A SIMPLE, DIRECT PROOF, USING SET-THEORY, OF FERMAT'S LAST THEOREM (FLT)

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Abstract. There is no confirmed, *simple* proof of FLT for each integral $n > 2$. Our simple proof of FLT is based on our algebraic identity, a function of two variables, denoted as $r^n + s^n = t^n$ for convenience. For $n \geq 1$ we relate (r, s, t) for which $r^n + s^n = t^n$ holds, with (x, y, z) for which $x^n + y^n = z^n$ holds. From these *true equations* we infer by *direct argument* (not by way of contradiction) that $\{(r, s, t)|r, s, t \in \mathbb{Z}, r^n + s^n = t^n\} = \{(x, y, z)|x, y, z \in \mathbb{Z}, x^n + y^n = z^n\}$ for any given n such that these sets are nonempty. Also, we show, for $n > 2$, that $\{(r, s, t)|r, s, t \in \mathbb{Z}\} = \varnothing$. Hence, for $n > 2 : \{(x, y, z)|x, y, z \in \mathbb{Z}\} = \varnothing$.

1. Introduction

FLT states, for integral $n > 2$, that $x^n + y^n = z^n$ does not hold for (x, y, z) with integers $x, y, z \geq 1$. No accepted *simple* proof of FLT exists for each $n > 2$.

We propose a *direct proof*, i.e., not by way of contradiction, for each $n > 2$.

2. Our Algebraic Equation : The Basis of our Direct Proof

Our argument begins, for integral $n \geq 1$, with a function of two variables, a *true statement* that is shown, below, to be sufficient for this proof, our equation (1) :

$$\left((4q^n)^{\frac{1}{n}}\right)^n + \left((p - 2q^n)^{\frac{1}{n}}\right)^n = \left((p + 2q^n)^{\frac{1}{n}}\right)^n. \tag{1}$$

For all integral $n \geq 1$: With *algebraic identity* (1) we restrict q to all positive rational values; unrestricted p has all positive real values such that $p > 2q^n$.

The values of rational q are *sufficient* for our argument, per Prop. 3.1, below.

Denote $4q^n$, $p - 2q^n$, and $p + 2q^n$, respectively, by $r^n, s^n, t^n \in \mathbb{Z}$, for convenience.

For $n \geq 1 : r^n + s^n = t^n$ is a *true statement* with (r, s, t), such that $r, s, t \in \mathbb{Z}$, for which $r^n + s^n = t^n$ *holds*. Clearly, $r^n + s^n = t^n$ is true for *non-trivial* $n = 1, 2$.

3. The Direct Argument Using Elementary Set-Theory

For $n \geq 1 : x^n + z^n = t^n$ is a *true statement* with (x, y, z), such that $x, y, z \in \mathbb{Z}$, for which $x^n + y^n = z^n$ *holds*. Clearly, $x^n + y^n = z^n$ is true for *non-trivial* $n = 1, 2$.

A false statement in our argument would be an equation of the form $\alpha^n + \beta^n = \gamma^n$ which, especially for $n = 2$, does not hold for (α, β, γ) such that $\alpha, \beta, \gamma \in \mathbb{Z}$.

We intend to infer, for any given $n \geq 1$ such that the following sets are nonempty, that $\{(r, s, t)|r, s, t \in \mathbb{Z}, r^n + s^n = t^n\} = \{(x, y, z)|x, y, z \in \mathbb{Z}, x^n + y^n = z^n\}$.

Should we confirm this equality it would show with $n = 3$, as the prime example for values of $n > 2$, that $\{(x, y, z)|x, y, z \in \mathbb{Z}, x^n + y^n = z^n\} = \varnothing$ - - - because, for $n > 2$, it is shown in Sect. 4, below, that $\{(r, s, t)|r, s, t \in \mathbb{Z}, r^n + s^n = t^n\} = \varnothing$.

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3.1. **For Integral $n \geq 1$, Formal Sets Essential To Our Proof. . . .**

Let $A \supset C$ be $\{(r,s,t)|r,s,t \in \mathbb{R}, r,s,t > 0, r^n + s^n = t^n\}$.

Let $B \subset A$ be $\{(r,s,t)|r,s \in \mathbb{R}, t \in \mathbb{Z}, rs, t$ are coprime, $r,s,t > 0, r^n + s^n = t^n\}$.

Let $C \subset B$ be $\{(r,s,t)|r,s,t \in \mathbb{Z}, r,s,t$ are coprime, $r,s,t \geq 1, r^n + s^n = t^n\}$.

Let $D \supset F$ be $\{(x,y,z)|x,y,z \in \mathbb{R}, x,y,z > 0, x^n + y^n = z^n\}$.

Let $E \subset D$ be $\{(x,y,z)|x,y \in \mathbb{R}, z \in \mathbb{Z}, xy, z$ are coprime, $x^n + y^n = z^n\}$.

Let $F \subset E$ be $\{(x,y,z)|x,y,z \in \mathbb{Z}, x,y,z$ are coprime, $x,y,z \geq 1, x^n + y^n = z^n\}$.

Let $G \supset J$ be $\{\frac{rs}{t}|\frac{rs}{t} \in \mathbb{R}, \frac{rs}{t} > 0, (r,s,t) \in A\}$.

Let $H \subset G$ be $\{\frac{rs}{t}|\frac{rs}{t} \in \mathbb{Q}, \frac{rs}{t} > 0, (r,s,t) \in A\}$.

Let $J \subset H$ be $\{\frac{rs}{t}|\frac{rs}{t} \in \mathbb{Q}, \frac{rs}{t} > 0, (r,s,t) \in B\}$.

Let $K \supset M$ be $\{\frac{xy}{z}|\frac{xy}{z} \in \mathbb{R}, \frac{xy}{z} > 0, (x,y,z) \in D\}$.

Let $L \subset K$ be $\{\frac{xy}{z}|\frac{xy}{z} \in \mathbb{Q}, \frac{xy}{z} > 0, (x,y,z) \in D\}$.

Let $M \subset L$ be $\{\frac{xy}{z}|\frac{xy}{z} \in \mathbb{Q}, \frac{xy}{z} > 0, (x,y,z) \in E\}$.

3.2. **Formal Propositions Essential To Our Argument.**

Proposition 3.1. *For any given n such that H, L are nonempty sets, $H = L$.*

Proof. Our argument involves not isolated r,s,t,x,y,z but, rather $\{(r,s,t)\}$ and $\{(x,y,z)\}$, exclusively for which $r^n + s^n = t^n$ and $x^n + y^n = z^n$ respectively hold.

Note that $G \supset J$; $K \supset M$. *For any given n*, due *solely* to varying unrestricted real p : *By inspection*, $\frac{(4q^n)^{\frac{1}{n}}(p-2q^n)^{\frac{1}{n}}}{(p+2q^n)^{\frac{1}{n}}} \in G$, or $\frac{rs}{t} \in G$, takes every possible real value, thus, takes every value of $\frac{xy}{z} \in K$ - - - A *big idea* since $A \neq D$, per Sec. 4.

Term $\frac{xy}{z} \in K$ takes every value of $\frac{rs}{t} \in G$ since $x^n + y^n = z^n$ is the most general such triple-nth-power formulation. So, for any given n : $\{\frac{rs}{t} \in G\} = \{\frac{xy}{z} \in K\}$.

Thus, for any given n with H, L nonempty, $\{\frac{rs}{t} \in H \subset G\} = \{\frac{xy}{z} \in L \subset K\}$. □

Values of rational q are sufficient for Prop. 3.1 to be true, as follows :

Non-rational values of q *do not apply* since values taken by p, q, with $q \in \mathbb{Q}$ being *independent of determining Prop. 3.1*, are *sufficient* for our proof of Prop. 3.1.

Proposition 3.2. *For any given n such that B, E are nonempty sets, $B = E$.*

Proof. If $\{\frac{rs}{t} \in H\} = \{\frac{xy}{z} \in L\}$, then, for any given n for which J, M are nonempty, $\{\frac{rs}{t} \in J \subset H\} = \{\frac{xy}{z} \in M \subset L\}$.

Hence, $\{rs|(r,s,t) \in B\} = \{xy|(x,y,z) \in E$; $\{t|(r,s,t) \in B\} = \{z|(x,y,z) \in E\}$.

Consequently, $\{rs,t|(r,s,t) \in B\} = \{xy,z|(x,y,z) \in E\}$ □

Proposition 3.3. *For any given value of n such that set B is nonempty, the elements of B are : $r = \left(\frac{w^n \pm \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $s = \left(\frac{w^n \mp \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, and $t = w$.*

Proof. For any given value of n with nonempty set J, notate taken-as-known values of $\frac{rs}{t} \in J$ by $\frac{v}{w}$ for which v, w are positive coprime values, such that $v \neq w$.

Thus, $\{\frac{rs}{t}\} = \{\frac{v}{w}\}$. Hence, $\{t|(r,s,t) \in B\} = \{w\}$, and $\{rs|(r,s,t) \in B\} = \{v\}$.

Solving $t = w$ and $rs = v$ simultaneously with $r^n + s^n = t^n$ results in $(r^n)^2 - (r^n)(w^n) + v^n = 0$ and $(s^n)^2 - (s^n)(w^n) + v^n = 0$.

The *solution* in J is $r = \left(\frac{w^n \pm \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $s = \left(\frac{w^n \mp \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $t = w$. □

Proposition 3.4. *For any given value of n such that set E is nonempty, the elements of E are : $x = \left(\frac{w^n \pm \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $y = \left(\frac{w^n \mp \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, and $z = w$.*

Proof. For any given value of n with nonempty set M, notate taken-as-known values of $\frac{xy}{z} \in M$ by $\frac{v}{w}$, with coprime v, w, as with proposition 3.3.

Thus, $\{\frac{xy}{z}\} = \{\frac{v}{w}\}$. So, $\{z|(x,y,z) \in E\} = \{w\}$, and $\{xy|(x,y,z) \in E\} = \{v\}$.

Solving $z = w$ and $xy = v$ simultaneously with $x^n + y^n = z^n$ results in equations $(x^n)^2 - (x^n)(w^n) + v^n = 0$ and $(y^n)^2 - (y^n)(w^n) + v^n = 0$.

The *solution* in M is $x = \left(\frac{w^n \pm \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $y = \left(\frac{w^n \mp \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $z = w$. □

Proposition 3.5. *For any given n for which C, F are each nonempty, $C = F$.*

Proof. Per propositions 3.3-3.4, for any given value of n such that B, E are nonempty, $\{r|(r,s,t) \in B\} = \{x|(x,y,z) \in E\}$; $\{s|(r,s,t) \in B\} = \{y|(x,y,z) \in E\}$; and $\{t|(r,s,t) \in B\} = \{z|(x,y,z) \in E\}$. Hence, for any given n with nonempty C, F :

$\{r|(r,s,t) \in C \subset B\} = \{x|(x,y,z) \in F \subset E\}$;

$\{s|(r,s,t) \in C \subset B\} = \{y|(x,y,z) \in F \subset E\}$.

We have shown, above, that $\{t|(r,s,t) \in B, t \in \mathbb{Z}\} = \{z|((x,y,z) \in E, z \in \mathbb{Z}\}$.

Therefore, $\{(r,s,t) \in C\} = \{(x,y,z) \in F\}$. □

4. Results and Conclusion

With the triple $((4q^n)^{\frac{1}{n}}, (p-2q^n)^{\frac{1}{n}}, (p+2q^n)^{\frac{1}{n}})$, term $(4q^n)^{\frac{1}{n}}$ reduces to $2^{\frac{2}{n}}q$.

It logically follows, for $n > 2$, with $q \in \mathbb{Q}$, that $\{2^{\frac{2}{n}}q \in \mathbb{Q}, r^n + s^n = t^n\} = \varnothing$.

Thus, for $n > 2$, it is true that its subset, $\{2^{\frac{2}{n}}q \in Z \subset \mathbb{Q}, r^n + s^n = t^n\} = \varnothing$.

[So, for $n > 2$, sets $A \neq D$. For example, with $n = 3$, equation $x^n + y^n = z^n$ holds for $x = 1$, $y = 2$, and $z = 9^{\frac{1}{3}}$; though, for $n = 3$, equation $r^n + s^n = t^n$ does not hold with $(4q^n)^{\frac{1}{n}} = 1$, $(p-2q^n)^{\frac{1}{n}} = 2$, and $(p+2q^n)^{\frac{1}{n}} = 9^{\frac{1}{3}}$. But, the fact that $A \neq D$ is irrelevent, since $C = F$ is possible either with $A \neq D$ or $A = D$.]

Thus, for $n > 2$, equation (1) does not hold for (r,s,t) such that $r, s, t \in C$.

Per proposition 3.5, for any given value of n such that sets C, F are nonempty, $(r,s,t) \in C = (x,y,z) \in F$.

Ergo, by using our simple, direct argument we conclude the following :

For $n > 2$: $x^n + y^n = z^n$ does not hold for (x,y,z) such that $x, y, z \in \mathbb{Z}$.

QED