3}^{3}k_nr^n \tag{10}$$

where k_n are constants which can be determined from physical considerations.

Now we will discuss in more details the case of physical interactions that are associated with the decomposition of 3-cells from a differentiable manifold and show that the physical interactions that are associated with the evolution of the geometric processes can be formulated in terms of general relativity. Physically, we can visualise with a complete picture how 1-cells and 2-cells are formed and released from a 3-dimensional manifold, but for the case of forming and releasing a 3-cell from a 3-dimensional manifold M such complete visualisation seems to be beyond our physical ability, except for local observation. Mathematically, the forming and releasing of a 3-cell from a 3-dimensional manifold M can be expressed as a decomposition in the form $M = M\#S^3$. We assume that the physical interactions associated with the forming and releasing of 3-cells are geometric processes that smooth out irregularities of the intrinsic geometric structure of the manifold. The geometric irregularities can be viewed physically as an inhomogeneous distribution of matter in space and the forming and releasing of the S^3 cells as an expansion. A similar geometric process that smooths out an inhomogeneous distribution of a substance can be realised on the surface of a 2-dimensional sphere. In order to smooth out the irregularities, 1-cells in the form of circles can be formed and released from a position with dense substance and the geometric process is viewed as a local expansion. With this realisation, the geometric process of decomposition of 3-cells S^3 to smooth out irregularities of the distribution of matter in the observable universe can be formulated in terms of general relativity in which the change of intrinsic geometric structures of the manifold is due to the change of mathematical objects that define the manifold. These mathematical objects are perceived as physical entities like the energy-momentum tensor and the equations that describe the changes can be obtained from mathematical identities, such as Bianchi identities, or, more physically, Einstein field equations of general relativity

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \tag{11}$$

with the Robertson-Walker metric of pseudo-Euclidean kind [11]

$$ds^2 = c^2 dt^2 - S^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{12}$$

or with the Robertson-Walker metric of Euclidean kind [12]

$$ds^2 = c^2 dt^2 + S^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{13}$$

where $k = -1, 0, 1$.

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