

Refutation of the Löwenheim–Skolem theorem

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We assume the apparatus and method of Meth8/VL4, with the designated *proof* value of \top . The 16-valued proof table is row-major and horizontal.

LET $p, q, r, s: \kappa \text{ lc_kappa}, M, N, \sigma \text{ lc_sigma};$
 \sim Not; $\&$ And; $+$ Or; $>$ Imply, greater than; $<$ Not Imply, less than;
 $=$ Equivalent; $@$ Not Equivalent; $\#$ necessity, for every; $\%$ possibility, for one;
 $(p@p)$ 0, zero; $(s>(p@p)) \mid \sigma$; $(q>(p@p)) \mid M$; $(r>(p@p)) \mid N$; $\sim(p<q)$ $(p\geq q)$.

From: en.wikipedia.org/wiki/Löwenheim–Skolem_theorem

In its general form, the Löwenheim–Skolem theorem states that for every signature σ , every infinite σ -structure M , and every infinite cardinal number $\kappa \geq |\sigma|$, (1.1)

$$\#(s\&((s\&q)\&(\sim(p<(s>(p@p)))))) ; \quad \text{FFFF FFFF FFNF FFNF} \quad (1.2)$$

there is a σ -structure N (2.1)

$$\%(s\&r) ; \quad \text{CCCC CCCC CCCC TTTT} \quad (2.2)$$

such that $|N| = \kappa$ and

$$\begin{aligned} &\text{if } \kappa < |M| \text{ then } N \text{ is an elementary substructure of } M; \text{ [and/or]} \\ &\text{if } \kappa > |M| \text{ then } N \text{ is an elementary extension of } M. \end{aligned} \quad (3.1)$$

$$((r>(p@p))=p)\&(((p<(q>(p@p)))>(q<r)) \text{ [}\&\text{, +]} ((p>(q>(p@p)))>(q>r))) ; \quad \text{FTFT FTFT FTFT FTFT} \quad (3.2)$$

Eqs. 1.1 implies 2.1. (4.1)

$$\#(s\&((s\&q)\&(\sim(p<(s>(p@p))))))>\%(s\&r) ; \quad \text{TTTT TTTT TTCT TTTT} \quad (4.2)$$

Eqs. (1.1 implies 2.1) implies 3.2. (5.1)

$$\begin{aligned} &(\#(s\&((s\&q)\&(\sim(p<(s>(p@p))))))>\%(s\&r)) > \\ &(((r>(p@p))=p)\&(((p<(q>(p@p)))>(q<r))+((p>(q>(p@p)))>(q>r)))) ; \end{aligned} \quad \text{FTFT FTFT FTFT FTFT} \quad (5.2)$$

Eq. 1.2 as rendered is *not* tautologous, and not contradictory.

Eq. 4.1 is *not* tautologous due to one \subset falsity value.

Eq. 4.2 is *not* tautologous, and the same result table as Eq. 3.2.

This means the Löwenheim–Skolem theorem is refuted.