Refutation of constructive Brouwer fixed point theorem

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We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: + Or; & And; > Imply, greater than; = Equivalent.

We construct the Brouwer fixed point theorem (BFPT) as implications of four variables:

the antecedent is the relationship of their rank orders; (21.1)
((p>q)>r)>s ; TFTT FFFF TTTT TTTT (21.2)

the consequent is disjunction of relational pairs of variables (away tautologous); and (22.1)
(((p>q)+(p>r))+(p>s))+(((q>r)+(q>s))+(r>s)) ;
TTTT TTTT TTTT TTTT (22.2)

the implication is always tautologous. (23.1)
(((p>q)>r)>s) > ((((p>q)+(p>r))+(p>s))+(((q>r)+(q>s))+(r>s))) ;
TTTT TTTT TTTT TTTT (23.2)

Eq. 23.2 as rendered is tautologous, and on its face appears as a constructive proof of BFPT.

However there is problem as to completeness because the antecedent is composed of the totality of ordered combinations.

Consequently, the connective is equivalence. (24.1)
(((p>q)>r)>s) = ((((p>q)+(p>r))+(p>s))+(((q>r)+(q>s))+(r>s))) ;
TFTT FFFF TTTT TTTT (24.2)

Eq. 24.2 is *not* tautologous, and refutes BFPT using a constructive proof.

Remark: If the consequent is taken as a multiplicity of ordered combinations, the equivalence connective and the implication connective share the same table result which deviates further from Eq. 24.2 with another F contradictory value. (25.1)
(((p>q)>r)>s) [= or >] ((((p>q)&(p>r))&(p>s))&(((q>r)&(q>s))&(r>s))) ;
TTFF TTTT TFFF TFTT (25.2)

We conclude that BFPT is mislabeled as a theorem, as non constructively based on set theory, and correctly named as the Brouwer fixed point conjecture (BFPC).