The Ababou Isomorphism Theorem

S.Fushida-Hardy

Abstract

We construct an isomorphism $\mathfrak{I}$ between the category $\mathbb{A}\mathrm{b}$ of Ababou Constants and $\mathfrak{ab}$ of affine bundles. We explore some special cases, namely the image of $(\mathbb{Z}, \mathrm{ab})$ under $\mathfrak{I}$, and prove that the distinguished Ababou constant is composite.

Preliminaries

ab denotes the *distinguished Ababou constant*, in other words the least upper bound on the integers. This is the primary object of interest in Mohamed Ababou's *Theory of Numbers have and end*, in which he claims that the natural numbers do indeed have an "end".

The category $\mathbb{A}\mathrm{b}$ contains Ababou structures as its objects, and Ababou maps as its arrows. By an *Ababou structure*, we mean an ordered ring equipped with an upper bound, which we call the *Yeet* of the ring. Well known examples include $(\mathbb{Z}, \mathrm{ab})$ with the standard ordering [3].

An *Ababou map* is a ring isomorphism $\varphi : R_1 \to R_2$ with the property

$$\varphi(c_1) = c_2$$

where c_1, c_2 are the Yeets for R_1, R_2 respectively.

The category $\mathfrak{ab}$ contains affine bundles as its objects and affine bundle homomorphisms as its arrows. An affine bundle is a fibre bundle

$$\pi : E \to B$$

whose standard fibre is an affine space. Given affine bundles $\pi_1 : E \to B$ and $\pi_2 : F \to C$, an affine bundle homomorphism is a pair (φ, ψ) of continuous maps preserving the affine structure such that the following diagram commutes:

$$\begin{array}{ccc} E & \xrightarrow{\varphi} & F \\ \Big\downarrow{\scriptstyle \pi_1} & & \Big\downarrow{\scriptstyle \pi_2} \\ B & \xrightarrow{\psi} & C \end{array}$$

In other words, it is a pair of maps preserving the fibres. By "preserving the affine structure", we mean that given a point $p \in B$, φ_p is an affine space homomorphism (that is, a semilinear map).

For example, we may take $E = \mathbb{R}^2$ and $F = \mathbb{R}^3$ viewed as trivial vector bundles over $\mathbb{R}$. Let φ be the inclusion map $(x, p) \mapsto (x, 0, p)$, and ψ the identity map. Then it is clear that $\psi \circ \pi_1 = \pi_2 \circ \varphi$, so the corresponding diagram commutes. Each map is continuous, and the inclusion is linear. Thus (φ, ψ) is an affine bundle homomorphism. (It is in fact a vector bundle homomorphism, which is a stronger condition.)

Construction of the $\mathbb{A}\mathrm{b} \to \mathfrak{ab}$ Functor $\mathfrak{I}$

We construct $\mathfrak{I} : \mathbb{A}\mathrm{b} \to \mathfrak{ab}$ as follows:

1. The empty Ababou structure is mapped to the empty bundle. Let $\mathscr{A}$ be the trivial Ababou structure. Then $\mathfrak{I}(\mathscr{A})$ is the trivial bundle - the bundle with standard fibre $\{0\}$, with base space $\{0\}$. We extend this to define $\mathfrak{I}$ on finite fields $\mathbb{Z}_p$ (with an ordering inherited from $\mathbb{Z}$, with yeet p): $(\mathbb{Z}_p, p)$ is mapped to the trivial bundle $\pi : \mathbb{R}^n \times \{0\} \to \mathbb{R}^n$, where n corresponds to the natural number such that p is the nth prime. For example,

$$\mathfrak{I}(\mathbb{Z}_{57}, 57) = \pi : \mathbb{R}^{17} \times \{0\} \to \mathbb{R}^{17}.$$

   For yeet $> p$, $r = \text{yeet} - p$ is the rank of the fibre. Finally, Ababou structures that are not rings are mapped to affine bundles that are not vector bundles in the natural way.

2. We now define the action of $\mathfrak{I}$ on Ababou maps. Let $\hat{a} : R_1 \to R_2$ be an Ababou map such that $\hat{a}(c_1) = c_2$, where c_1, c_2 are the yeets of R_1, R_2 respectively. Let ρ be the natural group representation mapping into $\mathbb{R}$. Then $\rho(c_1)$, $\rho(c_2)$ are the dimensions of the base space of the domain and codomain of $\mathfrak{I}(\hat{a})$ respectively. At each point in the base space, $\mathfrak{I}(\hat{a})$ canonically inherits semilinearity, so $\mathfrak{I}(\hat{a})$ is an affine bundle homorphism as required.

3. We now check that $\mathfrak{I}(\mathrm{id}_R) = \mathrm{id}_{\mathfrak{I}(R)}$. By definition, id_R maps between two ababou structures with equal yeet. Thus $\mathfrak{I}(\mathrm{id}_R)$ is a map between bundles with the same base space. Moreover, $\mathfrak{I}(\mathrm{id}_R)$ is clearly the identity on the fibres of bundle. Thus identity morphisms are preserved.

4. Finally we check that composition of morphisms is preserved. This is an easy consequence of the adjoint functor theorem.

The existence of an inverse functor to $\mathfrak{I}$ is a well known corollary of the Ababou limit theorem. Its construction is left as an exercise to the reader, but an outline of the construction is given in, e.g., [5]. Therefore, we have an isomorphism between the category of Ababou constants $\mathbb{A}\mathrm{b}$ and the category of affine bundles $\mathfrak{ab}$. □

The Ababundle, $\mathfrak{I}(\mathbb{Z}, \mathrm{ab})$

The fundamental object which sparked the “Theory of Numbers have an End" is the intergers along with the distinguished Ababou constant, written $(\mathbb{Z}, \mathrm{ab})$ in modern terminology. We now have access to the functor $\mathfrak{I}$ to study the structure of $(\mathbb{Z}, \mathrm{ab})$ in the realm of affine bundles, a well developed theory. A similar functor was used to better understand FÜD in [2]. From our construction of $\mathfrak{I}$, we observe that $\mathfrak{I}(\mathbb{Z}, \mathrm{ab})$ is an affine bundle with base space isomorphic to $\mathbb{R}^{\mathrm{ab}}$. However, it is not a trivial bundle (in that the total space of $\mathfrak{I}(\mathbb{Z}, \mathrm{ab})$ cannot be expressed as a product $\mathbb{R}^{\mathrm{ab}} \times F$ where F is the standard fibre of the bundle). We observe this because otherwise $\mathfrak{I}(\mathbb{Z}, \mathrm{ab})$ could be turned into a trivial vector bundle (by choosing an origin). This would in turn mean that $(\mathbb{Z}, \mathrm{ab})$ is a field, which is absurd. Thus $\mathfrak{I}(\mathbb{Z}, \mathrm{ab})$ is a nontrivial affine bundle, so ab must be composite. This agrees with earlier conjectures by R.Attema, such as “ab is likely composite", his reasoning being the vanishing density of primes for large n [4].

Futher Research

It is conjectured that there are deep links between ab and the discrete nature of physics at the quantum scale [1]. As the exact value of ab is unknown, the precise structure of the Ababundle is also unknown. However, many believe the 6 dimensional Calabi-Yau manifold is a submanifold of the base space of the Ababundle. If this is the case, the theoretical “quantum limit of observation" can be calculated by considering the dimensions of the Ababundle.

It is likely that a careful adaptation of the Ababundle to spacetime and quantum field theory will remove the need for renormalisation and regularisation. In current formulations, many unphysical results are obtained, such as self-interactions resulting in infinite quantities. However, the finite rank of the Ababundle will also provide an upper bound to the energies caused by self-interactions.

In the realm of mathematics, one of the greatest achievements of the decade was the proof by S.Shen that all series converge [6]. Now with access to the theory of affine bundles the exact nature of the convergence may be determined.

W.Kim, who originally introduced the the category of Ababou constants $\mathbb{A}$b, has famously been working in isolation for a number of years, most likely in an attempt to calculate the homology modules of the long exact Ababou sequence. This result is likely to aid in his research. His original 2014 paper is cited here for completion [7].

References

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